New Hampshire

PreK-16

Numeracy Action Plan For the 21st Century

New Hampshire PreK-16 Numeracy Action Plan for the 21st Century

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I. SUMMARY (ABSTRACT)

In the spring of 2008, quantitative literacy was established as a priority for New Hampshire students by the New Hampshire State Board of Education. In response, Lyonel B. Tracy, Commissioner of Education, called for the establishment of a New Hampshire Quantitative Literacy Task Force to create a statewide plan for numeracy. The intent was to investigate and document strategies that would cross grade levels and content areas so that every teacher, regardless of content area, would become a teacher of numeracy in his or her particular discipline.

The New Hampshire Department of Education selected the New Hampshire Impact Center, under the direction of Dr. Brian Beaudrie, associate professor of mathematics at Plymouth State University, and Emily Ricard, associate director of the New Hampshire Impact Center, to spearhead the effort with ongoing assistance and support from the New Hampshire Department of Education and the New England Comprehensive Center.

Stakeholders from the Department of Education, local school districts, parent and community groups, federally funded research centers, classroom teachers, and institutions of higher education were invited to join the Quantitative Literacy Task Force. The Task Force was comprised of two teams: the Writing Team (charged with writing the statewide plan for numeracy) and the Advisory Team (providing input and expertise throughout the writing process). The charge of the Task Force was to create a quality numeracy instruction available to every student in New Hampshire, from preschool through postsecondary education. The charge of the Task Force was to create a comprehensive statewide quantitative literacy plan that would mirror the *New Hampshire PreK–16 Literacy Action Plan for the 21st Century* (2007) by the summer of 2009 and to present it to the New Hampshire State Board of Education for approval and endorsement. The plan would then be introduced to educational leaders, teachers, and other interested parties throughout the state via dissemination workshops, published documents, and CDs. Recommendations and guidance cited in the Numeracy Action Plan would then be implemented in schools and districts throughout New Hampshire with the support of professional organizations, institutions of higher education, New Hampshire Department of Education, and the members of the Numeracy Task Force.

The Task Force examined the current body of research surrounding quantitative literacy development and instruction as well as student and teacher learning. Quantitative literacy, considered a specific type of literacy, involves many similar instructional components and goals as literacy instruction. As a result, a conceptual framework which visually represents the layers of support necessary for improved student learning and performance directly mirrors the framework included in the *New Hampshire PreK–16 Literacy Action Plan for the 21st Century*. The research on numeracy instruction and the practical applications of what is effective, and what is not, provided the foundation for the New Hampshire Numeracy Action Plan, which was designed to assist school and district leadership, as well as classroom teachers, in implementing a comprehensive numeracy program. The major foundational components include:

- Shared beliefs about learning
- Essential understandings about 21st century numeracy
- Clearly articulated standards and goals for mathematics instruction
- Underlying principles of the quantitative process and the development of the quantitatively literate student
- Essential components of effective numeracy instruction
- A culture of collaboration among the state, school districts, families, community organizations, students, and institutions of higher education

While this plan does not attempt to do all things for all people, it does attempt to define what is necessary to create and sustain a comprehensive numeracy program within school systems, and then to identify resources that will support that effort. It is hoped that this action plan will be used by school administrators to lead district efforts to adapt and strengthen their quantitative literacy programs to best meet the quantitative literacy challenges

of 21st century schooling. It is also hoped that teachers of mathematics and teachers of all disciplines will see this document as a resource to guide them as they work to ensure their students have the skills required to be quantitatively literate.

The New Hampshire Numeracy Action Plan is divided into seven major sections. At the end of **Section V**, there is a reference list and guiding questions that can be used for reflection or group study. The **Appendix** contains a collection of tools, lists, surveys, and reference documents that will be helpful to administrators, teachers, and parents.

II. INTRODUCTION

The New Hampshire Department of Education has defined school standards (ED 306) and established curriculum frameworks (RSA 193-c) to guide school districts as they create their own local curricula. These frameworks include Grade-Level Expectations (GLEs) and Grade-Span Expectations (GSEs) that establish what students should know and be able to do.

As it works to support and assist districts to improve student achievement and prepare them for life after schooling, the traditional advisory role of the Department of Education is changing. With the advent of school and district accountability at the local, state, and federal levels, the Department of Education must (by statute) establish a statewide system of support. New Hampshire's *Follow The Child* initiative adds a dimension of accountability that is defined by the success of each child. With all of this in mind, as well as a sincere desire to provide schools and districts guidance in the area of literacy, the *New Hampshire PreK–16 Literacy Action Plan for the 21st Century* was published in 2007. Following its dissemination, the Department of Education realized a need to create a companion plan, the Numeracy Action Plan (February 2010).

Building on the best, most current research in all areas of numeracy, learning, and leadership, the Numeracy Action Plan has been developed for use by school and district leadership, superintendents, principals, teachers and instructional coaches of all disciplines, curriculum directors and coordinators, teacher leaders, and parents. It attempts to define what is necessary to create and sustain a comprehensive numeracy program and to then identify the required resources to support that effort.

A. The Rationale and Vision

Despite substantial gains in early quantitative achievement, the performance of students in New Hampshire mimics the national statistics that achievement declines throughout the grades. A review of both the National Assessment of Educational Progress (NAEP) and New England Common Assessment Program (NECAP) scores confirms this fact. Research also shows that there is a correlation between students who struggle in mathematics and those who drop out of high school and college (Beaudrie et al., 2007). Furthermore, according to the National Center for Education Statistics Web site, there continues to be an achievement gap in quantitative ability between students in regular education classes and those with educational disabilities (http://nces.ed.gov).

The body of current research related to quantitative literacy is still in its infancy, focused primarily on the post-secondary level. However, the reasons supporting the importance of being numerate are so compelling that it is critical for primary and secondary schools to prioritize revamping their curricula to include numeracy. In order to support New Hampshire schools and districts, a stakeholder task force representing the Department of Education, districts and schools, community and parent groups, and institutions of higher education was formed to study the research on quantitative literacy and best practices in numeracy instruction. The New Hampshire Quantitative Literacy Task Force developed the action plan as a resource to provide guidance for parents, school districts, teachers, institutions of higher education, and pre-service education programs in the following areas:

- Current descriptions of quantitative literacy and numeracy
- The importance of being quantitatively literate
- Numeracy development and numeracy education
- Effective data-driven instruction (how to recognize it, foster it, and help it flourish)
- Professional development for teachers and school leaders in creating a learning community where a culture of numeracy can exist across grade levels and disciplines
- Methods for families and community organizations to support numeracy development and achievement

B. Audience and Focus

Although numeracy involves much more than problem-solving and data interpretation, these two components are a large part of this document. The Quantitative Literacy Task Force felt strongly about three things: the focus must be on problem-solving and data interpretation because these are the foundation for using numeracy in any and all areas of life; the Numeracy Action Plan should be directed at school and district leaders because real change cannot occur without the strong direction and focus of leadership; and the plan should be written as a resource for teachers to use as they work to help their students develop numeracy skills. Kolata (1997) supports this focus:

> Every high school graduate should be able to read and understand charts and graphs, but that is only the beginning—like saying they should know how to write a sentence. They should also understand logical arguments and logical fallacies. They should understand the nature of evidence. They should understand such things as the difference between absolute and relative risk. Quantitative literacy, in my view, means knowing how to reason and how to think, and it is all but absent from our curricula today (p. 28).

For decades, many mathematics classes have approached math as a set of rules and procedures to be memorized and used out-of-context. In order to move toward more effective numeracy instruction, a paradigm shift is necessary. To change day-to-day teaching practices will take focused leadership, collaborative problem-solving, and collective accountability. It will take a culture that understands that students engaged in real-world, contextual problem-solving will achieve at higher levels and be better prepared for college and beyond. Strategic school and district-wide examination of curricular alignment and instructional practice, led by a well-formed quantitative literacy team, is needed in order for numeracy instruction to be effectively implemented.

The Numeracy Action Plan outlines the importance of quantitative literacy throughout formal schooling and beyond. It also provides suggestions, guidance, resources, and references which can be used by schools, districts, and teachers as they work toward helping students become quantitatively literate.

C. Section II References

Beaudrie, B. et al. (2007). Making the transition from high school to college and the workforce Retrieved March 30, 2009, from www.plymouth.edu/graduate/nhimpact/documents/MaTHSC_research_report.pdf

Kolata, G. (1997). Understanding the News. In L. A. Steen (Ed.) Why numbers count: Quantitative literacy for tomorrow's America. New York: College Entrance Examination Board.

III. FOUNDATION

A. What is Numeracy/Quantitative Literacy/Mathematical Literacy?

Even though many political, economic, and educational entities agree that having a quantitatively literate population is crucial to society's well-being, "quantitative literacy" and "numeracy" are still relatively new topics. Finding a readily accepted, consistent definition of either term is virtually impossible. The urgency to improve quantitative skills together with the lack of a clear-cut definition confounds the path to increasing numerical ability. Determining the best course of action is difficult when the desired result is not easily defined. Furthermore, the ability to reason quantitatively hinges on the ability to think abstractly, which is in and of itself complicated to grasp and define.

For all practical purposes, the terms "quantitative literacy," "mathematical literacy," "quantitative reasoning," and "numeracy" can be considered synonymous[1]. However, they do seem to have subtle differences based on context and regionalism. Quantitative reasoning tends to refer to specific skills and coursework in a professional or academic setting. Quantitative literacy, mathematical literacy, and numeracy describe a set of quantitative skills required in everyday settings, not solely professional or academic ones. The term numeracy is the preferred term in England and several of its commonwealth countries (e.g., Australia), while quantitative literacy appears to be the term used most often in the United States.

Even though a concise, consistent, complete definition for quantitative literacy may not exist, a few common components or ideas are apparent when analyzing an aggregation of definitions from various sources (see **Appendix A**). These include:

- **Quantitative literacy involves real-life situations**. Citizens must be quantitatively literate in order to understand the mathematics encountered in everyday life (e.g., unit conversions, sizes and measurements, polling and other statistical data, probabilities of everything from disasters to winning the lottery).
- **Quantitative literacy involves problem-solving**. Simply memorizing a formula, inputting values, and obtaining a solution is not considered problem-solving. To be considered quantitatively literate, an individual must be able to analyze a situation involving quantities for which a solution *may not be readily apparent*, devise a strategy to solve it, carry out that strategy, and then reflect on the solution to determine if it is reasonable from a quantitative perspective.
- **Quantitative literacy involves a synthesis of several skills**. As the definitions show, there are several components to quantitative literacy and each, in its own way, contributes to the whole of being quantitatively literate. This seems to imply that quantitative literacy is not a separate skill in and of itself. Rather, it is infused in mathematics, and indeed in most areas of curricula and everyday life. Therefore, it cannot be taught as a separate skill that can be mastered by the end of an instructional unit. The teacher of mathematics (and, to some extent, all teachers) must realize these facts and work constantly to develop quantitatively literate students. This perspective parallels current trends seen in literacy instruction where all educators, regardless of their specialized discipline, must also be teachers of reading and writing. Thus, all educators must also be teachers of quantitative literacy. For example, a science or literature teacher may highlight patterns that occur in nature or in poetic verse, respectively.
- **Quantitative literacy involves responsible citizenship.** As U.S. citizens are expected to be able to read and write proficiently, so should they be expected to be quantitatively literate. The drawbacks to being quantitatively illiterate (innumerate) parallel those of being illiterate: if one is unable to sufficiently understand and

[1] This document will use these four terms interchangeably and synonymously.

analyze everyday quantitative data such as budget deficits or interest rates, he or she is incapable of participating and contributing in society.

However, while an illiterate person would be viewed by society as being uneducated, being innumerate is far more culturally acceptable. As professor of psychology, Louise Hainline (2001) commented:

> [W]e've more than once heard highly educated, competent people. . . .confess without evident embarrassment or discomfort that they really never could do mathematics. This is a disclaimer you'll hear from lots of people when the use of quantitative concepts is raised in polite company, people who often spent a lot of time and money to be well-educated. At times, it almost seems like a statement of honor or comradeship that distinguishes the normal people from the "dweebs". . . .So I have been interested in understanding why people who would never admit in public that they just can't master English grammar or can't marshal a coherent argument feel only slightly uncomfortable admitting that they just don't "get" applications of mathematical concepts.

The National Mathematics Advisory Panel (2008) discusses how people are so quick to dismiss poor math achievement: "Much of the public's self-evident resignation about mathematics education (together with the common tendencies to dismiss weak achievement and to give up early) seems rooted in the erroneous idea that success in mathematics is largely a matter of inherent talent, not effort" (p. xx). People believe that one is either inherently good at math or one is not. They do not feel that effort and attitude can compensate for a lack of natural mathematical ability. Therefore, people do not exhibit a sense of failure when math does not come easily.

Appendices A, B, and C include aggregated descriptions of quantitative literacy.

B. Historical Perspective

The idea of quantitative literacy as a separate, distinct concept, and as a topic that ordinary citizens can be adept at, began to evolve in the latter part of the twentieth century. Although very little research related to quantitative literacy can be found prior to 1950, the shift from a qualitative worldview to a quantitative one can be traced through Medieval Europe. In *Quantification and Western Society 1250–1600*, Crosby (1997) elaborates on some of the more pivotal developments toward quantification. Although "quantification" and "quantitative literacy" are not the same thing, the idea that everyday phenomena could be quantified made it necessary for data to be collected, represented, and interpreted, all requiring quantitative literacy.

During Medieval times, the opportunity to advance quantitatively and mathematically was stymied by an insistence to search numbers for meaning or patterns to phenomena rather than use them for calculations. Numbers were not "neutral" or merely quantitative; they had moral or emotional value. For example, the number three (3) represented the Trinity. Furthermore, Medieval Europeans were not concerned with accuracy, as tools and units had not been developed to precisely measure certain physical characteristics.

Around 1250, Europeans began to move from a qualitative to quantitative way of describing and comprehending their world. This movement toward quantifying information was largely responsible for Europe's political, social, and economic superiority in the subsequent centuries.

From the 13th to the 15th centuries, European societies made considerable quantitative advances. Musical notes and measures were introduced as multiples or divisions of a length of music. The introduction of grammatical punctuation and printed spaces made reading quick, personal, and informative. Time was quantified with the development of the mechanical clock (mid-fourteenth century); artists, following the methods written about by Ptolemy, embraced perspective painting (around 1400); marine charts were mapped in degrees (1494 in Spain and Portugal); and businessmen, drowning in complicated fractions, were saved by decimals and double-entry book-keeping (fifteenth century).

The shift toward quantification led to new terms and quantitative units. Previously, everything was described heterogeneously and qualitatively. Now, homogeneous units were developed. Bruegel's painting, *Temperance* (1560), showed people using uniform units of measurement. In the late sixteenth century, the French (following Réne Descartes) began using numbers to represent knowns and unknowns, which led to the beginning of seeing numbers as symbols of quantities, irrespective of qualities—similar to modern-day algebraic notation.

With the advent and eventual adoption of quantification, the development of the field of mathematics accelerated. Mathematics became a tool and often a requirement of ordinary citizens' vocations and lives, and numerical data became "the dominant form of acceptable evidence" (Steen, 2001, p. 3). Although the term had not yet been coined, the concept of numeracy was used by Thomas Jefferson and Benjamin Franklin to justify and support new political ideas such as democracy (Cohen, 1982).

As early as the first half of the twentieth century, quantitative literacy was evident in New Hampshire through the work of L. P. Benezet[2]. Benezet, former superintendent of the Manchester School District in the 1920s and 1930s, was disturbed by the inability of students in his district to properly use the English language. He concluded that the students needed to improve their literacy skills in order to not sound "like a group of half-wits." In order to facilitate his ideas, Benezet (1935) proposed postponing formal instruction in arithmetic for the first six grades:

> "In the fall of 1929, I made up my mind to try the experiment of abandoning all formal instruction in arithmetic below the seventh grade and concentrating on teaching the children to read, to reason and to recite. . . .my new Three R's."

Although Benezet spoke of postponing mathematics instruction, the students in these experimental classrooms were still doing mathematics, it was just not the drill of arithmetic operations that characterized a traditional elementary mathematics curricula of that day and age. Instead, teachers in these classes were told to have the students get "much practise [sic] in estimating heights, lengths, areas, distances, and the like." The students in these classes were measuring, estimating, making numerical connections to the real world and, above all, developing number sense. In essence, they were becoming quantitatively literate. Benezet himself might have used that term to describe what he was doing, but the term "quantitative literacy" would not be coined for another quarter-century.

The results of his experiment were astounding. Not only did the children in the experimental classes become quantitatively literate: "The Manchester children, who had not learned tables, but had talked a great deal about distances and dimensions recognized the fact that 2,500 feet was about a half a mile, while the children in the larger city who were fresh from their tables had little conception of the distance," but they also did not languish behind their peers, despite their lack of traditional instruction. Their performance on standardized tests, which focused on standard arithmetic procedures and not on quantitative skills, was comparable to that of their peers who had a more traditional mathematics curriculum.

> ". . . .by the middle of April, however, all the classes were practically on a par, and when the last test was given in June, it was one of the experimental groups that led the city. In

[2]Benezet's work was originally published in *The Journal of the National Education Association*: Vol. 24, No. 8, Nov.1935, pp. 241-244; Vol. 24, No. 9, Dec.1935, pp. 301-303; Vol. 25, No.1, Jan.1936, pp. 7-8.

> other words these children, by avoiding the early drill on combinations, tables, and that sort of thing, had been able, in one year, to attain the level of accomplishment which the traditionally taught children had reached after three and one half years of arithmetic drill."

Benezet also reported some other side effects of becoming literate, in the quantitative sense: "It was refreshing to go into one of these rooms. A happy and joyous spirit pervaded them. The children were no longer under the restraint of learning multiplication tables or struggling with long division. They were thoroughly enjoying their hours at school."

What the New Hampshire superintendent had inadvertently discovered—a discovery that would sit dormant for practically half a century until Paulos' groundbreaking book *Innumeracy: Mathematical Illiteracy and Its Consequences* (1988)—was that quantitative literacy matters. Becoming quantitatively literate not only helps students with typical "school" mathematics, but it allows those who are quantitatively literate to no longer dread and struggle with mathematical situations; instead, they understand and relish them.

Regardless of Benezet's and others' efforts, the terms "numeracy" and "quantitative literacy" did not surface until the mid-twentieth century as "numbers [became] the chief instruments through which we attempt to exercise control over nature, over risk, and over life itself" (Steen, 1999, p. 3). The term "*numeracy*" was first used in the Crowther Report (1959) on the education of children ages 15–18 in the United Kingdom. This report described numeracy as 'the mirror image of literacy' (par. 398) intended to include 'not only the ability to reason quantitatively but also some understanding of scientific method' (par. 419 (a)). In this context, **both literacy and numeracy signify abilities to communicate at a substantial level, especially about issues that arise in daily life.**

In *Mathematics Counts* (Cockcroft, 1982), two characteristics of a numerate individual are identified: the ability to use mathematics in everyday life and the ability to understand and appreciate information presented in mathematical terms. This report jump-started the mathematical phase of numeracy, and over the past two decades, this context has dominated numeracy in Western Europe and the U.S.

The discrepancy between quantitative skills required of citizens and actual quantitative ability was well apparent by the mid to late twentieth century. Terms such as "math anxiety" and "innumeracy" have highlighted this epidemic (Paulos, 1988; Tobias, 1978). This gap was further amplified by an increase in the use of data and charts by media outlets to report on current events. Additionally, research has shown that simply having math anxiety can actually lead to poor math performance (National Mathematics Advisory Panel, 2008; Tobias, 1978). The National Mathematics Advisory Panel (2008) reports "Anxiety about mathematics performance is related to low mathematics grades, failure to enroll in advanced mathematics courses, and poor scores on standardized tests of mathematics achievement. It also may be related to failure to graduate from high school" (p. 31).

Quantitative literacy was brought into mainstream consciousness with Paulos's *Innumeracy: Mathematical Illiteracy and Its Consequences* (1988). In his best-selling book, Paulos documents individual and societal costs associated with the inability to understand quantitative information.

Several educational and government institutions responded to this innumeracy and math phobia. Universities began to research and compile curricula and texts that included mathematics for all "normally" prepared first-year college students (University of Kansas, 1954; University of Virginia, 1958). The Mathematical Association of America (1979) convened a Minimal Competency Panel to survey college graduation requirements. In 1980, the National Council of Teachers of Mathematics (NCTM) published *An Agenda for Action* that recommended that problem solving become the focus of K-12 mathematics. In 1982, Stephen White and the Sloan Foundation launched the decade-long New Liberal Arts Program to bring attention to the concern that quantitative reasoning skills were absent, but equally important, in humanities and social sciences. *A Nation at Risk* (1983) called for higher standards and application of math in everyday situations. A National Assessment of Educational Progress (NAEP) survey assessing literacy introduced three types of literacy: prose, document, and quantitative (NAEP, 1985). In 1989, NCTM published the initial Standards for School Mathematics, calling for K-12 students to become confident problem-solvers who value mathematics. The 2000 NCTM Standards echoed this and provided additional support to ensure that all citizens have the quantitative understanding necessary to thrive. In addition, many government committees discussed concerns about the international competitiveness of the American workforce due to its lack of quantitative skills and preparation relative to its counterparts around the world.

The NCTM's first published standards in 1989 incited the beginning of the "Math Wars," an ongoing debate between those who believe math curricula should be skills and formula based, and those who favor an inquiry and problem-solving approach. Over the last two decades, these highly politicized debates have greatly altered math curricula, standards, and pedagogy. In 2006, President Bush created the National Mathematics Advisory Panel to end the "Math Wars." The Panel was unsuccessful, concluding that an exclusive use of either approach was not recommended. Additionally, the increase in school and teacher-based accountability has led districts to focus on standardized testing.

The abundance of literature and research concerning quantitative reasoning in the 1990s is a testament to how ubiquitous the topic became. Many articles focused on the growing divide between the skills required to be occupationally competent and competitive and the skills most Americans truly possessed (Steenken, 2003; Steen, 2002). Other articles focused on the differences between the quantitative abilities of American students and workers as opposed to their international counterparts (Carnevale & Desrochers, 2003; Rosen, Weil & Von Zastrow, 2003). Several mainstream, popular books highlighted the need for strong quantitative literacy skills in order to understand an author's argument (e.g., Gladwell, 1996; Schell, 1999). As everyday citizens have become increasingly involved in their political and economic worlds, the need to make sense of these worlds by analyzing numerical and statistical data has become more significant. Finally, modern advertising frequently attempts to capitalize on consumers' weak quantitative skills by touting a product's characteristics using deceptive, albeit quantitative, marketing data.

As driven by data as U.S. society may have been in the late 1980s (circa Paulos), it has become even more so today. This suggests that the need for schools to produce students that are quantitatively literate has increased as well. However, instruction in schools designed to keep pace with this has been mixed at best. One possible reason for the discrepancy is the recent focus on testing and accountability which puts significant pressure on schools to ensure that students perform well on standardized tests. Therefore, teachers focus on material that will be tested, which is not necessarily the material that will help to develop numeracy. Quantitative literacy is largely absent from assessment and accountability (Steen, 2004). Teaching numeracy is almost certainly being sacrificed for simply acquiring skills with the hopes that students will perform well on tests.

While requirements stipulated through P.L. 107-110 (*No Child Left Behind Act of 2001)* have imposed constraints at state levels regarding accountability and assessments within specific grade levels for reading and mathematics, New Hampshire has attempted to develop statewide assessments in mathematics for grades 3 through 8 and 11 that promote quantitative literacy. This is done through the New England Common Assessment Program (NECAP), which involves a four-state collaboration among Vermont, Maine, Rhode Island, and New Hampshire, to develop assessment items based on a set of common mathematics standards.

Because NECAP items are coded to grade level and grade span expectations as stipulated in the *K-12 Mathematics New Hampshire Curriculum Framework,* these items are aligned to both content and process standards. By the very nature of the process standards, students are expected to problem-solve and synthesize mathematical concepts and skills. These are two common components that help define quantitative literacy. "Since it is crucial that process standards, such as problem-solving, reasoning, proof, communication, connections, and representations, are not seen separate from content standards, the process standards have been imbedded throughout the content strands" (*K-12 Mathematics New Hampshire Curriculum Framework,* 2006, p. 8). An example of this is presented in the first big idea of the functions and algebra strand that states students will "identify and extend to specific cases a variety of patterns represented in models, tables, sequences, graphs or problem situations or writes a rule in words or symbols for finding specific cases . . . and writes an expression or equation using words or symbols to express the generalization of a linear relationship" (M:F&A:6:1). While students are learning the content associated with linear and nonlinear patterns, "instruction is also focusing on improving their abilities in problem solving, reasoning, and communication; furthermore, students are looking for and making appropriate connections as they understand and use multiple representations of mathematical ideas" (*K-12 Mathematics New Hampshire Curriculum Framework,* 2008, p. 8). Since this standard is also used to develop items on statewide assessments for NECAP, it is probable that quantitative literacy, in its early stages, is being developed when educators prepare students for NECAP testing.

Another aspect of NECAP that demonstrates a connection to quantitative literacy is the coding of assessment items to Norman L. Webb's *Depth of Knowledge* classification system:

> A fundamental criterion used to develop the NECAP grade level and grade span expectations is that the expectations should explicitly indicate cognitive demand (how content interacts with process) and that there should be a mix of cognitive demand levels at all grades. One should not assume that students at lower grades do less cognitively demanding work. The cognitive demand or depth of knowledge required by an expectation or an assessment item is related to the number and strength of connections of concepts and procedures that a student needs to make to produce a response, including the level of reasoning required along with self-monitoring. (Retrieved November 9, 2009 from www.ed.state.nh.us/education/doe/organization/curriculum/NECAP/documents/MathDOKlevels.6pages.pdf)

Norman Webb's system is based on four levels of classification. The levels can be summarized as follows:

Level 1	Recall
Level 2	Skill/Concept
Level 3	Strategic Thinking
Level 4	Extended Thinking

Cognitive demands at levels 3 and 4 of this system promote quantitative literacy. These levels require reasoning, planning, drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; using concepts to solve problems; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs. These are all elements that promote the development of quantitative literacy.

Overall, the *K-12 Mathematics New Hampshire Curriculum Framework* guides both classroom instruction and statewide assessment. As presented in the rationale for the framework,

> The definition of basic skills in mathematics must change to include mathematical problem solving, reasoning, the ability to communicate, and the use of appropriate technology in addition to being able to compute. Individuals need to be able to apply their understanding of mathematics to solve real-world problems for which there are not simple formulas and standard procedures. Individuals need to be able to use their knowledge of mathematics to make sense of complex situations and then communicate that understanding to others. In-

> dividuals need to be able to solve tomorrow's problems, as well as today's. (*K-12 Mathematics New Hampshire Curriculum Framework,* 2006, p.5)

It is this very philosophy that guides test development for NECAP and classroom instruction in New Hampshire schools and districts; thus promoting a connection to common components of quantitative literacy.

Irrespective of test development and classroom instruction philosophy, educators are still mandated to prepare their students for testing. The short-term gains of this practice are uncertain at best, as there is no definitive proof that drilling students on discrete, low-level topics helps their test performance. However, **the long term effects of a society of quantitatively illiterate graduates cannot be understated**. As retired General Electric engineer William Steenken succinctly stated, "Businesses would be 'ecstatic' if graduates were quantitatively literate" (Steen, 2002, p. 8).

C. The Importance of Numeracy in Today's World

During the last few decades, the importance of developing quantitative literacy has gained momentum. The need for a quantitatively literate society has accelerated as the world has become more global and increasingly dependent on statistical data. Academic and professional success, along with global competitiveness, is now highly dependent on one's ability to analyze numerical data and reason quantitatively. In addition, many day-to-day tasks require numeracy (e.g., understanding the implications of a tax code change, weighing the costs and benefits of a certain health care plan, or listening to the latest marketing ploy). According to Davidson and McKinney (retrieved May 16, 2009 www.ac.wwu.edu/~dialogue/issue8.html):

> In our information-rich—some might say information-overloaded—society, "[quantitative reasoning] skills are especially important. We may no longer need to perform quantitative calculations by hand, but we do need to interpret them and judge their accuracy and reasonableness. Few people are trained to work with complex mathematical concepts, but all educated citizens should be able to understand mathematics well enough to develop informed opinions about quantitative concepts.

Being quantitatively illiterate is costly, both from an individual and a societal standpoint. Conversely, exhibiting a comfortableness with numbers and data has many benefits, the most important of which are opening educational and employment opportunities; creating a competitive economy and competent, capable workforce; and participating, protecting, and understanding today's culture, society, and environment.

The lack of quantitative reasoning ability is of great concern. When an individual possesses strong quantitative reasoning skills, academic, professional, social, and cultural doors are opened. Historically, it was assumed that one only needed a solid numeracy foundation for an education or profession in a math or science field; however, it is now readily apparent that these skills are critical to living a fuller, richer life. Many hobbies and sports (e.g., sewing, golf, photography, and billiards) have a quantitative component. In addition, the media's increasing reliance on data and statistics makes it imperative that people can accurately interpret that information.

Educational and Employment Opportunities

> I advise my students to listen carefully the moment they decide to take no more mathematics courses. They might be able to hear the sound of closing doors (Caballero, 1989, p. 2).

Achieving high levels of mathematical knowledge and developing strong quantitative skills leads to an increase in educational and economic opportunities, which leads to higher incomes, more security, and additional stability. For the same level of post-secondary education, the wage premium is highly correlated to strong quantitative skills (Carnevale & Desrochers, 2003). Unlike Europe, the United States does not guarantee wages and benefits; therefore, if one lacks adequate numeracy skills, he or she will bear the cost of being underemployed as a result.

From the 1950s to today—roughly the same period over which the numeracy movement has gained prominence—many jobs have shifted from unskilled to skilled. Jobs now routinely require employees to think critically and to problem-solve. **Technological advances have made some jobs more complex and have eliminated other jobs entirely. Furthermore, the fastest growing, highest-paid professions undoubtedly require quantitative reasoning**.

Since the 1990s, the advent of emerging technologies like the World Wide Web has increased the shift from unskilled to skilled. Jobs that were once considered unskilled or semi-skilled now require employees to possess quantitative abilities, while jobs that do not require quantitative skills are being routed overseas, a term known as *outsourcing*. As Jaithirth Rao, founder of the Indian company MphasiS, stated in Friedman's *The World is Flat* (2005):

> We are in the middle of a big technological change. . . .it's easy to predict [the future] for someone living in India. In ten years we are going to be doing a lot of the stuff that is being done in America today. . . .We must deal with it and talk about it honestly. . . .Any activity where we can digitize and decompose the value chain, and move the work around, will get moved around. Some people will say 'Yes, but you can't serve me a steak.' True, but I can take the reservation for your table sitting anywhere in the world, if the restaurant does not have an operator. We can say 'Yes Mr. Friedman, we can give you a table by the window.' In other words, there are parts of the whole dining-out experience that we can decompose and outsource. If you go back and read the basic economics textbooks, they will tell you: Goods are traded, but are consumed and produced in the same place. And you cannot export a haircut. But we are coming close to exporting a haircut, the appointment part. What kind of haircut do you want? Which barber do you want? All those things can and will be done by a call center far away (pp. 15–16).

Using and interpreting numbers and data is no longer only required of those in Science, Technology, Engineer, Mathematics (STEM) fields. All disciplines incorporate quantitative reasoning in some manner, and having quantitative shortcomings will likely negatively affect one's academic and occupational opportunities and achievements. Also, regardless of the discipline, being quantitatively literate promotes intellectual understanding: "Numeracy and the ability to communicate are essential to intellectual development, and lack of those skills hampers efforts in all directions" (University of Guelph, 2005).

Furthermore, compelling research shows that students are more likely to graduate from college if they took four years of rigorous math courses while in high school. These students are being exposed to more math, more problem-solving, and more quantitative reasoning, which have a positive impact on their collegiate success, no matter their intended course of study (Beaudrie et al., 2007).

The Economy, Global Competitiveness, and the Workforce

Rosen, Weil, and Von Zastrow (2003) draw attention to the disparity between what businesses know they need and what they will actually place on their agendas. The business sector cares deeply about having a well-educated, quantitatively literate workforce. However, they are not yet championing the importance of teaching numeracy. "...[E]xisting practices in mathematics education do not adequately address this competency as it is required in the workplace. ...The challenge, then, is how to bring quantitative literacy into the business agenda for education reform" (p. 43).

As businesses have found that many job applicants lack the skills for even entry-level jobs, they have responded in two ways: by investing heavily in training, in effect paying for our schools' failures to educate after the fact and lobbying for the H-1B Visa Bill to import more skilled workers from foreign countries. Corporations are being proactive by influencing policy and instituting company initiatives and benchmarks. However, because the business and education communities define quantitative literacy differently, they are not on the same page regarding how to fix the shortcomings.

Despite having relatively low mathematical scores internationally (based on TIMMS), the U.S. has been extremely successful globally on an economic level. Two reasons for this success, the size of the U.S. population and the American culture, are completely unrelated to how math is taught or how numerically literate the populace is. The U.S. simply has more top students and more competitive companies which tend to breed competition and even more success. Also, the American culture is more adaptable and open; it tends to make the best use of its talent or imports talent where there is a need. This advantage will eventually fade as other countries become more flexible and no longer adhere to strict cultural doctrines (Carnevale & Desrochers, 2003). Within the last 15 years India and China have gone from having a socialist style marketplace structure to one that is more capitalistic in nature (Friedman, 2005). **The U.S. competitive advantage will erode as laborers from more quantitatively literate countries begin to compete for the same jobs, contracts, and funding**.

Participating, Protecting, and Understanding Society and Culture

Quantitative skills are used daily, often to solve a problem unrelated to an individual's professional or academic pursuits. For example, in order to center a picture exactly on a wall, problem solving must be used. This seemingly simple household task can be further complicated if the picture is to be hung from two fixed brackets on the back of the frame, rather than from a single wire. A trip to the grocery store is full of quantitative tasks, from estimating the best value to comparing nutrition labels. "Pure mathematics is abstract and context-free, yet 'unlike mathematics, numeracy does not so much lead upward in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life's diverse contexts and situations' (Orrill, 2001, p. xviii)" (Ginsburg, Manly, & Schmitt, 2006). Spring (2000) explains the importance of numeracy on participating and contributing to today's society, as well as its connection to literacy: "Literacy and numeracy should be tools that help people navigate through the complexities of modern life while freeing their creative powers to imagine better ways of structuring social, political, and economic organizations" (p. 109).

Having a strong conceptual understanding of large and small numbers (e.g., the size of a stimulus bill or the margin of error in a very close election) helps to make sense of the world. These essential skills enable people to make informed decisions regarding risks, regards, costs, benefits, and choices in their daily lives. This number sense aids in planning, handling, and monitoring resources in terms of time, money, and supplies. Solid quantitative skills are essential to understanding numbers embedded in text, especially in persuasive text. These skills help the individual to understand the problem and make an educated, informed decision about what action needs to be taken. Furthermore, numeracy skills are crucial if one wants to confidently question data.

While it may be difficult to concretely define quantitative or mathematical literacy and numeracy, **illiteracy and innumeracy have a definite negative impact on the safety and well-being of nations throughout the world**. "The eminence, safety, and well-being of nations have been entwined for centuries with the ability of their people to deal with sophisticated quantitative ideas" (National Mathematics Panel Report, p. XI). For much of the twentieth century, the United States was a world leader in "mathematical prowess." America will lose that leadership in the twenty-first century, however, if innumeracy or "an inability to deal comfortably with the fundamental notions of number and chance" continue to prevail (Paulos, 2001, p. 3). It is without question that quantitative literacy impacts the safety, quality of life, and prosperity of a nation.

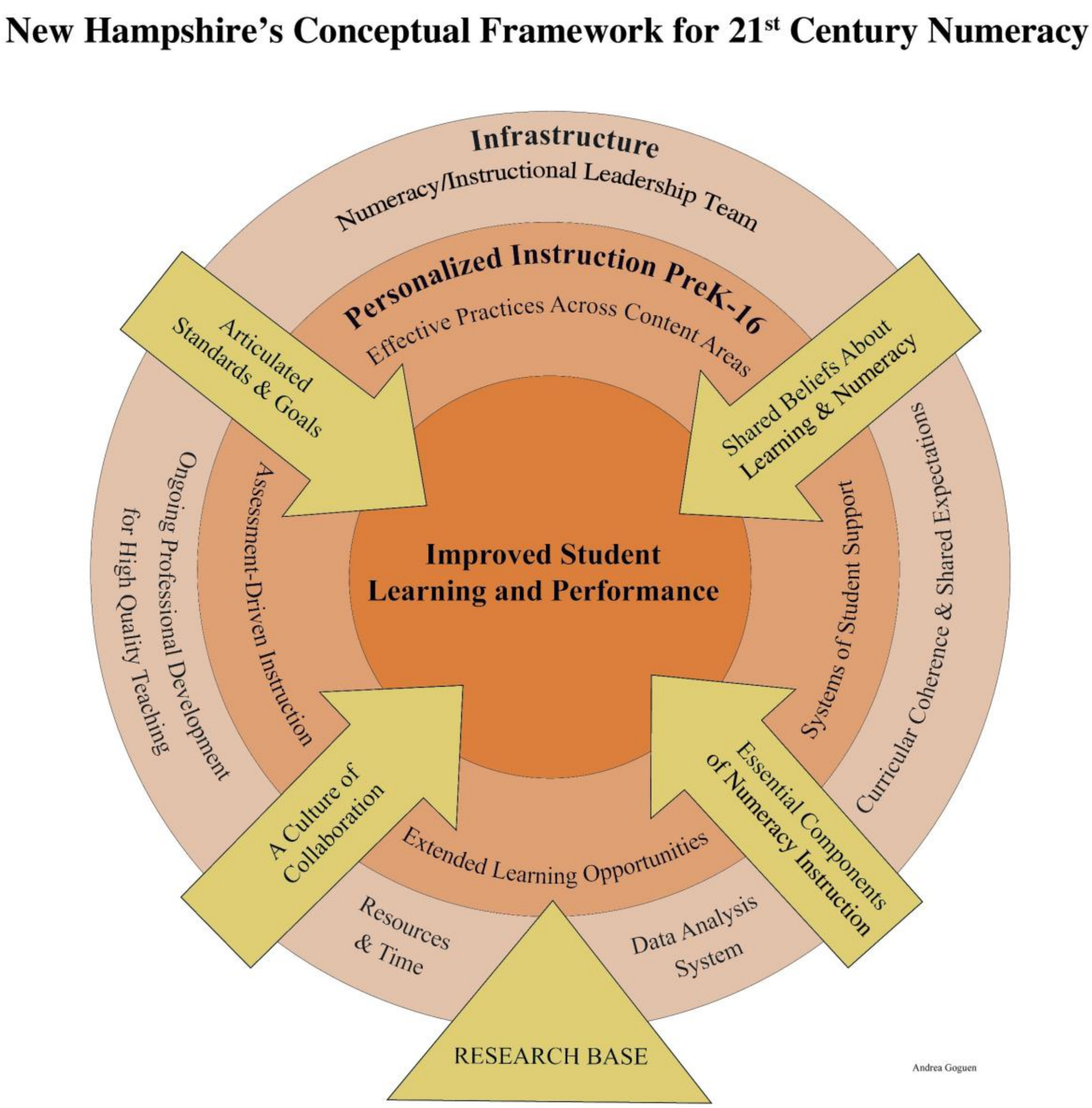
Steen (2004) summarized the importance for quantitative skills:

> "Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world" (p. 2).

D. The Conceptual Framework for the Numeracy Action Plan

Figure 1 represents New Hampshire's Numeracy Action Plan for the 21st Century. With the student at the plan's core, this conceptual framework reflects New Hampshire's *Follow The Child* philosophy in that it focuses on improved student learning and performance. The concentric circles represent the different layers of professional practice, resources, and infrastructure that are necessary to implement and maintain a comprehensive PreK–16 numeracy program. The arrows represent those components that cut across the layers and permeate the entire system. These components need to be established in order to sustain improved student learning and performance. Research provides the foundation for the framework.

Figure 1: New Hampshire's Conceptual Framework for 21st Century Numeracy



E. Students' Rights and Responsibilities

The process of becoming quantitatively literate does not happen in a vacuum. The responsibilities of achieving quantitative literacy do not fall solely on the shoulders of the teacher or the student. Both participants must be committed to creating a culture of mutual respect and effort for instructional gains to be achieved. Students must realize that teachers cannot simply transfer knowledge onto students. Students must be active, willing participants who exhibit a strong work ethic in order to become quantitatively literate.

The list below is not exclusive to mathematics and quantitative literacy and can be transferred to any discipline.

Students' Rights and Responsibilities in a Quantitatively Literate Classroom

Rights	Responsibilities
1. I have the right to be challenged academically and to encounter thought-provoking quantitative problems.	1. I have the responsibility to put forth my best effort and to approach problem-solving with an open mind.
2. I have the right to ask questions whenever I have them and I have the right to receive responses that facilitate learning.	2. I have a responsibility to listen and attempt to understand the answer.
3. I have the right to be a part of a community of learners and to be treated with respect by my teacher(s) and peers.	3. I have the responsibility to contribute to the community and to be respectful of its members.
4. I have the right to be a part of a classroom with different levels of ability and different types of learners.	4. I have the responsibility to participate in different types of mathematics activities.
5. I have the right to appropriate and sufficient classroom resources that help promote quantitative literacy.	5. I have the responsibility to take proper care of my classroom resources.
6. I have the right to a classroom climate that is conducive to developing numeracy skills.	6. I have the responsibility to positively contribute to the classroom climate.
7. I have the right to be encouraged and allowed to take risks while developing my numeracy and problem-solving skills.	7. I have the responsibility to take those risks.
8. I have the right not to understand mathematics and quantitative reasoning, and I have the right to say that I do not understand.	8. I have the responsibility to attempt to do my work and to try to understand the concepts before saying that I do not understand.
9. I have the right to learn at my own pace and to not feel stupid if I am slower than someone else.	9. I have the responsibility to realize that the teacher of the class must move at the pace of the class and not necessarily at my individual pace.
10. I have the right to be given help through additional tutoring or supplemental resources.	10. I have the responsibility should I be doing poorly in math class, to seek out additional help and resources.
11. I have the right for my work to be assessed in an honest and timely manner.	11. I have the responsibility to produce neat, concise, and thorough work on time.
12. I have the right to evaluate my math instructor(s) and how they teach.	12. I have the responsibility to evaluate my teacher(s) fairly and honestly, without vindictiveness.
13. I have the right to dislike mathematics.	13. I have the responsibility to keep an open mind and not to allow my dislike of math to influence my behavior or disrupt the learning of others.
14. I have the right to insist that *all* of my teachers, not just math teachers, strive to help me become quantitatively literate.	14. I have the responsibility to understand how numeracy is infused in school topics and necessary in my daily life.
15. I have the right to become a quantitatively literate citizen.	15. I have the responsibility to become a quantitatively literate citizen.

F. Habits of Mind

In 2003, the Pew Charitable Trusts and the Association of American Universities collaborated on Understanding University Success, a project led by Dr. David Conley, director of the Center for Educational Policy Research at the University of Oregon. The project identified and discussed various Standards for Success (termed S4S in the document) believed to be necessary for students to succeed in college (Conley, 2003).

The authors discussed Standards for Success in six content areas: English, mathematics, natural sciences, social sciences, second languages, and the arts. In mathematics alone, the authors identified 84 Standards for Success. In addition to these standards, the authors also defined sub-skills, or "Habits of Mind." Conley (2003) states:

> The faculty and staff members who participated in the process of developing these standards represent a wide range of academic viewpoints. One of the most dominant themes raised by participants is the importance of the habits of mind students develop in high school and bring with them to university studies. **These habits are considered by many faculty members to be more important than specific content knowledge** (emphasis added). The habits of mind include critical thinking, analytic thinking and problem solving; an inquisitive nature and interest in taking advantage of what a research university has to offer; the willingness to accept critical feedback and to adjust based on such feedback; openness to possible failures from time to time; and the ability and desire to cope with frustrating and ambiguous learning tasks (p. 8).

From 2005 to 2007, the Making the Transition from High School to College project at Plymouth State University adapted these ideas to construct the Mathematical "Habits of Mind." Beaudrie et al. (2007) state, in addition to citing the above, that the Mathematical "Habits of Mind" stress that "the emphasis of learning should be on the ability to use one's knowledge to solve a problem, present one's point of view, or obtain a reasonable conclusion based upon data given" (p. 2). This statement defines what it means to be a quantitatively literate individual.

The appendices contain two similar versions of the Mathematical "Habits of Mind": **Appendix E** is written for upper elementary and middle school students and **Appendix F** is more appropriate for high school and college-level students as it delves more deeply into the necessary Mathematical "Habits of Mind." While the "Habits of Mind" do not completely encompass all the components of what it means to be quantitatively literate, there are marked similarities between the "Habits of Mind" and being quantitatively literate. **In fact, it could be stated that being quantitatively literate is in and of itself a habit of mind.**

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IV. A CULTURE OF COLLABORATION

Numeracy instruction does not occur only in mathematics classrooms by mathematics teachers. Many integral players, both in and out of schools, contribute to the effective acquisition of quantitative literacy skills. These players are not autonomous; their roles are commutative and intertwined. They must work together to achieve the mutual goal of improved numeracy understanding. State and local governments, together with the schools and school districts, are responsible for choosing and implementing curricula, determining standards, and requiring valid assessment and accountability. School administrators, teachers, families, and community organizations all support the work of math teachers and their students. The Parent Teacher Association (PTA) is a classic example of many players coming together for the common goal.

Hill and Crevola (1997) created a general design for improved learning outcomes (Figure 2) which depicts how various components must work collaboratively to achieve increased student understanding. Each component is directly affected by at least one integral player in the school governance or leadership, or in the lives of its students. For example, monitoring and assessment is developed and mandated at the state level and administered by the districts, the schools, and the teachers. Once scored, the test results are reviewed by the state, the district, the principal, the teachers, the parents and, oftentimes, the student.

Figure 2: General design for improving learning outcomes (Hill & Crevola, 1997)

This section discusses the roles of the following players as they all work together to help students develop numeracy:

- The State
- School Districts
- School Administrators
- Teachers
- Families
- Community Organizations
- Students
- Institutions of Higher Education

The Role of the State

In the U.S., the responsibility and function of education is directed by the individual state. The Institute for Educational Leaderships' Task Force on Leadership (2005) "recognizes that state education leadership reflects a diverse collection of values, systems, and often incompatible practices, but suggests that state leaders should be guided by a set of principles that can support more effective leadership." In New Hampshire, leadership begins at the state level. By setting policy and offering resources that support literacy and numeracy achievement, the State Board of Education and Department of Education assure that all other stake holders work in harmony toward a common goal. Knowing that student achievement depends on effective instruction in every classroom in every school, the state education leaders will:

- Set clear standards (NH Curriculum Frameworks) and negotiate targets in a way that district and school leaders understand and own them.
- Allocate resources for the establishment of a strong assessment system (NECAP and local assessments) which is a measure both of and for learning.
- Recognize that 21st century literacy and numeracy requires the identification of funds to support schools in their movement forward with the integration of information and communication technologies.
- Foster and support a system of innovation that empowers schools and communities to develop effective assessment systems, classroom practices, and school and home partnerships.
- Support districts in the development of strong leadership teams charged with the development and implementation of a district numeracy plan.
- Monitor progress and intervene in districts and schools where students are not being well served.
- Keep educators informed about useful publications, innovative ideas, and meaningful data.

Most of the decisions in the state of New Hampshire, including teacher certification, school approval standards, state curriculum frameworks, and state-wide assessment systems, are determined at the state level by the Department of Education, the legislature, and the judicial system. Although each state is responsible for its own educational programs, the state of New Hampshire goes a step further toward decentralization. Rather than the state Department of Education, individual school districts and towns are responsible for a considerable portion of the decisions, implementation, and facilitation involved in the public education system. Therefore, it is the responsibility of the Department of Education to introduce, promote, and facilitate efforts that will support quantitative literacy.

The New Hampshire Department of Education is comprised of Bureaus within three divisions. While all Divisions and Bureaus have some role in supporting numeracy education, the primary responsibility rests with the Bureau of Accountability which oversees curriculum, instruction, assessment, accountability, and school improvement.

The Bureau of Accountability is charged with improving student achievement by providing educational leadership through accountability. This Bureau is responsible for numerous roles within the state's school system that have an effect on whether or not New Hampshire students become quantitatively literate. The Bureau helps coordinate, implement, and monitor the statewide assessments. The Bureau coordinates and oversees the statewide efforts to develop and revise state curriculum standards, the New Hampshire Curriculum Frameworks. These standards can encourage an emphasis on numeracy not only in mathematics but also across all curricula. In addition, the Bureau assists schools and districts in developing and revising local curriculum and improving instructional practices. When providing assistance, the Department of Education can help schools incorporate the state standards into local curriculum, evaluate program materials, and promote effective teaching practices that lead to an interdisciplinary, problem-solving approach to numeracy. Moreover, the Bureau offers technical and targeted assistance to schools and districts in need of improvement.

The Bureau of Accountability is also responsible for the administration of Title II Part B – Mathematics/Science Partnership Grant Program. This program is intended to encourage institutions of higher education, local school districts, and elementary and secondary schools to participate in professional development activities that increase the subject matter knowledge and teaching skills of mathematics and science teachers. Each year the New Hampshire Department of Education awards approximately $800,000 on a competitive basis to support projects in the state.

In addition to the Bureau of Accountability, other bureaus within the Department of Education have the ability to support numeracy instruction. The Bureau of Integrated Programs provides leadership, technical assistance, and professional development to schools and organizations as part of specific federal grant programs. The following programs could impact numeracy instruction:

- Title I Part A – Helping Disadvantaged Children Meet High Standards
- Title II Part A – Preparing, Training, and Recruiting High Quality Teachers and Principals
- Title II Part D – Enhancing Education Through Technology
- Title IV Part B – 21st Century Community Learning Centers[3]

Title 1 Part A provides opportunities for children to acquire the knowledge and skills necessary to meet the state proficiency standards by offering supplemental support to students and professional development to teachers. Clearly, numeracy should be a part of all supplemental support systems. The Title II Part A program could include numeracy in the standards for Highly Qualified Teachers and Principals.

Since technology plays such an integral role in facilitating numeracy (see **Section VI C**), schools, teachers, and students can benefit from the Title II Part D program. The 21st Century Community Learning Centers supported by Title IV Part B can foster home and school partnerships by bringing the community and students together in a forum where quantitative literacy could be explored.

The Bureau of Credentialing develops and implements policies and standards to improve the effective pre-service preparation of teacher candidates and continued professional development of certified educators. To strengthen the delivery of numeracy instruction, the state can require that teacher preparation programs and certification include numeracy coursework. In addition, the Department of Education can include numeracy as a professional development requirement for recertification. The Department can also encourage districts to include numeracy as a component of the district master professional development plan.

[3]The 21st Century Community Learning Centers program is discussed in greater length in the section discussing the role of the community.

The Division of Career Technology and Adult Learning implements policies and standards in NH CTE Centers and adult programs, often focusing on STEM (Science, Mathematics, Engineering, and Technology) initiatives that can promote quantitative literacy.

The Role of School Districts

District administrators (e.g., superintendents; curriculum, accountability and assessment personnel; English as a Second Language personnel; student services staff; and special education directors) are charged with the responsibility of leading their educational communities. A district's central office has the ability to set the tone, provide leadership and necessary support structures, establish the focus, and identify and encourage the best collaborative approach to address student achievement, especially in the area of numeracy. The district leadership team is responsible for steering the climate toward positive learning environments, promoting professional learning and development, setting high expectations for achievement, and modeling the communication that is required between and among schools. In addition, the leadership team identifies and supports principals in assisting their school staff as they develop individual plans based upon an identified district protocol.

By assisting individual schools with their ongoing efforts, the central office leaders serve as the intermediaries between the state and the schools to ensure state mandates are employed in meaningful, manageable forums. Professional expertise needs to be cultivated with an emphasis on achieving higher learning expectations.

District leaders can support teachers in helping students to develop strong quantitative literacy skills by setting the vision and identifying the mission for the district; defining student achievement to include achievement in numeracy, as a primary focus; identifying and providing on-going professional development for ensured effective high-quality teaching practices; developing the protocol for quantitative literacy and setting the goals needed to support instruction and learning; assisting in the allocation of resources, especially technology and manipulatives that aid numeracy instruction; supporting schools in creating plans for attaining numeracy achievement; promoting numeracy acquisition as a partnership with the entire community; encouraging all staff members to serve in leadership capacities; sharing ideas, programs, and professional development regarding quantitative literacy with educators on a regular basis; encouraging and helping to develop systematic processes of assessment, intervention, and monitoring of student progress; and annually reviewing results, celebrating successes, and providing forums across the district to update plans for continued work in subsequent years.

Classroom and special education teachers are the personnel responsible for putting theory into practice. All school personnel, in conjunction with a strong principal and visible support from the central office leaders, are instrumental in creating the climate that fosters a personalized education for each student. With all members of the learning community working in concert, the students are the recipients of improved learning and performance.

The Role of Schools

In order to increase teacher awareness, schools should offer professional development for all teachers concerning the definitions and distinctions between mathematics and quantitative literacy. These programs would help the school community reflect on the numeracy needs of students and recognize opportunities within and across the curriculum for developing numeracy. This professional development could be broadened to serve as a time to undertake collaborative planning for addressing numeracy among teachers.

Other factors that would facilitate students' development of quantitative literacy include the following:

- A positive school culture that allows for a balance of leadership and teacher input
- High expectations that all students can be numerate

- The creation of benchmarks for the development of numeracy across grades
- Assessments designed to gather and report information about students' numeracy skills
- Data about numeracy achievement that will inform planning and target setting relative to curriculum decisions about numeracy
- Communication with parents and communities pertaining to the goals for numeracy in learning and teaching
- The development of partnerships with professional associations and industry groups that have the interest and capacity to build quantitative literacy in schools
- Encouragement and support for attendance at professional development activities related to numeracy
- Prioritization of the budget to enable attendance for school leaders at numeracy-related programs

When the factors above have been established, schools will more effectively fulfill their role in introducing and developing students' quantitative literacy skills. Schools will be able to accelerate their progress to this end by assigning a numeracy coordinator to assist in the acquisition and development of classroom resources; provide direction, support, and assistance to classroom teachers; deliver in-school professional development; and develop benchmarks and assessments for numeracy. **Appendix N** includes additional summaries of factors that contribute to effective schools.

The Role of School Administrators

School leaders need to promote and maintain a focused commitment to the development of numeracy as a critical goal for all students. When school administrators emphasize numeracy as a sustained, school-wide goal, teachers in and outside of mathematics education become aware of the importance of quantitative literacy. They are more likely to develop an appreciation for the significance of their role in preparing numerate citizens. Also, they may seek additional opportunities to develop numeracy across the curriculum.

Some skeptics may be concerned that numeracy is yet another layer on top of an already overcrowded curriculum. However, addressing numeracy is not meant to add another layer to the curriculum: it is meant to broaden the entire curriculum. Steen (2001) states:

> So the call for numeracy in schools is not a call for more mathematics, nor even for more applied (or applicable) mathematics. It is a call for a different and more meaningful pedagogy across the entire curriculum. Numbers arise everywhere, so the responsibility for fostering intelligent numeracy should spread broadly across the entire curriculum. Quantitative literacy must be regarded as much more than the responsibility of the mathematics classroom alone (pp. 10-16).

Steen (2001) clarifies that topics which advance students' quantitative literacy are not so much additions to mathematics as they are applications and interpretations of the mathematics, "They are, to a great degree, mathematics set in context."

Teachers will need appropriate support in order to facilitate effective lessons that will help students to develop the capabilities to lead numerate lives. All administrators and teacher leaders should be familiar with the ideas that underlie standards-based mathematics education as their understanding of the nature of mathematics, learning, and teaching affects the support they will provide their teachers. By understanding these underlying ideas, they will be in a position to support classroom environments that foster numeracy. Appropriate support involves providing ongoing professional development opportunities that incorporate a numeracy focus, providing access to relevant resources, and providing the time and space for teachers to plan with colleagues and to reflect on the effectiveness of their practices.

Appendices N through U can be useful to administrators as they work toward incorporating numeracy across the curriculum.

The Role of Teachers

Throughout the past decade, teaching literacy has increasingly become the responsibility of all teachers, regardless of content. The phrase "Reading (or Writing) Across the Curriculum" is found in all types of educational settings and spans grade and ability levels. Teachers are now encouraged or required to participate in professional development to learn how to incorporate literacy into their lesson plans. Teaching quantitative literacy is slowly beginning to follow the same path.

Numeracy is incorrectly assumed to be math class- and content-specific. Teachers in other content areas often feel that teaching quantitative literacy is solely the responsibility of the mathematics teachers. Additionally, students are reluctant and even uncooperative when asked to think about a history or science question from a quantitative perspective. They ask, "This isn't math class. Why do I have to do this?" However, by including numeracy in their classes, teachers have the ability to foster deeper, more meaningful learning in other content areas. According to the Australian Department of Education, Employment, and Workplace Relations (2005), "[N]umeracy is not just knowing and doing mathematics in the mathematics lesson, but...it underpins learning in other areas of the curriculum. Teachers came to see numeracy as important, and at times essential, for understanding and learning in cross-curricula situations."

As with literacy, all teachers have a similar responsibility to help students develop into quantitatively literate citizens. As discussed in **Section III**, every discipline has a quantitative component, and each of these disciplines is made richer because of it. Teachers seem to ignore this component for a variety of reasons. Some teachers are short on the time needed to cover their own curriculum, while others are not entirely comfortable in their own quantitative ability or their ability to incorporate quantitative topics into their lessons. Other teachers avoid numeracy for pedagogical reasons: Students bring different attitudes and mathematical backgrounds into the classroom that can cause teachers to shy away due to the pedagogical complexities that may arise.

However, by ignoring the numeracy component, teachers forgo problems that have the potential to increase learning in their own content area. By introducing quantitative problems, teachers can help students develop perseverance and other critical "Habits of Mind" (**Section III F**). Students will not only be better-prepared to succeed in college or in the workforce, but they will also become more knowledgeable citizens.

Becoming numerate requires learning how to take on three roles: the fluent operator, the learner, and the critic (Australian Government Department of Education Science and Training, 2005). These roles are crucial in any discipline, and acquiring these skills is enhanced through quantitative problems. Teachers need to help students recognize numeracy in problems and help guide them through the relationship to their content-specific topic.

The Australian Government Department of Education Science and Training (2005) outlines several strategies[4] that teachers can use to help students develop their quantitative literacy skills: capturing the numeracy in the moment, being aware of possible numeracy demands when planning, allowing students to work it out, giving time, questioning, motivating, listening purposefully, and debriefing the numeracy.

By incorporating numeracy, teachers will be able to enrich their own content and lessons. They will also help their students to become more critical and perceptive, further enriching their learning and that of their classmates.

[4]Numeracy Demands and Opportunities Across the Curriculum can be found at www.dest.gov.au/NR/rdonlyres/7B31EC08-F53D-4B04-8F9B-716D29ADA5E0/4582/wa_brochure.pdf.
The brochure includes helpful examples for how to introduce numeracy in different content areas.

Section VI C discusses the role of mathematics teachers in depth.

Teachers may find **Appendices B-F (posters) and J-M** particularly useful as they help their students to become quantitatively literate.

The Role of Families[5]

Research on parent involvement consistently shows that when parents are actively engaged with their children's education their children do better in school, both academically and socially. Parents can, and most often do want to, contribute to the academic achievement of their children. With mathematics, however, many adults do not feel confident, especially when new math techniques are being taught and used in the classroom.

Teachers and schools have a variety of strategies available to encourage parents, regardless of their confidence with math, to positively engage their children. Schools and teachers can send messages home about a student's progress and include simple math activity ideas, or let parents know about an upcoming test and how to help their child prepare. Parents can also be made aware of resources, such as math web sites, community learning centers, and tutoring services which are sometimes offered free of charge.

Teachers can help involve parents in numerous ways. They can supply parents with ideas for applying mathematical thinking in a variety of real-life situations. For example, at the grocery store, students could calculate prices on a cost-by-weight comparison; at a restaurant, they could determine the gratuity based on percentage; and at a baseball game, they could continually update a player's batting average after each at bat. Teachers and parents can highlight problem-solving skills by modeling solutions to daily problems and by involving students in the process. By employing multiple methods to solve problems, teachers can foster student acceptance of these processes.

In New Hampshire, the Parent Information and Resource Center (NH State PIRC) offers free Family Math Nights at local schools. This program, for parents and children in elementary and middle school, provides families with a hands-on way to connect math skills to everyday life activities. Parents and children circulate among activity tables while learning how fun and practical math can be. This event utilizes a "train-the-trainer" format whereby teachers and parents can observe how the event is run and become equipped to facilitate future Family Math Nights.

Thankfully, many parents take an active role in their child's education and how their local curriculum is shaped. **In order for students to develop numeracy, schools need to incorporate it into their curriculum; and the schools need parental support to do so**. Regrettably, some parents can be reluctant to replace a traditional mathematics curriculum with one that will be more effective at teaching numeracy. For example, parents often push for a calculus-driven sequence even though research shows it is ineffective in teaching numeracy (Steen, 2004). Parents need to realize the powerful role they have in both instigating and inhibiting change that can help their students become quantitatively literate.

A little effort with parents can go a long way. Simply letting parents know they are important partners and asking for their help—and, giving them ideas and strategies on exactly how they can help can make parents feel included and informed. The Framework for Leveraging Parent and Community Involvement for Increasing Numeracy Skills in Students K-12 (**Appendix H**) outlines various areas of involvement along with the positive results that can occur as a result of that involvement. A list of resources for parents is included in **Appendix I**. Additional in-

[5]Prepared by NH State Parent Information & Resource Center www.nhpirc.org (800) 947-7005

formation on parent involvement research and training and technical assistance in building or strengthening family engagement efforts can be found at nhpirc.org, nhparentsmakethedifference.org, or by calling (603) 224-7005.

The Role of the Community Organizations

When it comes to promoting student math achievement, the community just beyond the school walls is an often largely untapped resource. Taking an inventory of all the possible community programs shows the breadth of resources available to aid numeracy acquisition. Some of these resources include tutoring services, community learning centers, after-school programs, mentorship and apprenticeship programs, library programs, hobby clubs that may involve math skills, sports clubs that require statistics be kept, and the local parent-teacher organization. In addition to helping students directly, these programs and organizations can be useful in reaching out to parents on a consistent basis.

When a student seems to lack the motivation to learn math, his or her teacher can look for a community resource that may tap into the student's other interests and provide the opportunity to apply math knowledge. Encouraging a mentorship, such as with Big Brothers or Big Sisters, can help engage children in real-life math activities. Places of worship are another possible resource, as they are typically open to promoting community service. Often a church, mosque, or synagogue already has a youth group or youth mentoring program that combines academic support with social reinforcement. The Framework for Leveraging Parent and Community Involvement for Increasing Numeracy Skills in Students K-12 (**Appendix G**) outlines various areas of involvement along with the positive outcomes that occur as a result of those interactions.

Appendix I includes a list of supplemental community programs and resources.

The Role of the Student

The student is probably the most crucial player involved in his or her numeracy instruction. Without active involvement and motivation from the student, developing the necessary quantitative skills would be difficult, if not impossible. As discussed in **Section III E**, students should be made explicitly aware of their rights and responsibilities. In addition, they should also be aware of the "Habits of Mind", discussed in **Section III F** and in **Appendices E and F**.

In order to create a climate conducive to learning numeracy, students should be respectful, participating members of their class. They need to ask and attempt to answer questions, and hold high expectations for themselves, their classmates, and their teachers.

Not surprisingly, most students dislike mathematics, and they have the right to express this opinion. Yet, regardless of their opinions, students have the responsibility to keep an open mind and to not let their opinions affect their learning. **Ultimately, children need to realize that quantitative problem-solving is not exclusive to math courses—they can face similar problems in any discipline**. In order to foster even stronger numeracy skills, they should be receptive to open-ended, quantitative problems that they encounter in non-math courses.

The Role of Institutions of Higher Education

Before primary and secondary schools realized the value and importance of teaching quantitative literacy, institutions of higher education had already become involved. Many colleges and universities began offering courses and programs targeted at increasing students' numeracy skills, and many institutions started conducting research and publishing articles on the topic.

The current role of institutions of higher education in numeracy acquisition is two-fold. One, they should emphasize that being quantitatively literate is critical and that students will be required to attain minimum competencies. Two, they should send a clear message to the primary and secondary education systems (e.g., the students, parents, teachers, principals, and administrators) that quantitative literacy should be integrated into their curricula.

Within their own institutions, colleges and universities need to ensure that their graduates are numerate by incorporating rigorous standards related to quantitative literacy. The following list is a representative sample of the various ways an institution can play a role in developing a quantitatively literate student body. **Section VI H.2** includes a thorough compilation of the ways a college or university can be involved:

- Establish clear prerequisite competencies for entering students, sample core competencies for all students during the first two years (**Section VI H.2 Table II.1**), and competencies within the major program (**Section VI H.2 Table II.3**)
- Engage in cross-disciplinary, institution-wide on-going discussions about what the institution values in a quantitatively literate student and the role that quantitative literacy should play in each major and in each department
- Increase awareness in the college community--particularly among faculty and administration--of the value and role of quantitative literacy in everyday life and in being a knowledgeable, productive citizen
- Develop a clear assessment plan that measures the degree to which students are meeting the competencies and steps for improvement
- Establish clear oversight of the quantitative literacy plan and how the plan will be evaluated
- Prepare students to deal with the technological demands of a changing society

Many of the goals of quantitative literacy courses and programs (e.g., preparing students for productive citizenship, building "Habits of Mind," positive attitude, confidence, and developing an appreciation and ability for problem-solving) are very much the same as the goals in any other numeracy program from pre-kindergarten through post-graduate work. However, institutions of higher education play an additional unique and influential role: they can affect what primary and secondary educational institutions view as important in terms of their own curriculum and numeracy preparation. With the goal of attending college, many high school students and their parents attempt to create a transcript that would "please" college admissions officers. Thinking that colleges prefer the traditional algebra-geometry-calculus sequence, parents sometimes fight to keep school districts from adopting a more integrated, numeracy-focused curriculum. Research indicates that the traditional, calculus-driven sequence is not working to create quantitatively literate students (See **Section VI H**). Yet high schools and districts are holding onto the traditional curriculum because they believe it produces the academic background that colleges require of incoming students.

Institutions of higher education are still sending high school students, their parents, and the school administrators mixed messages. On the one hand, some colleges and universities require that students meet quantitative literacy competencies at various stages in their post-secondary education. But on the other, many college placement and entrance exams do not have a quantitative literacy component which implies that the numeracy skills of incoming students do not matter.

Institutions of higher education have the unique opportunity to provide academic opinion and research to which school districts and departments of education may respond. They have the ability as well as the responsibility to lead the charge toward creating numeracy education through the educational system. Furthermore, **with a college degree as a goal for many students and their parents, colleges and universities have the clout to affect pre-collegiate preparation by requiring that their applicants be numerate.**

A. Section IV References

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V. SUPPORTING NUMERACY: INFRASTRUCTURE AND IMPLEMENTATION

A. Highly Qualified Educators

Creating and maintaining the infrastructure to support students in becoming quantitatively literate is a major challenge for schools. To achieve this goal, the following factors need to be considered:

- Highly qualified teachers, in all content areas, need to understand the importance of numeracy instruction and know how to implement it in their classroom
- A numeracy or instructional leadership team must be created to provide guidance for all classroom teachers, specialists, and support staff at a school
- Alignment of content standards, curricula, and assessment will lead to curricular coherence and shared expectations
- Identifying student needs and monitoring the effectiveness of numeracy instruction will be achieved through ongoing and effective use of data
- Quantitative literacy instruction can be properly and effectively carried out with sufficient resources and time
- Effective implementation of quantitative literacy instruction can be achieved through ongoing professional development opportunities for teachers
- All instruction in numeracy must be supported by and reflect the New Hampshire Curriculum Frameworks in Mathematics (the Grade-Level and Grade-Span Expectations in Mathematics)

These elements, when coordinated across classrooms, grades, and schools, will create a powerful quantitative literacy plan that will incite the changes necessary to ensure that all students graduate from high school possessing the necessary skills in numeracy.

Without the fundamental infrastructure, schools may perceive an uneven development of quantitative literacy knowledge among students and only see anecdotal evidence of success. The establishment of these elements is crucial to helping all teachers, no matter their specialty or discipline, realize the value of quantitative literacy development in their students and to understand how they can effectively bring numeracy into their classrooms. This infrastructure will provide the greatest strength in supporting the goal of ensuring that all students are quantitatively literate. Despite conventional wisdom that school inputs make little difference in student learning, a growing body of research suggests that schools can make a difference, and a substantial portion of that difference is attributable to teachers (Darling-Hammond, 2000).

If it takes a village to raise a child, it takes an entire school of committed teachers to educate one. Each teacher has an important role to play and contribution to make in the effort.

<u>Classroom Teachers</u>
According to the National Math Panel Report (2008):

> A meta-analysis of the findings from seven large studies of variation in teacher effects found that 11% of the total variability in student achievement gains in mathematics across one year of classroom instruction was attributable to teachers (Nye, Konstantopoulos, & Hedges, 2004). . . . Sanders and Rivers (1996). . . .measure[d] the effectiveness of all math teachers in Grades 3, 4, and 5 in two large metropolitan school districts. . . .The progress of children assigned to these low- and high-performing teachers was tracked over a 3-year period. Children

> assigned to three effective teachers in a row scored at the 83rd percentile in mathematics at the end of fifth grade, while children assigned to three ineffective teachers in a row scored at the 29th percentile. Using a different methodology, Rivkin, Hanushek, and Kain (2005) came to a similar conclusion: The cumulative effects of children being taught by highly effective teachers can substantially eliminate differences in student achievement that are due to family background.

There is little doubt that the primary factor in student learning and success in mathematics is a highly effective classroom teacher. The teacher plays an important role in the students' mathematical foundation; what the teacher knows and knows well, he or she will usually be able to effectively transmit to students. However, if the teacher is unsure or apprehensive about a topic or concept, even one in his or her field of teaching, it seems to follow that his or her students also may not understand it well.

For the mathematics teacher, a deep, solid, conceptual understanding of quantitative literacy is essential if a school is to produce graduates who are quantitatively literate. However, **even if a teacher is aware of the facts and ideas about quantitative literacy, he or she may not make quantitative literacy an important part or emphasis in his or her everyday mathematics teaching simply because he or she does not know how**. After all, the concept of quantitative literacy has only been around for a few decades at most, implying that a majority of teacher training programs have more than likely not emphasized quantitative literacy or how to teach it. Additionally, most teachers tend to teach the way they were taught, unless post-degree professional development is completed in response to a perceived need. Since it is apparent that creating quantitatively literate citizens is indeed a need and some teachers may not have received specific training, professional development in this area may also be a necessity.

But what about teachers in other content areas? The term content area teachers refers to mathematics, science, social studies, and English/language arts teachers, as well as any other subject-specific teachers (e.g., art, music, culinary arts, and auto mechanics). More often than not, content area teachers outside of mathematics have had very little significant pre-service education in quantitative literary instruction. However, it is important for the quantitative literacy development of all students that teachers in all content areas understand how to effectively implement numeracy topics in their classrooms. So, it is also imperative that these content area teachers receive professional development in quantitative literacy.

What do content area teachers in the middle and secondary schools need to learn in order to support students' learning of quantitative literacy in their discipline? Content area teachers need to understand how their specific subject area interacts with the ideas and concepts of numeracy. They need support to help them develop strategies that will bring these ideas to the forefront of their teaching, to help demonstrate to students how quantitative situations arise in areas besides mathematics. They then need solid assessment strategies to know if their students have mastered these concepts. Also, they need supporting documents, samples, texts, and other materials that demonstrate clearly the quantitative literacy concepts that are related to their disciplines.

Elementary teachers are in a different situation than their content area colleagues in the higher grades. They must be generalists, having appropriate knowledge in a variety of topics and content areas necessary for their grade level(s) of teaching. This implies, when it comes to quantitative literacy, that they know how quantitative literacy interacts in a variety of content areas. However, beyond that, their needs are very similar to those of the content area specialists: support for developing strategies to help students become quantitatively literate and development of assessment strategies that will help them monitor student success.

Elementary Mathematics Specialists

In recent years, the use of teachers who specialize in mathematics at the elementary level has increased, the thought being that teaching mathematics demands a specialized knowledge at all grade levels (K–12), while most elementary teachers are generalists. Colleges in eight states now offer elementary mathematics specialist degrees as an increasing number of schools are looking to employ these specialists as a way to improve the mathematical performance of their elementary school students.

According to Skip Fennell, former president of the National Council of Teachers of Mathematics, there are two major models of elementary mathematics specialists, the lead teacher model and the specialized teacher model. The lead teacher model frequently involves a teacher in the role of the mathematics resource person at a school who works primarily with teachers at the school and rarely, if ever, instructs students directly on his or her own. They mentor elementary teachers when it comes to teaching mathematics, assisting them in planning and implementing lessons. They can also help teachers to properly interpret standardized testing data and design approaches to help improve students' achievement in mathematics. In contrast, the specialized teacher model gives one teacher primary responsibility for teaching mathematics to elementary grade students. Depending upon the size of the school, this teacher may teach mathematics at one grade level or at several. There seems to be more specialized teachers in the upper elementary (grades 4 or 5) than in the lower (Fennell, 2006).

A review of programs that certify elementary content specialists indicates that quantitative literacy training is not a specific requirement; therefore, it can be safely assumed that these programs do not prepare their graduates in the nuances of quantitative literacy teaching any better or worse than those that prepare future middle and high school mathematics teachers. Based on that information, it appears as if elementary mathematics specialists may need the same type of quantitative literacy professional development as their middle and secondary school colleagues.

Special Educators

Special educators are instrumental members of the numeracy team who focus their attention on students with identified special education needs. They provide curricular adjustments and strategies to meet the needs of students who learn differently or who may need support to demonstrate what they know.

In order to be effective teachers, special educators need to be trained in a wide variety of skills, materials, and strategies. Regular classroom teachers need to learn the skills and strategies that will support students with disabilities in order to ensure they gain as much quantitative knowledge as possible. In the same vein, special education teachers need to become as quantitatively literate as possible to support student learning in this area.

Paraprofessionals

In most schools, the paraprofessional's job is to assist the classroom teacher with duties impacting the students. This definition encompasses a wide variety of tasks, some which include individualized instruction and attention to those students who need it.

Paraprofessionals oftentimes will provide teacher-developed and -directed lessons in one-on-one or small group situations to help students "catch up" to their peers. When possessing the appropriate training and education, an effective paraprofessional can be a valuable asset to the teacher and the entire school.

Some paraprofessionals may have the advantage of having previous or current experience working in situations outside the school that demand quantitative knowledge; they can bring these situations into the school and provide a valuable benefit in that regard. However, as with all of the educators previously discussed, paraprofessionals may too need training in the aspects of numeracy in order to effectively implement it.

B. Numeracy and Instructional Leadership Teams

While it is vitally important to have knowledgeable, dedicated teachers interested in bringing quantitative literacy education to their schools, the school principal is the primary agent of change in most school buildings. **The principal is instrumental in setting the vision and tone for the school and enforcing its goals and mission**. According to the Alliance for Excellent Education (2004), "if one principal is not properly trained and up to the task of leadership, it will have a damaging effect on hundreds of students—an unacceptable thought. . . .The principals of tomorrow's high schools must be instructional leaders who possess the requisite skills, capacities, and commitment to lead. . . . Without leadership, the chances for systemic improvement in teaching and learning are nil" (p. 44).

In the role of developing a quantitatively literate school, the principal has several distinct roles and functions. The principal must act as an instructional leader who believes in the quantitative abilities of every student; develop personal understanding and knowledge of quantitative literacy and pass that knowledge along to others; be visible in the classrooms, showing support for teachers' development and delivery of lessons that demonstrate and teach quantitative literacy concepts; make decisions that support all students becoming quantitatively literate; continually assess the program to ensure progress; empower teachers to experiment with new lessons and activities that support numeracy; and foster the development of teacher leaders to support the quantitative literacy efforts.

A second key to the success of a school-wide initiative on improving students' numeracy development is the establishment of a numeracy/instructional leadership team. The ideal team should consist of faculty and staff from all disciplines who are involved in the quantitative literacy development of their students. The team members should include classroom teachers, specialists, paraprofessionals, and, when appropriate, parents and students. It should be given leadership and direction from the principal along with a clear focus, plan, and long- and short-term goals.

Section VII further discusses the key steps needed to create a local numeracy action plan.

C. Curricular Coherence and Shared Expectations

To define what each student should accomplish regarding mathematics and quantitative literacy, schools and districts need to examine their local curriculum from two different angles: the curriculum that is intended, that is "on the books," and the curriculum that is actually taught in the classrooms. These two "curricula" should be checked to ensure they align well with each other: that what is being taught is what is intended. As Fullan (2006) states, "variations in students' achievement are greater across classrooms within a school than across schools" (p. 55).

For alignment purposes, the local curricula should also be compared to the 2006 New Hampshire Curriculum Frameworks that include the Grade-Level Expectations (GLEs) for grades K–8 and the Grade-Span Expectations (GSEs) for grades 9–12 (www.ed.state.nh.us/frameworks). Once aligned, expectations for student learning need to be well articulated, rigorous, and focused at each grade level, as well as coordinated across grade levels and schools within a district. Each teacher of mathematics at every grade level must know what is expected of him or her and of each student in that grade level to ensure that learning potential is maximized.

Commonalities at each grade level and between the grades are important so that students are exposed to a consistent, coordinated, and coherent curriculum. Students often learn mathematical concepts at deeper levels when they are exposed to the concepts multiple times, by multiple teachers, and in multiple classrooms across the years of their schooling. Therefore, the student expectations need to be communicated not only to the teachers at each grade, but to the student and his or her parents as well.

Some ways to help ensure consistency across and within the grades are to use common tasks, common themes, a common curriculum, common assessments, common rubrics for scoring tasks, and even common formative and summative tasks and exams. Some districts have even developed common expectations for written work. In some districts well-defined expectations have been called "the guaranteed curriculum" (Westerberg, 2007). **In today's classrooms, the expectation for students learning in mathematics and quantitative literacy should be common and constant**; the items that vary could be the time allotted for the task and the instructional strategies employed by the teachers. A variety of strategies will benefit all students and additional time may be required in some situations with some students.

D. Assessment and the Effective Use of Data

To determine whether students have learned what they are expected to learn, a variety of summative, formative, and benchmark assessments must be employed by teachers. All of these assessment techniques need to be used together in order to provide a complete picture of what the student is capable of achieving.

Summative assessments are given for the purpose of grading or writing an evaluation on the student's ability or understanding of a concept. They are given to students at a particular point in time to determine if each student learned what they were supposed to over a certain period of time. National assessments (e.g., NAEP), state assessments (e.g., NECAP), and classroom tests given at the end of chapters or the end of terms fall into this category, as do quizzes, homework, portfolios, presentations, research papers, and projects. Oftentimes, when politicians or the general public speak of "assessment," they are generally referring to summative assessment.

Formative assessment is conducted for the purpose of gathering knowledge about a student's ability or understanding of a concept; it is not performed for the purpose of establishing a grade or written evaluation. Formative assessments are informal; they are the day-to-day assessments that teachers make of their students *during active instruction*. Teachers use this form of assessment to help them understand what a student knows and does not know about the present concept they are teaching.

Teachers assess formatively in many ways: by observing students as they work on a problem or concept, by verbally asking questions in the course of a lesson to determine the depth of students' knowledge, by having students honestly self-evaluate their understanding of a topic, and so on. Formative assessments provide a feedback loop between teacher and students; the teacher, gathering information about how his students are doing, adjusts his teaching in response to that information. The students, seeing the adjusted approach to the topic, reapply the concept. The teacher once again gathers data formatively to determine if the students understand the concept, and so on.

Benchmark assessments are a subset of summative assessments. They are assessments made in relation to a group of "benchmarks," or standards, which have been predetermined as necessary for students to achieve at a particular level for their age and grade. These assessments are not designed to determine how students compare to one another, but how they compare to the benchmarks for their age group. The NECAP test, which is a test based on the New Hampshire Curriculum Frameworks (the Grade-Level Expectations for grades K–8 and the Grade-Span Expectations for grades 9–12) is an example of a benchmark assessment, but districts and schools could design their own benchmark assessments as well.

Exemplary teachers will use a variety of formative, summative, and benchmark assessments to measure student progress and to guide instruction and student learning. Because they effectively use assessments, exemplary teachers know and understand the strengths and weaknesses of each student in their classroom. The challenge facing schools and districts is finding ways to collect and compare common points of data so this information can be shared with other teachers as the student progresses through the grades.

To meet this goal, common assessments should be used to measure progress at predetermined benchmarks with the data being reviewed regularly and collaboratively by grade-level, school-level and district-level teams. This data is reviewed to determine how a student is progressing and to inform teachers of the areas they may need to focus on for their students.

At any level, if decisions are driven by data, they have a better chance for sustainability (Westerberg, 2007). Furthermore, data-driven decisions are more likely to lead to improved student performance. Teachers should also be provided ample professional development opportunities to work with technology-based data tools (for example, the NH DOE's Performance Tracker Tool), for collecting and analyzing classroom data.

E. Sufficient Resources and Time

The days are long gone when teachers of mathematics needed only chalk and a blackboard to teach mathematics effectively. To create a successful learning environment in today's mathematics classroom, several resources are necessary. These resources include physical items (e.g., classroom supplies, manipulatives, calculators, Calculator Based-Rangers (CBR) and Calculator-Based Laboratories (CBL), interactive whiteboards, professional materials, software, and technical support) and strategically allocated financial resources that may also target additional sources of revenue to meet students' and teachers' needs. Schools should carefully consider what materials and resources can be best used to encourage student understanding of quantitative situations.

In order for today's teachers to learn how to use new curricula and materials to maximize their effectiveness in teaching numeracy, time and professional development support need to be utilized. A relatively inexpensive way to increase teacher knowledge and productivity is to allow teachers to have common planning times. Teachers need time to reflect on their practices and to talk to their colleagues in order to build a well-informed, supportive community of practitioners (PLCs). Common planning times will also allow for professional development opportunities to take place during school hours. By using the collective expertise of all teachers, responsibility for the success of students is no longer an individual burden, but one shared by all members of the mathematics faculty.

F. Professional Development

Professional development, as defined by NCLB (2001), "includes activities that improve and increase teachers' knowledge of the academic subjects the teachers teach, are an integral part of broad school-wide and district-wide educational improvement plans and are high quality, sustained, intensive and classroom-focused in order to have a positive and lasting impact on classroom instruction and the teacher's performance in the classroom," among other things. Professional development allows mathematics teachers to improve and enhance their mathematical knowledge and classroom practices and provides support and collaborative opportunities for those who sometimes feel isolated in their practice.

All of this can positively affect student learning outcomes. A recent comprehensive research review by the Johns Hopkins University School of Education's Center for Data-Driven Reform in Education found that teachers have a greater impact than new textbooks or computers when it comes to raising scores in mathematics. Robert Slavin, director of the Center, reports that "[t]he debate about mathematics reform has focused primarily on curriculum, not on professional development….Yet the research review suggests that in terms of outcomes on math assessments, curriculum differences are less consequential than instructional differences" (NCSM, 2009). This research implies that school districts, when searching for ways to improve student performance while facing budget constraints, may want to consider offering professional development in lieu of purchasing new materials.

The National Staff Development Council (2001) outlines the following standards for professional development. When considering professional development programs for teachers, schools should consider these context, process, and content standards.

Context Standards
Staff development that improves the learning of all students organizes adults into learning communities whose goals are aligned with those of the school and district (Learning Communities); requires skillful school and district leaders who guide continuous instructional improvement (Leadership); and requires resources to support adult learning and collaboration (Resources).

Process Standards
Staff development that improves the learning of all students uses disaggregated student data to determine adult learning priorities, monitor progress, and help sustain continuous improvement (Data-Driven); uses multiple sources of information to guide improvement and demonstrate its impact (Evaluation); prepares educators to apply research to decision making (Research-Based); uses learning strategies appropriate to the intended goal (Design); applies knowledge about human learning and change (Learning); and provides educators with the knowledge and skills to collaborate (Collaboration).

Content Standards
Staff development that improves the learning of all students prepares educators to understand and appreciate all students; create safe, orderly and supportive learning environments; and hold high expectations for their academic achievement (Equity); deepens educators' content knowledge, provides them with research-based instructional strategies to assist students in meeting rigorous academic standards, and prepares them to use various types of classroom assessments appropriately (Quality Teaching); and provides educators with knowledge and skills to involve families and other stakeholders appropriately (Family Involvement).

Professional development can take many different forms where teachers can strengthen their learning and then transfer that learning into their classroom practice (e.g., mentoring, coaching, active practice, peer feedback, or demonstrations of new or unfamiliar teaching, learning, or assessment strategies). Coursework, online learning, in-service workshops, webinars, and podcasts are also effective forms of professional development that help advance content and pedagogical knowledge. School improvement efforts may utilize the train the trainer[6] model, set up classrooms as demonstration sites, establish partnerships with higher education, and encourage action research projects[7].

In New Hampshire, there are many professional development opportunities for mathematics teachers at all grade levels. The list below contains only a few of the organizations that provide professional development in mathematics:

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[6]The train the trainer model is one where a teacher becomes an expert in a particular area and in turn trains other teachers at his or her school.
[7]An action research project involves a teacher conducting research in his or her school, usually with the guidance of a higher education partner, with the purpose of discovering ways to improve student learning.

- The New Hampshire Impact Center at Plymouth State University: www.plymouth.edu/graduate/nhimpact
- The Joan and James Leitzel Center at the University of New Hampshire: www.leitzelcenter.unh.edu
- The South Eastern Regional Education Services (SERESC), located in Bedford: www.seresc.net
- The Local Educational Support Center Network (lescn.org) regional sites located throughout New Hampshire:
  - Capital Area Center for Educational Support (CACES) in Concord: www.caces.org
  - Greater Manchester Professional Development Center (GMPDC) in Manchester: www.gmpdc.org
  - North Country Education Support Center (NCES) in Gorham: www.ncedservices.org
  - Seacoast Professional Development Center (SPDC) in Exeter: www.k12opensource.org/spdc
  - Southwest New Hampshire Education Support Center (SWNH-ESC) in Keene: www.swnhesc.org
  - Sugar River Professional Development Center (SRPDC) in Claremont: www.sau6.k12.nh.us/SRPDC/PDindex.htm
- Open-NH: New Hampshire's online Professional Education Network http://www.nheon.org/opennh/index.htm

Professional Learning Communities

When schools work to establish a true professional learning community, they begin the shift from each teacher individually responding to difficulties in their classroom to a community response. According to Southwest Educational Development Laboratory (SEDL) (retrieved June 19, 2009 www.sedl.org/change/issues/issues61/attributes.html), the literature on professional learning communities continually refers to the five following characteristics: supportive and shared leadership, collective creativity, shared values and vision, supportive conditions, and shared personal practice. By becoming as a professional learning community, schools have the opportunity to realize substantial improvement (Eaker, DuFour, & DuFour, 2002). They can also leverage the unique characteristics of a professional learning community to collaborate and achieve numeracy goals.

A professional learning community is enhanced when there is openness to improvement, trust, and respect; a foundation in the knowledge and skills of teaching, supportive leadership, and school structures; and events that encourage the school's vision and mission. Participation at all levels including administration, classroom teachers, special education teachers, and support staff is critical to the development of the professional learning community. According to Zemelman, Daniels, and Hyde (2005),

> the best [professional development] activities provide a mirror in which teachers see themselves in new ways. They draw on teachers' prior knowledge and abilities, and help them construct new approaches of their own. . . .They renew people's enjoyment of their own learning. And they provide space to reconceptualize what learning and teaching can be (p. 283).

Connection to GLEs and GSEs

In response to the required testing of all students mandated by NCLB (2001), the New Hampshire Department of Education together with the Departments of Education in Vermont and Rhode Island developed a common set of expectations in mathematics, reading, and writing for each grade. These expectations are found within the New Hampshire Curriculum Frameworks and are known as the Grade-Level Expectations (GLEs) for grades K–8 and the Grade-Span Expectations (GSEs) for grades 9–12 (www.ed.state.nh.us/frameworks). The GLEs and GSEs are not representative of the full curriculum; rather, they were meant to help focus the curricula, provide coherence and consistency across the state, and promote and support good instruction.

In addition, one of the greater purposes was for the GLEs and GSEs to serve as a framework for developing common assessments which would be administered in New Hampshire, Rhode Island, and Vermont, beginning in the 2005–2006 academic year. The expectations highlight concepts and skills that are not commonly or easily assessed on large-scale, national assessments. Thus, in order to both meet the NCLB requirements and to more accurately assess students on certain topics, the New England Common Assessment Program (NECAP) exams were developed using the outlined expectations as a guide. (The NECAP Collaborative now includes Maine.)

Knowing the GLEs and GSEs and incorporating them into daily instruction is important for every New Hampshire teacher of mathematics. First, the expectations are primarily based on the standards and principles outlined in the National Council of Teachers of Mathematics' Principles and Standards for School Mathematics document[8]. This document was heavily researched; therefore, the GLEs and GSEs are well-supported by current research. Using them to inform instruction can help lead to increased student achievement. Second, the questions on the NECAP exams are based on these expectations. Thus, if a teacher wants his or her students to perform well on the NECAP exam, he or she has an interest in preparing students to meet or exceed the expectations.

Proponents of numeracy instruction often cite current standardized testing as an obstacle to promoting and developing numeracy in children. Many of the large, standardized tests are not designed to assess students' quantitative literacy. Instead, the questions primarily involve rote memorization and procedures as well as abstract or non-contextual mathematics. As a result, teachers are less likely to spend time emphasizing quantitative literacy themes if they believe their students will not be tested on it. Yet, because of the strong connection between the GLEs and GSEs (upon which the NECAP test items are based) and the components of numeracy instruction, New Hampshire teachers do have the opportunity to seamlessly incorporate quantitative literacy into their lessons.

With an already over-crowded curriculum and the pressure of preparing for testing, teachers have good reason to be skeptical about the prospects of including numeracy into their daily instruction. However, **many connections already exist between the current curriculum, the GLEs and GSEs, and numeracy instruction, thereby making teaching numeracy more accessible and possible**. Teaching quantitative literacy is not one more topic on top of everything else that a teacher already needs to do. It is already being incorporated; teachers just need to be aware of how numeracy fits in with their current curricula in order to enhance numeracy education. Doing so can lead to improved student outcomes because of the strong connections between quantitative literacy and the expectations that are in place.

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[8] The GLEs and GSEs were also influenced by the PISA (Programme for International Student Assessment) recommendations, Achieve standards, NAEP (National Assessment of Educational Progress) standards, and Rhode Island community college entrance standards, among others.

In addition, students may even perform better on standardized tests if they are taught from a quantitative literacy perspective. L.P. Benezet's work as superintendent of the Manchester school system in the 1920s and 1930s provides compelling evidence. Benezet chose to postpone mathematics instruction so students could focus on literacy. They were, however, still learning mathematics through reasoning and discussion; in essence, they were becoming quantitatively literate. Even though standardized tests tend to focus on standard arithmetic procedures, these students performed at least as well as their peers who had received traditional mathematics instruction (see **Section III B**).

The connections between numeracy and the New Hampshire mathematics expectations can be seen in both the Frameworks and in the GLEs and GSEs. First, the New Hampshire Curriculum Frameworks outline six primary goals of mathematics instruction. The Department of Education states "these goals are closely aligned with those espoused by various national commissions and groups in their efforts to reshape the teaching and learning of mathematics" (New Hampshire Curriculum Frameworks, 2006). Upon closer inspection, these goals are very similar to those that would also be representative of the goals included in a strong numeracy-focused curriculum:

- All students will develop a firm grounding in number sense that includes computational fluency.
- All students will develop a basic understanding of key concepts and principles central to the study of geometry, algebra, probability, and data analysis, while appreciating the interrelationships of all areas of mathematics.
- All students will develop strong mathematical problem solving and reasoning abilities.
- All students will develop positive attitudes about mathematics.
- All students will develop the ability to use appropriate technology to solve mathematical problems.
- All students will develop the ability to communicate their understanding of mathematics effectively

Without explicitly creating a separate numeracy curriculum, the New Hampshire mathematics curriculum already promotes and supports several numeracy goals, such as problem-solving, effective communication, and positive attitudes. Teachers and schools are already inherently charged with developing quantitative literacy through the intended curriculum.

The Curriculum Framework also highlights characteristics of a learning environment that supports the goals for mathematics education. Again, these characteristics are ones that will likewise support numeracy instruction: students are actively involved in doing mathematics; problem-solving, thinking, reasoning, and communicating are everyday activities; central mathematical concepts are understood; an appropriate balance between application and acquisition of knowledge and skills exists; manipulatives are used, when appropriate, to connect conceptual to procedural understanding; technology is used in appropriate ways; the curriculum is coherent and well-articulated across grade levels; and assessment is an integral part of instruction.

The above characteristics further highlight the relationship between teaching quantitative literacy and the existing curriculum. For example, both require active involvement, technology, problem-solving and communication, and conceptual understanding. **Sections VI A and VI B** further discuss the goals and components of numeracy instruction.

The connections are even more compelling when looking at the GLEs and GSEs. First, the strands of the expectations are prominent goals and topics throughout numeracy education. The four content strands of the GLEs and GSEs include number and operations; functions and algebra; data, statistics, and probability; and geometry and measurement. The two process strands of the GLEs and GSEs are problem-solving, reasoning, and proof; and communications, representations, and connections.

In addition, when explaining its rationale concerning the expectations, the New Hampshire Department of Education's Curriculum Frameworks (2006) state:

> Since it is crucial that process standards are not seen as completely separate from content standards, the process standards have been imbedded throughout the content strands. . . .This mirrors classroom instruction as in most classes, as students are learning content knowledge, instruction is also focusing on improving their abilities in problem solving, reasoning, and communication; furthermore, students are looking for and making appropriate connections, and they are able to understand and use multiple representations of mathematical ideas (p. 8).

This statement supports teaching several skills and topics at once, in unison, rather than teaching them independently. A student should be exposed to topics that incorporate both content and process strands simultaneously. Additionally, the expectations call for students to make connections and to use multiple representations, all of which are paramount components of effective numeracy instruction.

The activity below, "What is the difference between a million, billion, and trillion?", illustrates how one problem can be used to both teach quantitative literacy and to meet Grade-Level (or Grade-Span) Expectations. This activity touches upon many of the ideas central to both the GLEs and GSEs and to numeracy instruction for all grade levels. These ideas include conceptualizing rational numbers and their relative magnitude, making estimations, identifying and extending patterns, communicating understanding, and exploring connections. This exercise can be modified for any grade or ability level by using different numbers or scenarios.

What is the difference between a million, billion, and trillion?
By Dr. Brian Beaudrie

Topics: Number Sense, Quantitative Literacy

NCTM Standards: Number and Operations; Measurement; Reasoning and Proof; Communication; Representation; Connections

New Hampshire Grade Span Expectations: Number and Operations; Geometry and Measurement; Problem Solving, Reasoning and Proof; and Communication, Connections and Representations.

Directions: Have students estimate the following. After each, give them the answer, and ask any of the follow up questions, if appropriate:

Question: How long ago was one million seconds?

Answer: One million seconds would take up 11 days, 13 hours 46 minutes and 40 seconds.

Follow up questions: How many million seconds have passed since the beginning of the month? Of the year? Since school began?

Question: How long ago was one billion seconds?

Answer: One billion seconds is a bit over 31 and one-half years.

Follow up questions: Are you a billion seconds old? Do you know someone who is?

Do you know someone who is two billion seconds or three billion seconds old?

Question: How long ago was one trillion seconds?

Answer: One trillion seconds is slightly over 31,688 years. That would have been around 29,679 B.C., which is roughly 24,000 years before the earliest civilizations began to take shape.

Follow up questions: Were you surprised by the size of the difference between one million seconds and one billion seconds? Between one billion seconds and one trillion seconds?

Question: If one million seconds takes up about 11 and one half days, how long is one million minutes?

Answer: One million minutes takes one (non-leap) year, 329 days, 10 hours and 40 minutes.

Follow up questions: How far back in time would you have to go to get to one million minutes? How long will it take from today until we go another one million minutes?

Question: How far back in time would we have to go to get to one billion minutes?

Answer: One billion minutes would take a bit over 1,902 years. To go back one billion minutes would put us a few years after 100 A.D.

Follow up question: What was happening one billion minutes ago?

Question: How far back in time would we have to go to get to one trillion minutes?

Answer: One trillion minutes was about one million, nine hundred thousand years ago…long before any human walked the earth.

Question: Now, let's look at hours…will you live one million hours?

Answer: It's possible, although you'd have to be very, very healthy! One million hours would take about 114 years to complete.

Directions: Read each question aloud. Have your students estimate the answers first; then have them calculate them, using calculators when appropriate.

Question: You've just won one million dollars! But, the rules require that you need to spend it all in one year. How much do you have to spend per day?

Answer: Spending one million dollars in a year would require spending about $2,739.73 per day.

Question: This means that to spend one billion dollars in a year, you'd need to spend how much per day?

Answer: Spending one billion dollars in a year would require spending about $2,739,726.03 per day.

Question: The United States of America's National Debt as of July 20, 2009 at 5 p.m. Eastern Standard Time was $11,612,146,250,911. Say this number out loud.

Answer: Eleven trillion, six hundred-twelve billion, one hundred forty-six million, two hundred fifty thousand, nine hundred eleven dollars.

Note (for the teacher): It is important to have students practice saying very large (and very small) numbers, since this helps them develop a feeling for quantity and the relationships between quantitative magnitudes. Note that they state them correctly. One common mistake occurs by saying just the digits, for example: "eleven…six one two…one four six…." Another common mistake involves adding the word **and** incorrectly (for example) "six hundred **and** twelve billion."

Note (read to the class): This information can be found at brillig.com/debt_clock/.
Keep in mind the National Debt isn't how much we are spending as a country…this is how much *more* we are spending as a country than we are taking in through taxes, fines, and tariffs. In 2008, our government collected 2.5 trillion dollars…which, if you spent just that, you'd have to spend almost $80,000 per second, or over one million dollars in the time it probably took you to read these two notes out loud!

Question: If we paid back one billion dollars on this National Debt each day *without adding more debt and without paying any interest,* how long would it take to pay it off?

Answer: It would be paid off somewhere around September 16, 2041.

Question: To pay back one billion dollars a day, how much do we need to pay back per second?

Answer: Paying back one billion dollars a day would mean paying back $11,574 per second.

Question: The Gross Domestic Product (GDP) of the state of New Hampshire is approximately 50 billion dollars per year. If all New Hampshire people and businesses donated everything they ever earned to paying off the national debt under the same circumstances described above, how many years would it take?

Answer: It would take over 232 years.

Million: 1,000,000	Quintillion: 1,000,000,000,000,000,000
Billion: 1,000,000,000	Sextillion: 1,000,000,000,000,000,000,000
Trillion: 1,000,000,000,000	Nonillion: 1,000,000,000,000,000,000,000,000,000,000

Appendix U includes sample problems from the 2007 NECAP exams to exemplify the strong connection between the GLEs and GSEs and numeracy instruction. These problems show how teachers are perhaps already using problems that can be expanded to teach quantitative literacy as well. These problems also incorporate many of the important components of numeracy instruction. Similar problems are also included in **Sections VI D through VI G**.

G. Guiding Questions

Below is a collection of guiding questions to be used for various audiences including administrators, numeracy/instructional leadership team members, teachers, parents, families, community organizations, and institutions of higher education. This collection is by no means exhaustive; however, it does include many questions to help one begin reflecting and thinking about his or her numeracy instruction and teaching practice. It also provides questions to facilitate thinking about a school or district's goals, curriculum, teaching practices, and assessment. These questions were compiled throughout the writing of this action plan. Also, the ideas for many

questions came from two sources: the *New Hampshire PreK–16 Literacy Action Plan for the 21st Century* (2007) and *Strategies for Numeracy Across the Curriculum* (High Schools That Work, Southwest Ohio, 2007). In addition, *Strategies for Numeracy Across the Curriculum* also includes many valuable resources to help guide the development and implementation of an action plan
(retrieved 11/18/09 from
www.hstwohioregions.org/sitefiles/Strategies%20for%20Numeracy%20Across%20the%20Curriculum1.ppt).

For the principal, superintendent, or numeracy/instructional leadership team:

- What evidence do you have that teachers are modeling problem-solving strategies?
- How does the school and the faculty address varying stages of numeracy skills development?
- Are extended learning opportunities available?
- How do you assist and encourage teachers to collaborate so that quantitative literacy can be truly cross-disciplinary?
- What home and community influences affect literacy achievement in your school or district?
- Do you have a Quantitative Literacy or Instructional Leadership team in place? If so, who are the members and what are the team's goals? If not, who would be the members and what should its goals be?
- What are the challenges your school or district faces in trying to implement a numeracy action plan? How could you overcome these obstacles?
- Do you have extended learning opportunities available for students? If so, how are these opportunities linked to the curriculum and to developing numeracy skills?
- As you begin to implement a quantitative literacy program in your school or district, what are some "excuses" that faculty members or colleagues may offer? How can you respond?
- What organizational obstacles may impede implementing a quantitative literacy program in your school or district? How can these obstacles be mitigated or eliminated?
- What additional research could you provide to support adopting a quantitative literacy action plan?
- What are your school's or district's goals with respect to quantitative literacy?
- What data and statistics can you use to measure whether your students are developing critical numeracy skills? Whether your school or district is meeting your goals?
- What resources do you need to implement a quantitative literacy program in your school or district?
- How do you ensure that your quantitative literacy program is cross-disciplinary and not just the responsibility of the mathematics department?

At a school's faculty meeting or at a district's meeting:

- What are the shared beliefs about numeracy in your school or district and how are they communicated?
- What steps has your school or district taken to make connections between the New Hampshire GLEs and GSEs and your district's curriculum? What still needs to be done?
- What guidance does your school or district provide all faculty and staff in selecting appropriate and varied activities and manipulatives to meet the needs of all learners?
- Do your teachers have the time and resources they need to incorporate quantitative literacy topics into their daily lessons?
- Are teachers encouraged to collaborate?
- What support systems are in place to help struggling students?
- Are there extended learning opportunities? Are parents and teachers aware of them?
- What are your shared numeracy goals? How can you achieve them?
- What assessment data do you have? What does it tell you about your students' quantitative literacy skills? How will you adjust your instruction and your goals given this data?

For classroom teachers:

- Are you a good numeracy role model?
- Do you make numeracy a part of daily instruction?
- Do you provide time for students to make connections between numeracy and the lesson's objectives?
- Do you incorporate logical reasoning and problem-solving daily, as related to your content?
- Are you able to provide resources, such as calculators and rulers, to enhance the students' experience in making quantitative connections?
- Do you share articles with your students that show how mathematics is used in your field as well as in other fields?
- Do you encourage students to incorporate data collection and analysis as part of their class projects?
- Do you encourage multiple representations?
- Do you help guide students through cross- and inter-disciplinary connections?
- Do you read professional articles about incorporating numeracy into your content area?
- Do you avoid sharing your dislike of mathematics?
- What one change could you make toward embedding problem-solving strategies into your daily instruction?
- What one change could you make toward encouraging your students to discuss real-world problems and to communicate their thinking?
- How do you address varying stages of numeracy skills development?
- Are there opportunities to collaborate with other teachers so that quantitative literacy can be truly cross-disciplinary? Do you take advantage of these opportunities? If not, why?
- How do you motivate students to enjoy mathematics? How do you motivate students to not avoid mathematics?
- How do you motivate and encourage students to develop and use "Habits of Mind"?
- How do you motivate and encourage students to not be afraid of problem solving? To be creative in their problem-solving?
- How do you motivate and encourage students to communicate mathematically?
- Are there shared expectations for student work and achievement across grades and classrooms? Are these expectations communicated to the faculty, students, and parents?
- Is your time organized to maximize student achievement?
- How do you collect and use data regarding your students' numeracy skills and performance?
- Do your students know what their individual learning goals are?
- What support systems are in place for struggling students? Do you know how to refer a student to these support systems?
- Do you consider your school to be a professional learning community? If so, why? If not, why? What would contribute to its becoming a professional learning community?
- Have your numeracy instructional practices been assessed? Have you had the opportunity to receive feedback on your teaching practices?
- Have you been offered professional development opportunities with respect to teaching quantitative literacy?
- Do you have access to the technology necessary to support numeracy instruction? If not, what is missing?
- How will you know if your students are developing quantitative literacy skills? Can you identify both formal and informal assessments?
- What are the challenges you face in trying to incorporate numeracy into your daily lessons? How could you overcome these obstacles?

- Do you remember a teacher who did a particularly good or poor job incorporating real-world, contextual problems into their class?
- What are common quantitative or mathematical misconceptions in your content area? How can you use these misconceptions in your teaching strategy?
- What steps can you take to remedy these misconceptions?
- Are you reinforcing any misconceptions?
- Are there any areas of mathematics that you are not confident in teaching? If so, how can you become more confident?
- Can you think of a topic in your content area where multiple representations would increase understanding?
- Is there a topic in your content area that involves very large or very small numbers? How can you help students understand the magnitude and significance of these numbers? How would this help students understand the concept better?
- Are there topics in your content area that involve proportional reasoning? Are there topics that would be enhanced by including proportional reasoning?
- Do you ever need to perform calculations in your classroom? In your content area? In your daily life? Do you perform these calculations mentally, on paper, with a calculator? Do you share your calculations and thought processes with your students?
- Are you ever faced with an open-ended problem during class or within your content area?
- How can you encourage problem-solving?
- Are mathematical topics in your content area addressed from a traditional mathematics perspective or from a quantitative literacy perspective?
- Which topics lead to higher-order thinking? How can you extend problems to include more higher-order thinking?
- What resources and support do you need to be able to teach from a quantitatively literate perspective?

Following a quantitative literacy-focused professional development:

- How has your understanding of numeracy expanded?
- How can you select appropriate and varied activities and manipulatives to meet the needs of all learners? Where can you find these activities and manipulatives?
- List characteristics of a quantitatively literate person.
- List characteristics of innumeracy.
- How can you incorporate quantitative literacy into your daily lessons?
- How can you assess your students' progress toward becoming numerate?
- How can you collaborate with teachers in other disciplines?
- What if you are not comfortable with your own quantitative literacy skills?

For personal reflection:

- List characteristics of a quantitatively literate person.
- List characteristics of innumeracy.
- List some of your own experiences with quantitative literacy or illiteracy.
- Are you quantitatively literate? Why or why not?
- Do you love math? Do you hate math? How do your feelings affect your teaching?
- Are you a good numeracy role model?
- Are your students aware of your dislike of math? How might this be affecting their learning and your teaching?

- Are you comfortable or uncomfortable with your own quantitative literacy skills? Do you shy aware from quantitative topics in your own discipline?
- How can you become more confident when dealing with quantitative topics?

For parents, families, and community organizations:

- What home and community influences affect literacy achievement in your school or district?
- Are the numeracy goals of your school or district learning clearly communicated? If so, how? If not, what information do you feel is missing?
- What strategies are in place to facilitate extended learning partnerships between schools, parents, and the community? Have mutually beneficial partnerships been formed with other area schools, libraries, community programs, and local colleges and universities?
- Are there support systems for struggling students?
- Are you aware of school- and district-wide assessments? How did your school or district perform? Are you aware of how the school or district is adjusting its goals and plans given that data?
- How can you help develop students' numeracy skills?
- What resources are available to you to help you become more quantitatively literate?

For institutions of higher education:

- Do you require that students take a quantitative reasoning or literacy class?
- Is there a quantitative reasoning or literacy requirement? Is it a single class or a program of classes?
- What quantitative skills are tested on both entrance and placement exams?
- What quantitative skills does the institution believe are important for its graduates to have? Do the required courses work toward ensuring that students will develop these skills?
- Does the institution cite numeracy as an institution-wide, cross-disciplinary goal? If so, how is it working to fulfill this goal?
- What is your impression of the skills of students entering your institution? After their second year? Upon graduation?
- Do you have any relationships or partnerships with elementary or primary schools? Do you communicate about quantitative literacy goals?
- Is there a quantitative component or requirement to your teacher training programs?
- How are new teachers prepared to address numeracy in their classrooms?
- Are any members of your institution currently involved in quantitative literacy instruction?
- Are there any obstacles to incorporating more numeracy-focused classes?

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VI. QUANTITATIVE LITERACY FOR PREK–16

A. General Goals for Numeracy Instruction PreK–16

Literacy has been one of the preeminent goals of education for decades. Without being literate, a person cannot fully participate in society nor acquire additional knowledge through reading. Over the past decade or so, numeracy has been slowly following the same path. Without being quantitatively literate, one's ability to fully understand, interpret, and utilize everyday data is hindered. As stated by the National Research Council (2001):

> To function in today's society, mathematical literacy—what the British call "numeracy"—is as essential as verbal literacy. These two kinds of literacy, although different, are not unrelated. Without the ability to read and understand, no one can become mathematically literate. Increasingly, the reverse is also true: without the ability to understand basic mathematical ideas, one cannot fully comprehend modern writing such as that which appears in the daily newspapers (p. 7).

Therefore, a school system and a society have a vested interest in helping students develop strong literacy and numeracy skills. This overarching goal is necessary if people are going to be able to comprehend the world around them.

As most of the current research and curriculum involving quantitative literacy is at the post-secondary or adult education level, clearly defined goals for numeracy instruction in the primary and secondary levels are not readily available. However, given the importance of quantitative literacy (outlined in **Section III C**), numeracy instruction should work toward fulfilling several general goals. These goals[9] include:

- Quantitative literacy should be a priority for all mathematics educators.
- Teachers should focus on having their students understand concepts instead of rote memorization of facts and procedures.
- Students should develop a comfortableness and proficiency with quantitative data.
- Students should develop "Habits of Mind" related to numeracy.
- Students should develop real-world, contextual problem-solving where they are not afraid of the unknown.

<u>Quantitative Literacy Should Be a Priority for All Mathematics Educators</u>
Many mathematics education programs and school districts are beginning to embrace the importance of teaching numeracy as a necessary part of any curriculum. The New Hampshire Curriculum Frameworks (2006) clearly show the state's emphasis on developing numeracy skills within its math programs (see **Section V G**).

<u>Understanding Concepts Instead of Rote Memorization of Facts and Procedures</u>
For many years, being "good" at math entailed being able to memorize formulas and compute quickly. But, those skills have become less important with the advent and prevalence of computers and calculators. Mathematics instruction needs must now focus on developing skills and abilities that cannot be performed by a calculator. These skills include, but are not limited to, data analysis, communication and discussion, critical thinking, problem-solving, and data representation. Many students feel that having a calculator empowers them mathematically, and

[9]These goals were developed by the Numeracy Action Plan Writing Team. They are similar to the five strands of mathematical proficiency described in Adding It Up: Helping Children Learn Mathematics (Center for Education, National Research Council, 2001). These strands are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition.

some even attempt to use their calculator to hide their weak mathematical ability or phobia. However, a calculator is only as effective as its operator and in order for a calculator to produce the correct answers, one needs to have a deep understanding of the concepts.

In addition, some students incorrectly believe that they are "bad" at math if they cannot quickly calculate an answer or if they have difficulty memorizing formulas. **Students and teachers should not confuse arithmetic with quantitative literacy instruction**. By removing the pressure to simply get an answer, teachers can help students recognize the true purpose and value of quantitative reasoning. This helps build student confidence. Therefore, a goal of numeracy instruction must be for students to develop a deep understanding of concepts so they are able to use technology correctly and employ critical thinking skills confidently. Simply memorizing facts, definitions, and formulas will not create knowledge that is transferable to understanding future topics.

Developing a Comfortableness and Proficiency with Quantitative Data

An important goal of numeracy instruction is for students to become more comfortable when faced with quantitative data. The United Kingdom's Department of Children, Schools and Families outlines a framework for teaching mathematics: years 7, 8, and 9 (retrieved April 26, 2009 nationalstrategies.standards.dcsf.gov.uk/node/42765):

> Numeracy is a proficiency which is developed mainly in mathematics but also in other subjects. It is more than an ability to do basic arithmetic. It involves developing confidence and competence with numbers and measures. It requires understanding of the number system, a repertoire of mathematical techniques, and an inclination and ability to solve quantitative or spatial problems in a range of contexts. Numeracy also demands understanding of the ways in which data are gathered by counting and measuring, and presented in graphs, diagrams, charts and tables.

As with any activity (e.g., waterskiing, oil painting, baking, or checkbook balancing) people tend to spend their time enjoying activities they are good at; thereby, they are likely to improve in those activities. Conversely, human nature compels people to avoid activities in which they do not excel. As a result, children and adults alike tend to shy away from quantitative reasoning because they are not comfortable with numbers and their ability to reason using quantitative data. **In a worst-case scenario, some people are so uncomfortable with numbers that they become frozen with fear**. Assuming that "practice makes perfect," fostering a comfortableness with quantitative data is a crucial goal of numeracy instruction so that students will not avoid quantitative reasoning and problem-solving.

Developing "Habits of Mind"

Along with the goals of developing conceptual understanding and a comfortableness with numbers, numeracy instruction also needs to promote the "Habits of Mind" (**Section III F** and **Appendices E and F**). Both explicitly and implicitly, the "Habits of Mind" include many behaviors that a quantitatively literate person routinely practices. In addition, by possessing and using these behaviors, students will become more comfortable with numbers and problem-solving. They will therefore incorporate quantitative analysis into their everyday activities. It is the goal of numeracy instruction to help students develop these skills and use them in both their academic and non-academic approach to quantitative reasoning and problem-solving. Habits that teachers should help students develop are persistence, curiosity, logical thinking skills, estimation, and confidence.

Developing Real-World, Contextual Problem-Solving

Much of traditional mathematics instruction is abstract and non-contextual. Students have difficulty grasping mathematical concepts out-of-context, and they fail to see the connection of those concepts to their lives and to other topics. Their ability to comprehend and solve problems is greatly increased if problems are in context and applicable. In addition, **true quantitative literacy hinges on the ability to analyze and interpret real-world problems so that informed decisions can be made**: the "[a]bility to function in the global economy requires that numeracy instruction be linked to concrete social situations as opposed to abstract rote learning. . . .Students should learn that numbers exist because of a social need" (Spring, 2000, p. 105). Ginsburg (2008) also addresses how the problem-solving processes can aid numeracy acquisition:

> In class, solve real problems that may be complex and messy and worry less about getting a "correct" solution than about the problem solving processes used. Talk about alternative strategies, the choices that could be made, and the benefits and drawbacks of each. . . . People need to learn how to make numerical decisions when the numbers don't come out even, and even when all the information is not clearly defined. Being a flexible problem solver means knowing what to try when the obvious doesn't work or seems to lead to a dead end (p. 18).

Although Ginsburg also discusses communication and perseverance, both critical traits of a numerate person, she feels that numeracy instruction requires real-life, open-ended, contextual problems.

Being quantitatively literate is now required to keep academic and professional doors open; thus, schools and teachers have a responsibility to teach these skills. In order for students to develop the appropriate skills, an effective numeracy program should work toward fulfilling the goals for mathematics education.

B. Essential Components of Numeracy Instruction PreK–16

For numeracy instruction to fulfill its goals, several essential components should be addressed. These essential components[10] are:

- Numeracy instruction is not the same as mathematics instruction
- Numeracy instruction is interdisciplinary
- Students should be encouraged to communicate about problem-solving methods and data analysis
- Teachers should teach several skills and concepts in unison, not succession

Numeracy is infused in everyday problems and activities, regardless of complexity or discipline. Students of all levels face situations, both in and out of the classroom, that require strong quantitative literacy skills. As long as the content and the teaching method are age-appropriate, most characteristics of "good" quantitative literacy instruction are the same regardless of the grade level being taught.

Numeracy Instruction and Mathematics Instruction are not the Same

Although there has been a recent movement toward bridging the goals of math instruction with those of numeracy instruction, significant distinctions remain. Many people, including teachers, incorrectly believe that by learning mathematics, one will also develop numeracy. But this is indeed not the case: "Numeracy is not the same as mathematics, nor is it an alternative to mathematics. Rather, it is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society" (Steen, 2001, p. 115). Distinct differences exist

[10]These components were outlined by the Numeracy Action Plan Writing Team; however, they are very similar to components that were also outlined by Lynda Ginsburg (2008) retrieved August 1, 2009 www.ncsall.net/fileadmin/resources/fob/2008/fob_9a.pdf#page=14. Ginsburg and her colleagues also cite three major components of numeracy: context, content, and cognitive processes (Ginsburg, Manly, Schmitt, 2006).

between "numeracy" and "mathematics," yet they are definitely linked. **Whereas math instruction tends to focus on abstract, non-contextual procedures, numeracy instruction involves applying mathematical concepts in a contextual, applied setting**. However, to solve any contextual problem, one needs an understanding of mathematical content and procedures. "Numeracy is not so much about understanding abstract concepts as about applying elementary tools in sophisticated settings" (Steen, 2001, p. 108). These "elementary tools" are acquired through mathematics instruction, whereby the application of these tools is a prominent goal of numeracy instruction.

The acquisition of quantitative literacy skills is inextricably linked to the acquisition of math skills and the connection between the two needs to be explicit: "Numeracy takes years of study and experience to achieve. Thus numeracy and mathematics should be complementary aspects of the school curriculum. Both are necessary for life and work, and each strengthens the other. But they are not the same subject" (Steen, 2001, p. 108).

Teachers of all disciplines should be aware of the connection between both mathematics and numeracy in order for them to more effectively balance the two. Yet they need to be careful not to believe that teaching mathematics is teaching numeracy[11].

Numeracy Instruction is Interdisciplinary

As discussed in **Section III**, quantitative literacy is necessary to lead a complete, full, and informed life. Furthermore, quantitative literacy is not limited to mathematical and scientific fields—being able to accurately interpret and analyze data is required in all disciplines, professions, and pastimes. Unlike mathematics instruction, numeracy instruction needs to be interdisciplinary in scope. It must incorporate contextual problems from various topics and be integrated into other curricula, not just within math classrooms. "Whereas mathematics is a well-established discipline, numeracy is necessarily interdisciplinary. Like writing, numeracy must permeate the curriculum. When it does, also like writing, it will enhance students' understanding of all subjects and their capacity to lead informed lives" (Steen, 2001, p. 115). Although Steen compares numeracy to writing, it can also be compared more broadly to literacy. Literacy is considered the interdisciplinary component to reading and writing, while numeracy can be seen as the interdisciplinary component to mathematics. Steen (1990) goes even a step further in this comparison, "[n]umeracy is to mathematics as literacy is to language" (pp. 211–231).

Spring (2000) describes the numeracy-literacy connection when discussing the Malaysian female factory workers' plight in a male-dominated culture and economy. He argues that in order for economic and social change to occur, people need to be both literate and quantitatively literate: "As I have suggested, literacy should prepare people for the dynamics of cultural change. The teaching of numeracy should be directed toward that same goal" (Spring, p. 105). Thus, **if a goal of education is to prepare students to be agents of change and to contribute to society, numeracy must be taught in addition to literacy in all disciplines.**

Communication about Problem-Solving Methods and Data Analysis

Another essential component of numeracy instruction is to promote student analysis and communication:

> Make certain that learners can talk about the meaning of what they are doing: what, why, and how. In explaining their thinking to someone else, whether to another learner or a teacher, people have to really understand what they are doing" (Ginsburg, 2008, p. 18).

[11]The distinction between math instruction and numeracy instruction is become increasingly muddied as math curriculums are placing greater emphasis on numeracy topics and pedagogy.

Students should be required to explain and defend their solutions verbally, pictorially, or mathematically. In doing so, they will have the opportunity to self-assess. They also need to be given various opportunities to communicate mathematically and to use a variety of representations, such as graphs, tables, charts, or equations. They should be able to read, interpret, and critique numerical and statistical data presented in a variety of formats. Teachers can ask students to read newspaper articles to find quantitative data, or students can be asked to defend an argument using statistical information.

Teach Several Skills in Unison, Not Succession

Although the study of math and numeracy is developmental and sequential, teachers should integrate several topics whenever possible, rather than teach topics one by one. Ginsburg (2008) writes about a "risky" numeracy instruction practice: "primarily dividing math content into distinct, non-overlapping topics" (p. 17). For example, fractions, ratios, decimals, and percents can be taught together, as different forms of the same set of numbers, the rational numbers. Also, graphs, equations, and tables can all be explored at the same time when students are learning about linear relationships. By integrating topics, students will have the opportunity to see the interconnectedness of topics and be able to transfer this knowledge:

> [M]any teachers and most workbooks address each topic in isolation, as if one had pretty much nothing to do with the others. Learners are somehow expected to make connections amongst them independently. . . .[m]uch evidence indicates that assumptions established and reinforced when studying one narrow topic are easy to apply but difficult to modify when studying another topic (Ginsburg, 2008, p. 17).

As students are learning a new topic, teachers can revisit prior knowledge to help students make connections between what they already know and what they are in the process of learning. Teachers can then build upon their new knowledge to solve more complex problems that require the synthesis of many skills and procedures.

When facing more complex problems, students often struggle because they fail to realize that a solution may require more than one step and mathematical concept. Students often use whatever concept they have most recently learned, and they do not recognize that true problem-solving is often a multi-step and multi-layered process. By including multi-step problems, teachers can help students break the habit of believing they have completed a problem.

Furthermore, not all skills being taught are necessarily math content-specific. Some skills that can be taught in unison are valuable for any content area and these skills are particularly important when developing numeracy. **The real world is not simple, and, by definition, real-world, contextual problems are not simple: They incorporate many skills and procedures**. For example, these problems encourage inquiry and discovery and they require student to face the unknown. Most often, the solution or path to the solution is not readily apparent. Thus, the student will need to devise a plan to solve the problem, perform the required functions, find a solution, and then reflect on whether the solution makes sense or is reasonable. Students should consider their assumptions and how these assumptions affect their methods or answer. In a particularly strong quantitative reasoning problem, there may be many ways to arrive at the same solution or a problem might have more than one correct answer. This is especially true when students interpret the problem differently or make different assumptions. Many of these skills reflect the "Habits of Mind" (**Section III F** and **Appendices E and F**).

C. Specific Ways Teachers Can Promote Numeracy Skills

In order for students to develop their quantitative reasoning skills, teachers need to be aware of how students think, including common misunderstandings and misconceptions, and determine how to help them. Various teaching techniques, together with contextual problems, can be employed to aid this process. Student interest may be achieved by using examples that are reasonable, relevant, thought-provoking, and which simulate real-world situations. To further assist students, teachers should consider problem-solving strategies, assessment, questioning, and other means of discourse. Teachers can also help students by differentiating instruction or assessment where appropriate. Strategies that might be helpful in developing numeracy and quantitative reasoning are to develop problem-solving skills; promote discourse and communication; encourage "Habits of Mind"; use technology; formatively assess; acquire thorough curriculum, content, and pedagogical knowledge; and conceptualize specific real-world topics (e.g., time and money).

Developing Problem-Solving Strategies

The ability to understand and solve open-ended problems is a fundamental skill that should be paramount in the mathematics instruction of all students. The NCTM's *Principles and Standards for School Mathematics* (2000) states that the heart of mathematics is problem solving. Assuming this premise, every mathematics teacher must develop and master his or her own problem-solving techniques in order to teach students to do the same. **Teachers at all levels must work to move students from basic problem-solving strategies, such as "guess and check," to more advanced strategies appropriate to their level of instruction**. In an ever-changing world, students must be prepared to be competitive in a variety of disciplines, not just math-specific ones. Understanding problem solving is, perhaps, the best way for teachers to increase their students' capabilities and competitiveness. By developing a comfort level with the unknown and the subsequent problem-solving required, students will expand their quantitative reasoning skills. As a result, they will be undaunted by difficult problems for which they do not see clear solutions and they will be prepared to attempt different ways to solve them.

A very effective way to approach problem-solving is to use real-world, contextual problems. Children learn best when they are able to make connections between their lives and their classroom mathematics. Teachers and parents can highlight problem-solving skills by modeling solutions to daily problems and by involving students in the process. By employing multiple methods to solve problems, teachers can foster student acceptance of these processes. Using instructional strategies, such as sharing a problem-solving thought process aloud with students can encourage these skills. This process provides the student with the mathematical language, an insight into the thought process used to arrive at a solution, and concrete examples to help encourage connections to abstract concepts.

Teachers can encourage problem-solving and discussion by incorporating mathematical modeling. Mathematical models, sometimes called equations, simulate real-life situations that can then be extended to predict future outcomes or behaviors (i.e., how many people in San Francisco would perish in a 8.0 magnitude earthquake?). Students can learn to be critical of the assumptions of a given model, or they can consider the model's limitations in predicting actual outcomes. Depending on the level of the students and the objective of the lesson, teachers may place more or less emphasis on the mathematics used to develop the model. Through the use of technology, students and teachers can focus more on their predictions and results rather than the calculations. However, to use models effectively for the purpose of developing numeracy, a teacher needs to be armed with appropriate questions about the model, its outcome, and how the model is developed mathematically.

Teachers can also help students further develop their problem-solving skills by extending problems. Extending problems involves presenting a problem, discussing it, adding to the problem to make it more complex or complicated, and then discussing how the problems and its solutions change as a result. This form of differentiation can lead to a rich discussion concerning the effects different input values or assumptions can have on an outcome. Also, the teacher will have the opportunity to assess the student's quantitative reasoning and communication skills through this exercise. The NCTM states that teaching in and of itself, is a problem-solving activity; and teachers must continually reflect on their practice to improve student learning.

Appendices L and M illustrate cognitive and metacognitive behaviors for non-routine problem-solving. Teachers may find this information helpful as they work toward understanding their students' thought processes and helping them develop greater problem-solving ability.

Promoting Discourse

One crucial component of developing a mathematical community that promotes quantitative literacy is to encourage student discourse. Vygotsky (1962) states that "speech is an expression of the process of becoming aware" (p. 16). For students to "become aware" of mathematical concepts, there must be ample and frequent opportunities to discuss their thinking. This discourse should occur among students as well as between teacher and students. Children will often look to teachers for the correct answers; therefore, it is important for teachers to avoid becoming the source of right and wrong during discussions. When asked if a solution is correct, the following teacher responses will help students take more ownership of their answers: "How can you decide?"; "Why do you think that might be right?"; "I see what you have done. How can you check that somehow?" (Van de Walle, 2007, p. 46).

Questioning one's answers is a central component of mathematical discourse. **The questioning process is integral to developing quantitative literacy as it helps increase the comfort level with numbers, the ability to communicate mathematical thinking, and self-assessment**. The questioning process is a connected one that begins by initiating language, then includes thinking and understanding, and ends with long-lasting learning. To facilitate more meaningful classroom interactions, key pedagogical techniques can be employed including using open-ended questioning techniques, encouraging students to explain how they arrived at solutions (have students explain both correct and incorrect solutions), and developing a repertoire of questions and using them frequently.

When students respond to open-ended questions, they tend to demonstrate more about their mathematical thinking; this becomes a window to their thinking. Consider the following example: a teacher asks: "How could you add thirty-seven cents and fifty-six cents?"

Students suggest several different strategies for doing so, including using a number grid; getting out real coins, making those two amounts, and putting them together; and adding the dimes first and then the pennies. As students describe their methods, the teacher asks clarifying questions, such as "Why would you start there on the number grid?" or "How did you arrive at 8 dimes[12]?" If the teacher had simply written the problem *37 + 56 = ?* on the board, in a vertical fashion, students would not have been as encouraged to use diverse methods and some may not have been able to solve the problem all together.

When students are asked to explain how they arrived at their solution, they reexamine the process they used to arrive at their answer. In so doing, they either validate their correct mathematical thinking, or they discover where they went wrong and self-correct. **Self-correction is very powerful and often leads to concept retention**. Through these processes, students receive a clear message that mathematics is a process of thinking and reworking as opposed to just finding an answer and moving on. This also helps to create a mathematical community of learners who value the process of learning and discussing mathematics.

[12] A student would arrive at 8 dimes if he or she added the 3 dimes from 37 cents and the 5 dimes from 56 cents.

To encourage discussion and critical thinking, most mathematics curricula, such as Everyday Mathematics and Investigations, include questions within each lesson to engage student thinking. In addition, teachers should also have a list of five or six general questions to ask repeatedly during mathematical dialogues. One source for such questions is the *Professional Standards for Teaching Mathematics* (NCTM, 1991). Examples are:

- How did you arrive at your answer?
- Have you solved a similar problem that might help with solving this one?
- Does anyone have the same answer but solved it a different way?
- Can you show or tell me another way to solve this problem?
- What strategy did you use to solve this problem?
- What would happen if we changed this part of the problem?

Posting a list of general questions in the classroom helps students to realize that they will be asked these questions every day during mathematics lessons. They will develop a familiarity with them and may even begin to ask themselves the same types of questions as they work to solve a problem on their own. One eventual outcome that helps to produce student-to-student discourse is that even young students begin to ask each other these questions. This interchange of ideas is an important component of developing qualitative literacy in students.

During the course of instruction, teachers can help students develop quantitative reasoning by paying close attention to their questioning type, technique, and style. The Center for Teaching Excellence cites many helpful suggestions regarding effective questioning. First, teachers must carefully plan their questions ahead of time to ensure questions are phrased well and are unambiguous, are in a logical sequence, and evoke the desired thought process from the students. With regard to developing quantitative skills, teachers should determine the lessons' content and procedural goal and construct questions to achieve that goal. Questions should be open-ended, provide for flexibility in student responses, and anticipate common misconceptions. Questions should not be the "yes or no" variety, and should be such that students wonder what the teacher is really asking or feel that they need to guess what the teacher is thinking. Teachers should also focus on asking questions that require higher cognitive demands that require students to make connections mathematically, rather than just supplied previously memorized content or procedures. Lastly, when preparing questions, teachers should also consider how they will handle and respond to incorrect responses.

Once the questions are prepared, teachers can help students to become quantitatively literate through their questioning delivery and style. Teachers should not provide leading introductions to questions, such as, "Don't you think the longest side of a right triangle is the hypotenuse?" Teachers should also avoid beginning a question by calling a student's name. Once a student has been singled out, the rest of the class usually feels that they can tune out and not listen to the question. To improve questioning effectiveness, teachers can call on students once the question has been asked in order to keep all students prepared to answer. This should only be done after sufficient wait time has been provided so that all students can have the time to think and devise an answer or reason. Finally, students are often motivated by their teacher's responses which can include reinforcement, further probing, adjustment, refinement, and rephrasing.

The question preparing process can be used to provide insight for planning future lessons, as questioning is a type of formative assessment. As students work through mathematical problems and share their methods, the teacher can make a number of observations. These include, but are not limited to:

- Do students demonstrate a clear understanding of the concept?
- Are students using multiple methods for solving problems?
- Do students use appropriate mathematics vocabulary in their responses?

- What level of understanding do students demonstrate (i.e., beginning, developing or mastery)?
- Are students able to find an answer independently?

In addition to questioning and discussion, teachers can encourage students to write about mathematics. "Writing Across the Curriculum" has become ubiquitous in professional development workshops of late, yet many math teachers continue to struggle to find ways to incorporate writing into their lessons. Teachers can use John Collins Type 1 and Type 2 problems to help facilitate vocabulary acquisition (Developing Writing and Thinking Skills Across the Curriculum: A Practical Program for Schools, 1996). Type 1 problems involve brainstorming. In a Type 1 problem, a teacher might ask students to break into small groups and list everything they know about a certain topic, such as parallelograms. Then, as the groups share their results with the class, they can compare their knowledge, thus leading to further discussion. Type 2 problems involve a short writing prompt, or a pop-quiz. For example, a teacher can ask his or her class to explain each component of the Pythagorean Theorem in written form. Another possible Type 2 activity could be asking students to generate their own problem, given certain parameters. By requiring students to write in complete sentences, it is relatively easy to add writing into a daily math lesson. Moreover, these activities can lead to increased language and vocabulary skills as well as the ability to write about number sense.

Developing a community of learners who will take risks as they learn mathematics is an important component of a quantitatively literate environment. It is a process during which teachers learn to listen to students, allowing them to struggle at appropriate junctions as they discover the excitement of sharing their thinking with others. If the goal is to create quantitatively literate students, the emphasis is taken *off* of telling students how to solve problems and placed *on* how to communicate their thinking and reasoning.

Encouraging the "Habits of Mind"

"A 'Habit of Mind' is knowing how to behave intelligently when you don't know the answer" (retrieved March 10, 2009 from www.habits-of-mind.net/). Mathematics frequently involves problems to which one does not know the answers; therefore, employing the "Habits of Mind" is critical to developing strong quantitative skills. Many students simply give up when an answer is not easily apparent; however, a student utilizing the "Habits of Mind" is likely to persevere, which is a key step to becoming a good problem-solver. A goal must be for each student to progress through school with the "Habits of Mind" that will allow the student to solve mathematics problems encountered in everyday life. These behaviors must become habits, a routine followed almost involuntarily, if students can be expected to apply these habits beyond the classroom.

If they are to help their students embody the "Habits of Mind," teachers should display mastery in the area as well. Teachers should lead and teach by example, including perseverance in problem-solving. Developing and understanding these "Habits of Mind" must become a primary focal point of pre-service training. This end cannot be served through a simple lecture or project; instead, a teacher certification program must embed the development of these behaviors in every facet of the training curriculum. It should be the goal of every instructor and the means by which every student comes to make intelligent decisions when looking for solutions to complex mathematical problems.

The "Habits of Mind" are also discussed in **Section III F** and in **Appendices E and F**.

Using Technology

Technology is becoming ever more prominent and important in mathematics curricula. Used properly, technology can be a valuable tool to help students develop their quantitative thinking. For example, spreadsheets can be used to quickly calculate a stream of interest payments. Students can then spend the bulk of class time analyzing, discussing, and making predictions based on these payments. The use of spreadsheets also encourages students to move away from focusing on the rote procedures and calculations and toward analyzing and discussing the results. However, in order to be quantitatively beneficial, teachers must know the proper way to use spreadsheet technology. If a student is simply asked to calculate the interest payment in the fifth year, they are only learning how to plug in data, set up an equation, and read the output. The teacher must guide the student with thought-provoking questions regarding the output.

Graphing calculators help students arrive at answers quickly so they can focus on the analysis instead of the procedures. Teachers should be cautious when using graphing calculators to ensure the desired outcome is more than just getting an answer. Students should use their results to think quantitatively, answer questions, and make predictions regarding their answer. Additionally, any graphing software (e.g., graphing calculators or dynamic graphing software) makes efficient use of time: students are able to graph a complex equation in seconds and then move onto developing their quantitative literacy by discussing aspects of the graph. Furthermore, teachers can use technology to differentiate instruction or assessment by using technology to make problems easier or more difficult. In general, students will not develop quantitative reasoning if class time focuses on procedures related to graphing or if the desired outcome is merely answer-driven.

Some teachers and parents are reluctant to allow students to use calculators, despite their usefulness, worrying that a reliance on calculators will impede students' ability to think quantitatively. However, research indicates that using calculators can actually enhance a student's numeracy acquisition (Demana & Waits, 1990; Kelly, 1985; Moschkovich & Schenfeld, 1993). Students who use calculators have higher math achievement, perform better on mental computations, have more positive attitudes, do not rely too heavily on their calculator, and experience a greater variety of concepts (Sutton & Krueger, 2002).

Formative Assessment

The National Mathematics Advisory Panel (2008) reports that formative assessment (i.e., the ongoing monitoring of students learning to inform instruction) is generally considered a hallmark of effective instruction in any discipline. Formative assessment can be used by teachers to shape and guide their instructional decisions and strategies to help students develop quantitative literacy skills: "Assessment needs to be a moving picture—a video stream rather than a periodic snapshot" (Heritage, 2007).

Teachers make decisions about the content or form of instruction using formative assessments. These assessments are conducted during learning to promote, not merely to judge or to grade, student success. It is a systematic process to continuously gather evidence and data on learning. Some of the data that can be gathered include information about a student's content knowledge: what they know and are able to do; mathematical disposition: confidence, response to challenges, and metacognition; and work habits (i.e., perseverance, contributions to group tasks, organization, and ability to work independently).

The data is used to identify a student's current level of learning and to adapt lessons to help the student reach the desired learning goal (Heritage, 2007). Teachers and students can use assessment-based feedback to make adjustments that will improve students' achievement of intended curricular aims.

There are countless ways a teacher can assess a student's understanding. Some formative assessment tools include classroom questioning; analysis of tests; quizzes and homework reflective writing assignments that describe their understanding of vocabulary or concepts; interviews (individually or in groups); portfolios or collections of student work; self and peer-assessments; quick checks during instruction; and exit slips or writing prompts.

When used effectively, these tools will help teachers identify those students who understand numeracy and those who still need additional or differentiated instruction to further develop their understanding. In formative assessments teachers should include quantitative literacy problems whenever possible to determine the level of understanding or mastery of those specific skills. Examples of strong quantitative literacy instructional strategies for each grade level are included in **Sections VI D through VI H**. Assessment and effective use of data are further discussed in **Section V D**.

To ensure that lessons are centered on the needs, strengths, and weaknesses of students, teachers must constantly adapt their teaching and redirect their instruction. Teachers should use their own problem-solving abilities to determine the best way to approach a new topic or how to correct student misconceptions. Teachers must have a repertoire of problem-solving strategies that includes tools such as the depth of understanding required within the context of a lesson, examples to promote higher order thinking, questioning techniques to explore student thinking, and routine analysis of student understanding.

To help students make adjustments toward improved numeracy, teachers can provide specific comments about students' errors and then make suggestions for improvement. Descriptive feedback and suggestions help students understand what they are doing well and give specific input on how to reach the next step. However, even the best assessment feedback will not produce the desired outcome if it is not provided in a timely manner. For students to fully comprehend the connection between their work and the suggestions for improvement, they need to receive the feedback as soon as possible.

If, through formative assessment, a teacher realizes that a student (or even the majority of a class) does not understand a concept[13], he or she may need to give an impromptu lesson before proceeding with the planned lesson. This review should not be a repetition of the original lesson. It may require techniques often employed by veteran teachers, including using a different approach, making connections to something the student understands, extending or strengthening the original lesson, or offering additional opportunities to practice.

Curriculum, Content, and Pedagogical Knowledge

In order to effectively teach numeracy, a teacher must have a thorough knowledge of the curriculum, the content, and the relevant pedagogical strategies. They must also possess a wide and deep knowledge of their content area that includes all components of their subject. Teachers who have such knowledge are better able to engage students with appropriate questions and activities aligned with standards. They are also aware of the areas in which students may encounter difficulties and are prepared to address them. A solid understanding of the curriculum promotes awareness of how and when to incorporate quantitative reasoning into lessons.

In order for teachers to acquire such a broad-based knowledge, they must have a high-quality preparation during which they develop a strong background in mathematics. Combining a fundamental knowledge of mathematics with an understanding of New Hampshire's Grade-Level and Grade-Span Expectations creates a solid foundation for teaching. These teacher qualities are essential for the development of quantitatively literate students.

Although knowledge of subject and curriculum is essential, it is not enough to produce an effective teacher. Berliner's (1986) *In Pursuit of the Expert Pedagogue*, states that "effective teaching is a dynamic mixture of expertise in a vast array of instructional strategies combined with a profound understanding of the individual students in class and their needs at particular points in time" (Marzano, 2007, p. 5). A teacher must be able to establish learning goals, set high expectations, and then engage and motivate students to achieve them. Establishing a rapport and

[13]For those students who have few or no learning errors, the teacher can provide enrichment activities to broaden and expand their learning.

getting to know students is an important step in this process. Teachers also need to move from a teacher-centered model of instruction to a student-centered model. In a teacher-centered class, student involvement and inquiry can be stifled. Conversely, when a class is student-centered, discussion and discovery tend to be more thorough and involved. Discussion and discovery are key elements of expanding quantitative reasoning skills.

The art of teaching involves having well-defined classroom procedures so that students know what is expected of them. It requires using proven teaching strategies and the ability to connect the present lesson with prior knowledge. Knowing how and when to arrange students in problem solving groups and when problems are better suited to working individually is critical for teachers. Depending on the group of students involved, certain problems will produce more insightful learning and discussion individually or collaboratively.

Teachers who are in-tune with their students can determine when they need additional time to think, rather than having more time to take copious notes. Oftentimes, students are so busy writing everything down that they do not think about what they are learning. Teachers need to allow sufficient time for students to mentally process and reflect upon new information. Additionally, providing students enough time to think gives them the opportunity to self-assess or to determine whether they have unanswered questions that need clarification.

For educators, the knowledge of the mathematical curriculum and content coupled with the art of teaching is crucial. Teachers need to be well versed in proper vocabulary and mathematics content as well as how to present the curriculum using best practices. In order to create quantitatively literate students, teachers can make the curriculum come alive using the "Habits of Mind," good questioning techniques, and problem-solving strategies. Other ways to make learning relevant and effective are rigorous expectations, use of group work and cooperative learning, Writing Across the Curriculum, formative and summative assessments, and data-driven instruction.

Conceptualizing Time and Money

Teachers can engaged students and expand their numeracy skills by utilizing real-world examples that are relevant and meaningful to them. Time and money are two topics that are especially effective even at a very young age. Children build conceptual frameworks of time as they build their repertoire of language. This process begins at home and is carried over to their preschool and primary school classrooms. Young children become aware of time through everyday conversation, such as "I'll help you in a just a few minutes" or "You can play for five more minutes." However, often these experiences do not give them a true understanding of how long a minute or an hour really is. In order for young children to understand certain lengths of time, they need many opportunities to experience time in real-life activities. One such activity that can help young children gain a concrete understanding of an abstract minute is to have them put their heads down on a desk or table and to close their eyes. Beginning when the teacher indicates, they are to use any strategy they wish to try to keep track of when one minute has passed. At that time, they raise their hand while still keeping their heads down (so all children have a chance to individually determine when to raise their hand). After a true minute has passed, the teacher calls out and everyone raises their head back up. Most students are very surprised at the actual duration of a true minute; most raise their hand long before a minute has passed. This exercise provides students with a concrete example of an abstract idea. Following this lesson with additional experiences involving counting seconds with a digital or sand timer helps students to become more cognizant of the passage of time. Children also learn to tell time by practicing on a regular, often daily, basis. Both at home and at school, students can tell the time at certain points in the day and record it.

Similar to time, money is also an accessible topic for teaching many numeracy skills. As they witness parents and adults purchasing items at grocery stores, restaurants, and gas stations, children are exposed to the concept of money at a very young age. Paying for items is a tangible activity that can be observed. However, where the money comes from and its value are concepts about which they are naïve. Often, children feel that money is infinite and available in endless amounts. Building an appreciation for the value of money is the first step toward their understanding of how money is obtained and then utilized. Since money is such an abstract concept for young students, providing them with money experiences is extremely important in the early elementary grades. Children often make incorrect assumptions about a coin's value based on its size or shape. Students have difficulty grasping that the "tiny" dime is worth more than the "larger" nickel. Presenting students with opportunities to repeatedly practice this concept (e.g., close examination of coins, multiple opportunities to count coins, assigning coins to numbers within daily routines, and providing concrete experiences using coins to purchase items) helps to provide students with a growing conceptualization of a very abstract idea.

Encouraging a home-school connection provides additional monetary experiences for children. Giving children a weekly allowance and asking them to count the money when they want to make a purchase is a great way to reinforce information learned at school.

The following problem can increase quantitative literacy by utilizing the concept of money:

> A first grade student goes to the school store with 75 cents to spend. The cost of items range from 10 cents to 99 cents. How many items can the student purchase?

Students are encouraged to solve this problem using the method of their choice. The likelihood of all students making the same choices and arriving at the same solutions is slim. This problem is a real-life situation and is meaningful and contextual to them, as they need to consider the cost of each item, choose which items to purchase, and not exceed the given amount of 75 cents. Students may choose to draw pictures, come up with concrete examples, set prices, or use coins to generate their solution. When students have completed the problem, they are then asked to explain their answers and defend their solutions. The teacher plays a key role in helping the classroom community see the variety of solutions, as well as scaffolding[14] the discourse that should happen within the community of learners. Solutions can then be analyzed to see if there are any patterns in the various answers. For example, some possible patterns could involve the most number of objects purchased, the least number of objects purchased, or the largest and smallest amount of change remaining.

Time and money are easily combined to solve real-life problems that are engaging for secondary school students. They may face problems about the speed by which a particular virus can be spread or the probability of winning the lottery. For instance, students may study compound interest in situations such as buying a car or house and even saving for retirement or paying off a credit card. These topics of interest connect to the typical high school curriculum, and are also essential to students' quantitative literacy as an adult. Since numeracy often deals with very large or very small numbers, time and money are excellent topics for teaching quantitative reasoning.

The next four sections (**Sections VI D through VI H**) discuss what numeracy looks like at each grade span. The table below outlines the "big ideas" covered in each grade level and highlights the connections across grade spans. These "big ideas" help connect the knowledge young children possess when they enter school with the knowledge they will need in subsequent grades and to enter the world of work and higher education. By mastering these important topics, early elementary students will gain the foundation necessary to become quantitatively literate and mathematically fluent.

[14]"Scaffolding" refers to a teaching strategy where, in order to learn new material, a teacher builds upon a student's prior knowledge. This new material is just beyond the student's current knowledge base; however, the prior knowledge is used as a bridge toward learning.

The "Big" Ideas

The Most Crucial Topics Taught at Each Grade Span

Grades Pre K – 2	Grades 3 – 5	Grades 6 – 8	Grades 9 – 12
▪ Number Conceptualization ▪ Conceptual Understanding of 2D space ▪ Identifying & Extending Patterns ▪ Interpreting & Communicating Data	▪ Multiplicative Reasoning ▪ Computational Fluency ▪ Equivalence (Including Measurement) ▪ Data Analysis ▪ Algebraic Reasoning	▪ Rational Numbers: Fractions, Decimals, Percents ▪ Geometry ▪ Data, Probability, & Statistics ▪ Functions & Rates of Change	▪ Number Sense ▪ Problems Solving / Applying Math Skills ▪ Communication & Representation

The "Big" Ideas Common Across all Grade Spans: Number Sense & Data Analysis/Representation

Grades Pre K – 2	Grades 3 – 5	Grades 6 – 8	Grades 9 – 12
Number Conceptualization ⇨	Multiplicative Reasoning ⇨	Rational Numbers: Fractions, Decimals, Percents ⇨	Number Sense
Interpreting & Communicating Data ⇨	Data Analysis ⇨	Data, Probability, & Statistics ⇨	Communication & Representation

D. Numeracy Skills in Grades PreK–2: Quantitative Literacy for Early Elementary School

Young children begin their formal education with many preconceived ideas about the world around them. Additionally, early elementary students have "differing levels of preparedness for learning" (National Research Council, 2005, p. 259). Teachers need to determine what concepts students already know and how to best build upon that knowledge. This process is necessary if students are to develop conceptual frameworks required for learning and for the transfer of knowledge. The National Academy of Sciences (2000) reports "Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught" (p. 229).

The most important concepts taught in the pre-kindergarten to grade two level are number conceptualization; conceptual understanding of two-dimensional space; identifying and extending patterns; and interpreting and communicating data.

These "big ideas" help connect the knowledge young children possess when they enter school with the knowledge they will need in subsequent grades. By mastering these important topics, early elementary students will gain the foundation necessary to become quantitatively literate and mathematically fluent.

Number Conceptualization

While many students begin early elementary school able to count from one to ten, very few have even a subtle understanding of what those numbers mean or represent. Most simply rattle off the digits in order from rote memorization. Number conceptualization, also referred to as number sense, is at the core of being able to understand mathematical concepts. "Teachers also recognize that children's ability to handle problems in other areas (e.g., algebra, geometry, measurement, and statistics) and to master the objectives listed for these standards is highly dependent on number sense" (National Research Council, 2005, p. 259). For grades pre-kindergarten through two, number sense includes a child's ability to grasp the following key concepts: visual representation of numbers, one-to-one correspondence, proper number sequence, ability to compare numbers based on relative magnitude, number conservation, and the properties of numbers.

Young children need to form an understanding of the relationship between a physical quantity, a number symbol, and the related number name (verbal or written). Children begin making the connection that quantities can be represented by visual clusters, groupings, names, or symbols. For example, "fourness" can be represented in a variety of ways. Visually, a four can be represented as four hearts or other objects (♠ ♠ ♠ ♠); as on a die (such as ::); or four fingers on one hand. Students also need to recognize that the symbol "4" represents four and is shorthand for representing four objects. Ultimately, young children will realize that these are all equivalent and acceptable, albeit very different, ways of representing the same quantity.

Once students have developed a concrete understanding of what a number is and what it represents, they can begin to understand the one-to-one correspondence of numbers as well as the proper order of numbers. For example, five comes after four when counting forward, and 20 comes before 19 when counting backward. These skills call for children to recognize that, when they are counting a group of objects, each number is matched with exactly one object, according to the correct numbering sequence. For example, if kindergarten students are counting pennies, they are encouraged by their teacher to count and touch each penny, move it aside, and go onto the next penny, until they eventually reach the total. Common mistakes young children make include recounting objects, assigning more than one number to an object, and missing objects:

> Children who make these errors are demonstrating some knowledge of counting. They are typically able to say the string of counting words in the correct sequence, and they know what must be done to figure out the answer to the question (e.g., touch the objects present while saying the words). What they do not yet understand is that chips must be touched in a certain order and manner to coincide precisely with their recitation of the counting words (National Research Council, 2005, p. 272).

The New Zealand Ministry of Education (retrieved February 13, 2009 from www.nzmaths.co.nz/) cites five developmental steps required to learn the sequence of numbers: counting from one, counting on, additive strategies, multiplicative strategies, and proportional strategies[15]. If a student can count objects from one successfully, then they know the names of numbers in order, and they understand that each object gets a single count (the one-to-one principle). Also, they understand the cardinality principle—that the final number in a count represents the quantity of the entire set. Furthermore, children at this stage understand that each subsequent number represents one more than the previous number (National Research Council, 2005). However, students have a very concrete

[15]Multiplicative and proportional strategies will not be discussed in the context of the early elementary grades.

understanding of the connection between numbers and quantity: "Students at this stage typically solve addition problems by physically representing each quantity (set) in a problem, then combining the sets and finally counting the combined set from one." When students employ the more advanced "count-on" method, they recognize the total count of one set and then use that number to add (or count-onto) the other set. They also realize that sets of objects can be represented by a single number or as a "collection of ones." At this point, students can grasp the concept of number conservation—they can determine that two collections of items have the same quantity even if they are arranged differently. Finally, when a student can view numbers abstractly, rather than representing objects in a set, he or she can partition and recombine to solve addition or subtraction problems.

In addition to sequencing numbers, young children begin to understand the relative magnitude of numbers and can make comparisons between two numbers. Initially, when a student is shown two piles of objects and asked which is bigger or has more objects, the response is usually correct only when the difference is visually obvious. At an early age, children do not make the connection between counting and making a quantity (or magnitude) comparison. Eventually (around five years of age), the student will know

(1) that numbers indicate quantity; and therefore,
(2) that numbers themselves have magnitude;
(3) that the word "bigger" or "more" is sensible in this context;
(4) that the numbers seven and nine occupy fixed positions in the counting sequence;
(5) that seven comes before nine when one is counting up;
(6) that numbers that come later in the sequence or higher up indicate larger quantities; and
(7) that nine is therefore bigger (or more) than seven (National Research Council, 2005).

For children to be able to accurately compare numbers, they need to be able to count up and understand number sequencing. In elementary school, students begin to formally compare quantities using words or symbols. They may indicate that two numbers are "less than," "more than," or "equal to" each other. They also learn that the equal sign (=) indicates "is the same as" and that the symbols < and > mean that the numbers on either side of the inequality symbol are not equal.

In order to develop mental fluency for comparing magnitude, young students can be exposed to benchmark numbers such as 5, 10, 25, or 100. Students begin to see patterns in the base-10 number system or group numbers together, rather than just memorize them sequentially, in order to make a quick comparison. For example, if a student is asked to compare 73 to 28, he or she might look at the first digits only, knowing that the 7 represents 70 and the 2 represents 20. Already knowing that 70 is greater than 20, the student may realize that comparing the second digits (the 3 and the 8) is unnecessary. Furthermore, students are not only asked to compare two numbers but may also be asked to determine the relationship between the two numbers, such as "60 is 10 more than 50" or "200 is 100 less than 300." This practice can also help students develop computational fluency.

Early elementary children can start to expand their understanding of the properties of numbers after mastering counting, number sequencing, and magnitude comparison. Some properties that concretely materialize during this grade span include the conceptualization of odd and even numbers, the commutative property of addition, the identity property of addition, and the associative property of addition.

One way to acquaint young children with the idea of odd and even numbers is to provide them with a number of objects, for example, 7 children. They can pair the children up in order to establish if there is a child without a partner. If that is indeed the case, then 7 is an odd number. If the objects are all paired up, as with 6 children, they can conclude that the number is even.

Young children can discover the commutative property of addition ($a + b = b + a$) when they use manipulatives, such as Cuisenaire Rods. They will see that a red and yellow rod train is the same length as a yellow and red rod train, illustrating that the order of the rods is irrelevant. This knowledge is particularly useful when children are beginning to memorize their addition facts, as understanding the commutative property cuts the number of facts to be learned in half. If the commutative property is understood, a student who knows that $4 + 3 = 7$ will also know that $3 + 4 = 7$. Both addition facts would not need to be memorized separately. Conversely, students become aware that this property does not apply to subtraction. When working with counters, they will observe that $4 – 3$ equals 1, but $3 – 4$ does not equal 1.

Young students begin to become aware of the identity property of addition ($a + 0 = a$) as they gain more experience with number stories and games. In the card game "Name That Number," student partners use number cards to arrive at a designated number. They create a number sentence using one or more operations and as many cards as possible. As students become fluent with their operations, they quickly recognize that 0 can be added indefinitely to any number sentence without changing the answer.

The concept of the associative property of addition [$(a + b) + c = a + (b + c)$] becomes useful to young children with repeated addition practice. When students are adding multiple addends, they discover they can use their knowledge of complements and multiples of 10 to arrive at an answer more quickly. For example, given the number sentence $3 + 5 + 7 = ?$, students who know complements of 10 will quickly add $3 + 7$ to get 10. Then they will add on the 5 to reach 15. Students who are insecure in their knowledge of grouping complements of 10 will typically add the numbers in the order given. As they become more quantitatively literate, they realize these properties of numbers assist them in solving addition problems accurately and efficiently.

Understanding place value is also a critical milestone for the early elementary student. First, students learn the 10 digits in the number system. Not surprisingly, most children begin counting at 1. Therefore, they tend to see 10 as a "digit" unto itself, rather than the compilation of two digits. A fundamental, yet difficult-to-grasp, concept is that the value of a digit in a numeral is dependent on its position in that numeral. Young children need ample opportunities to group objects and then count those groups as tens and hundreds. Both discrete and continuous materials can be used to help young students grasp place value concepts. Discrete materials include popsicle sticks, beans and cups, and number lines. Cuisenaire Rods and tens blocks are examples of continuous materials as they represent numbers and thus do not need to be counted and recounted. Representations such as ten frames, number grids, and arrays are also useful resources to practice grouping and thereby learn place value.

Sample Problems for Number Conceptualization

The following problem can be used to help students estimate, analyze patterns, make predictions, and communicate their reasoning:

> Kindergarten students each have a behavior chart that contains 5 rows of 5 boxes, making a total of 25 boxes. Each time their teacher sees a student following directions or working hard, he or she receives a sticker. Each student can earn up to 2 stickers per day. Students begin using estimation to predict how many days it will take to fill their chart. The teacher can ask, "About how many days would you predict it will take you to fill your chart? Explain your thinking."

For the kindergarten or primary grade level student, the above problem represents a real-life situation that is meaningful to them. It encourages students to estimate and to predict the solution. Each student makes different assumptions based on personal behavior and the methods used to make a prediction. Appropriate behaviors are rewarded with one sticker daily, excellent behavior is rewarded with two stickers daily, and misbehavior leads to no additional stickers. Students control the variable of behavior. If all the rules are followed, they earn a minimum of one sticker. Individual students begin solving the problem through counting rows of empty boxes. They can then decide from the number of empty squares how many days it will take them to fill the chart based on earning one sticker daily. If the variable changes to two stickers earned, they can estimate how many days it will take to fill their chart based on the number of remaining squares. The solution is based entirely on the assumptions students make about the variable of stickers earned. These estimations change daily as the chart fills. Students can revise their predictions as the chart fills and make decisions about how many days it will take to fill the chart based on the changing variable of stickers earned per day.

This problem allows students to synthesize multiple mathematical skills and use one-to-one correspondence, numeric or visual patterns, estimation, or a combination of methods to generate a solution. Given that each student makes his or her own assumptions, there is more than one solution to the problem, and it can be solved in a variety of ways. Finally, students are encouraged to provide proof for their prediction by using words, pictures, or tally marks.

The next problem is similar to the kindergarten behavior chart problem, but it is targeted for second grade. This problem requires students to keep track of the days they have been in school. On the 94th day of school, they are asked to come up with expressions that show the sum of 94 using at least three addends. Then students are asked to give a verbal proof for their expressions.

This problem provides students with a concrete, daily situation. Each day, they can be asked to show expressions that represent the sum of the number of the day. This activity provides the students with opportunities to exercise their mathematical skills and to see patterns, such as each subsequent day is $n + 1$ or every odd day is $n + 2$. Students model their computational skills and discuss their different ways of getting to the same solution. Students can also discuss their "strategy" for choosing their particular addends. For example, did they choose to start with a large number or a small number and why was that their strategy?
Teachers can offer a differentiated activity by changing the rules to make the problem easier or more challenging, depending on the students' skills. Some options for differentiating the activity are to change the operation or to change the number of numerals used in the expression. A teacher could require students to multiply two numbers and add a third number to arrive at 94. Another example of a more difficult problem might include coming up with the sum of 94 using 3 addends and a specific, given number. This encourages the student to decompose the number based on the given parameters and then devise an expression—a great way to introduce algebraic thinking at the early elementary level. To make this problem more open-ended, students could be asked to think of their own similar problem and to solve it with a classmate. For example, Johnny might ask Suzy to find different "names" or expressions for 94 using at least two operations in each solution. Suzy might ask Johnny to find number sentences where the difference between numbers is equal to 94. Partners then share all their different solutions with the whole class.

Conceptual Understanding of Two-Dimensional Shapes

Most preschool students can identify circles or squares, but have yet to formally compare and contrast the properties of two-dimensional shapes. The understanding of elementary geometry concepts now occupies a more prominent place in the curriculum for young students, in part because of a model developed by two Dutch educators, Pierre van Hiele and Dina van Hiele-Geldorf, in the late 1950s (Bassarear, 2005). They defined four levels[16], or stages, that children progress through as they learn about geometric shapes. These levels are:

- Level 1: Reasoning by Resemblance (Visualization). Students describe shapes by their appearance and not necessarily by mathematical qualities: "That is a square because it looks like a square."

[16]Some sources state that there are five Van Hiele levels. Also, different sources number and name the levels differently. The fifth stage is commonly referred to as "rigor." (retrieved July 15, 2009 from http://images.rbs.org/cognitive/van_hiele.shtml)

- Level 2: Reasoning by Attributes (Analysis). Students are able to describe shapes by their attributes. However, they do not apply relationships between figures: "That is not a rectangle because it is a square."
- Level 3: Reasoning by Properties (Abstraction). Students are able to look at many shapes and see the relationship between and among them based on common properties.
- Level 4: Formal Reasoning (Deduction). Students use reasoning to solve problems and justify solutions.

According to the van Hieles these stages are sequential: Each level must be mastered before a student can go on to the next one. Their theory may explain why many high school students struggle with proofs. Constructing a formal proof requires a student to be at Level 3. However, most students begin high school at a very low van Hiele level. Thus, as these stages must be mastered in order, older students are more likely to struggle. If children are exposed to two-dimensional shapes, their properties, and informal deduction at an earlier age, then they may have an easier time progressing through the van Hiele stages and through geometry in the later grades.
In order to learn the relationships and properties of shapes, young students need to be involved in activities where they handle shapes, find shapes around them, and name shapes. They also need to become aware of the similarities and differences among shapes so that they can sort and classify shapes. These activities must take place in order for students to work through the first two van Hiele levels.

Identify and Extend Patterns

Mathematics is the study of patterns—recognizing, extending, applying, and creating patterns. These concepts are inherent in all mathematical strands: concrete patterns, visual patterns, patterns in nature and designs, numeric patterns, and patterns in data. Young students explore patterns using a variety of materials and activities. They will notice concrete and visual patterns as they play with blocks, on their clothes, around the classroom, and by using typical mathematical manipulatives. Pattern blocks[17] are one commonly used manipulative. A typical activity using pattern blocks might involve identifying, copying, and extending a pattern, such as orange square, green triangle, orange square, green triangle, and so on. As students become proficient with patterns, they begin to recognize numeric patterns. These patterns can include counting numbers in an alternating odd and event pattern or that every tenth number ends in a zero. When students approach mathematical problems using their knowledge of patterns, it shows a developed ability that will assist them in solving myriad problems.

Sample Problem for Conceptual Understanding of Two-Dimensional Shapes and Patterns

At a very young age, students are able to identify patterns and two-dimensional shapes. They learn the name of two-dimensional shapes through repeated exposure in the world around them. The shapes can be seen, touched, held, and manipulated. Once the language related to shapes is learned, this knowledge can be utilized to help students understand patterns. The following problem can help students connect their knowledge of shapes with their elementary knowledge of patterns:

> Pre-Kindergarten students are given a variety of objects with which to show a pattern. Each child is then asked one or more of the following questions: What is your pattern? How did you come up with your pattern? Could your pattern be extended? How could you change your pattern?

Providing students with concrete materials to show a pattern allows them to represent their understanding of a pattern in any way they choose. The questioning process helps the teacher assess the students' level of understanding. As the student progresses from understanding simple two-object patterns to more complex patterns in-

[17] A set of pattern blocks contains 6 shapes in 6 different colors: orange square, green equilateral triangle, yellow hexagon, red trapezoid, and tan and blue rhombi.

volving three, four, or five objects, they begin to notice patterns in more abstract items, such as numbers and counting.

Read, Interpret, and Communicate Data

From a very early age, children are surrounded by data and graphs which can be seen in environmental print, on signs, on food packaging, and in books and newspapers. Early childhood classrooms provide experiences for collecting and displaying data that is meaningful to the student. These experiences can involve activities such as responses to a question of the day, polling likes and dislikes, listing birth dates by months, or displaying lost teeth or age charts.

As students begin elementary school, they participate in activities where they collect data, learn how to organize and display it, and then analyze it to determine what conclusions can be made or what patterns can be seen. There are numerous data collection activities connected to young students' lives. Some examples of these activities are counting the number of pockets on their clothing, the length of their arm span, or the number of pets at home. The data can then be aggregated into a table using tally marks and displayed on a variety of graphs. Finally, students can analyze visual representation to see if there are any discernable patterns from which to draw conclusions. Young students need to be given the opportunity to respond to questions about the data as well as make up their own questions concerning the data. They eventually need to be able to make their own graph, choose an appropriate title, and label the axes.

Sample Activity for Reading, Communicating, and Interpreting Data

First graders can be asked to devise a question that they will then use to survey the class to gather data. Once the data is collected, each student may choose a method for representing their data whether they use a tally chart, pie graph, bar graph, or line plot. Each student must title their chart, label all pertinent information, and share the results of their survey.

This example allows students to come up with a real-life question based on their interests. Each child is then given the chance to collect data and compile it in a way that is meaningful to them. This sharing of information invites students to explain their understanding of the gathered data and provides the teacher with an opportunity to question students and guide them in analyzing the facts. Upon sharing the data, the results can be sorted and analyzed for similarities, differences, or patterns.

E. Numeracy in Grades 3–5: Quantitative Literacy for Later Elementary School

> Most students enter grade 3 with enthusiasm for, and interest in, learning mathematics. In fact, nearly three-quarters of U.S. fourth graders report liking mathematics (Silver, Strutchens, and Zawojewski, 1997). They find it practical and believe that what they are learning is important. If the mathematics studied in grades 3–5 is interesting and understandable, the increasingly sophisticated mathematical ideas at this level can maintain students' engagement and enthusiasm. But if their learning becomes a process of simply mimicking and memorizing, they can soon begin to lose interest. Instruction at this level must be active and intellectually stimulating and must help students make sense of mathematics (NCTM, 2000).

A pivotal time when many students begin disliking math is during the later elementary grades. Many students who excel in early elementary school end up struggling during middle school, in part because mathematical concepts transition from the concrete to the more abstract. This transition begins during the later elementary years as students apply and build upon their knowledge. In order to bridge the conceptual gap between elementary and middle school, students in grades 3–5 need to master the following big ideas: multiplicative reasoning, computational fluency, equivalence (including measurement), data analysis, and algebraic reasoning.

Mastering the ideas listed above can ensure that students will have the numeracy foundation necessary for higher-level mathematical thinking. Furthermore, students are not only more engaged in contextual problem-solving, rather than memorization and procedures, but they are also able to understand more of the math involved. Through better understanding, students will feel successful and that they can make sense of the material.

Multiplicative reasoning

While the earlier grades focus on addition and subtraction, students in grades 3–5 focus on multiplication and division. At this age, students begin to transition from using only arithmetic thinking to using multiplicative reasoning. By developing multiplicative reasoning, students will understand the various meanings of multiplication and division; understand the effects of multiplying and dividing whole numbers; identify and use relationships between operations, such as division being the inverse of multiplication, to solve problems; understand and use properties of operations, such as the distributivity of multiplication over addition; and develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30×50 (NCTM, 2000).

Through accomplishing the above milestones, students will develop stronger number and operation sense. Students can transfer their knowledge of multiplication to their knowledge of its inverse operation (i.e., division) which segues directly into ratios and proportions. Multiplicative reasoning is important because it "provides foundational knowledge that can be built on as students move to an emphasis on proportional reasoning in the middle grades" (NCTM, 2000, p. 200). Proportional reasoning, which is used in countless everyday problems, is a key component to developing quantitative literacy. A complete understanding of proportions includes knowledge of the relationship between multiplication and division as well as being able to solve problems using these operations.

Throughout elementary school, students begin to deepen their understanding of multiplication by recognizing the multiplicative structure of the base-ten number system. For example, students need to understand that the number 484 is equal to $(4 \times 100) + (8 \times 10) + (4 \times 1)$. Multiplicative reasoning is further developed as students become familiar with geometric models for multiplication, such as a rectangular array; and they can adapt this model for computing the area of two-dimensional shapes and the volume of three-dimensional solids.

Sample Problems for Multiplicative Reasoning

In order to develop quantitative reasoning skills, students need exposure to both equal group and multiplicative comparison type problems. These types of multiplicative problems are very common in real-world applications. To pique students' interest, multiplicative problems should involve situations from their experiences.

Equal group examples:

- Mark has 4 bags of apples. There are 6 apples in each bag. How many apples does Mark have altogether?
- Mark has 24 apples. He wants to share them equally among his 4 friends. How many apples will each friend receive?

Multiplicative comparison examples:

- Jill picked 6 apples. Mark picked 4 times as many apples as Jill. How many apples did Mark pick?
- Mark picked 24 apples, and Jill picked only 6. How many times as many apples did Mark pick as Jill did? (Van De Walle & Lovin, 2006, pp. 57-60).

In order to increase a problem's likelihood of teaching numeracy, students should be asked for more than just an answer when solving these problems. Their understanding will be enhanced and readily accessible if they are asked to explain their reasoning by drawing a picture or providing an oral or written explanation.

The calculator is an appropriate tool to help show concretely how multiplication relates to addition. The following problem uses the calculator to help students discover this relationship through an open exploration. This activity also provides teachers with an opportunity to assess student understanding.

> **The Broken Multiplication Key:** Students are told to find various products on the calculator without using the multiplication key. Extension: Have students work in groups to find methods of using the calculator to solve division exercises without using the divide key (Van de Walle & Lovin, 2006, p. 65).

Through this process, students can see how multiplication (or division) leads to the same answer as "repeated addition" (or "repeated division").

The next problem could be used as an extension to further develop students' conceptual understanding of multiplication and to foster quantitative literacy:

> Without doing any multiplication, how could you find the difference between 16×7 and 16×8? Find the difference and show how you found it.

Students could use their understanding of multiplication as repeated addition to see that 16×7 represents 7 groups of 16, and 16×8 represents 8 groups of 16. Other students may use decomposition[18] to determine that $8 = 1 + 7$ and hence, 16 groups of 7 would be $16 \times 1 = 16$ less than 16 groups of 8.

Multiplicative reasoning can be further supported as students work on problems involving rectangular arrays. Constructing scale drawings is a concrete way of introducing rectangular arrays for this purpose. The following is an example from *Illuminations* (http://illuminations.nctm.org/):

> In the Junior Architects unit, as students design their own clubhouse, they identify two- and three-dimensional shapes; determine perimeter and area; create two-dimensional blueprints; and use a scale drawing to construct a three-dimensional model.

This problem involves many skills to connect multiplicative reasoning to geometry and measurement. It is engaging because of the context: Students could also do the same activity using their room or their playground. This activity is hands-on and when done cooperatively, it provides opportunities for sharing ideas relative to multiplicative reasoning.

Computational Fluency

If a student struggles with the basic operations, he or she will have difficulty focusing on the actual problem-solving or the interpretation of data which are key components of numeracy. Therefore, if a student is to be quan-

[18]Composition means that a whole number can be composed by adding two or more numbers (e.g., 3 + 5 = 8; 6 + 2 = 8; 2 + 2 + 4 = 8). Decomposition means that a whole number can be renamed as the sum of multiple addends (e.g., 8 = 3 + 5; 8 = 6 + 2; 8 = 2 + 2 + 4).

titatively literate, he or she needs to have a certain degree of computational fluency. Students need to recall basic facts quickly be fluent with both whole number operations and estimation (NCTM Focal Points, 2006). By having computational fluency, children can focus on how to solve and discuss problems instead of getting sidetracked with calculations. As stated in PSSM (2000), "*Fluency* refers to having efficient, accurate, and generalizable methods (algorithms) for computing that are based on well-understood properties and number relationships."

A large amount of instructional time is spent building fluency during grades 3–5. By the time a student reaches middle school, they are expected to be comfortable using the basic operations. According to NCTM's *Principles and Standards for School Mathematics* (2000), children in grades 3–5 need to

- develop fluency in adding, subtracting, multiplying, and dividing whole numbers;
- develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;
- develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience;
- use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals; and
- select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tools.

To be quantitatively literate, children need to be fluent with basic computational facts. The traditional approach has been to memorize them out of context. Therefore, "fluency" has often resulted from being a good memorizer, rather than understanding and using the operations efficiently. Memorizing puts the "facts" into the language portion of the brain. Instead, this knowledge needs to be in the mathematical thinking portion of the brain. When a student utilizes mathematical thinking, the child has true, retrievable "fact power."

Becoming computationally fluent does not occur in silos; each of the mathematical operations is related to the others. For instance, students can see multiplication as repeated addition and division as the inverse of multiplication. By having a deeper understanding of one operation, students can transfer this knowledge to other operations, thereby building computational fluency. Students will also discover and use mathematical properties, which will aid them in their progression to higher levels of mathematics. Students can use the identity properties, the zero property of multiplication, and the commutative property to decrease the number of single facts they need to know. In addition to working with whole numbers, students in this grade span are expected to become fluent in adding and subtracting decimals and fractions.

In *A Research Companion to PSSM* (NCTM, 2003), these basic facts are more appropriately called "single-digit multiplication and division." This puts the emphasis on understanding the operation rather than the facts, which are merely isolated bits of information to memorize. As stated by Van de Walle & Lovin (2006), "Fluency with the basic facts is developed through a strong understanding of the four operations and an emphasis on conceptual strategies for retrieving the facts. . . .Number relationships provide the foundation for strategies that help students remember basic facts" (p. 74). For example, children need to "see" multiplication facts through the use of concrete materials, arrays, successive addition, and by building upon facts (e.g., addition) they already know. To increase fluency, children should also have opportunities to look for and identify patterns in the multiplication tables, to practice skip-counting, and to use the inverse relationship between multiplication and division. These methods are more effective tools to use for learning facts than timed tests. In Burns's *About Teaching Mathematics* (2000), "it [using a timed test to learn basic facts] conveys to children that memorizing is the way to mathematical power, rather than learning to think and reason to figure out answers" (p. 157). Games such as *War* and *Concentra-*

tion are also a better alternative to timed tests for children to practice their facts once they have developed strategies for figuring them out.

In addition, children should have many opportunities to solve real-life problems and to generate their own algorithms to help develop their computational fluency. "Research has indicated that beginning with problem situations yields greater problem solving competence and equal or better computational competence" (NCTM, 2003). Students should have ample opportunities to solve problems using their own invented strategies before any standard algorithms are introduced. As students develop their own computational algorithms, teachers should evaluate their work, help them recognize efficient algorithms, and provide sufficient and appropriate practice so that they become fluent and flexible in computing. Teachers have ample opportunities to assess their students' current number and operational sense, allowing for any misconceptions to be addressed prior to acquiring the intermediate and standard algorithms. Students should come to view algorithms as tools for solving problems rather than as the goal of mathematics study. In addition, "research indicates that some algorithms are more accessible[19] to understanding than others and that understanding can be increased by quantity supports (e.g., manipulatives, drawings) to help children understand the meanings of the numbers, notation, and steps in the algorithms" (NCTM, 2003).

Elementary students can develop their fluency by using and sharing their own strategies both mentally and on paper. Performing computations mentally is not necessarily deemed more fluent than using pencil and paper. Many students have difficulty keeping track of several steps in their heads. Thus, jotting down ideas and numbers helps to facilitate the recording of their thinking as well as build their confidence, all leading to increased fluency. Furthermore, "mental math" and "estimation" are two frequently neglected areas that are critically important to developing computational fluency and numeracy. While students should not be discouraged from writing down their calculations, they should be encouraged to think about ways to mentally perform operations more quickly. Asking students to compute mentally opens the door for constructing their own methods and algorithms. "Traditional" algorithms do not translate well to mental computation because they are not configured the way children naturally think about numbers and operations. Children think left to right—the same way they read—and highest place value to lowest, whereas standard algorithms are the opposite. Allowing and encouraging them to add and subtract mentally builds upon their natural tendencies.

Estimation strategies, which are particularly important to developing numeracy, are another critical component of computational fluency. By making a reasonable estimate, students are able to think about their answers with respect to reasonableness or magnitude and estimations can often be performed more quickly and accurately than the actual calculation. Research into how children learn to estimate is in its infancy. Computational estimation (i.e., approximating the numbers and then computing with the rounded numbers) requires two steps and can be difficult for children in this grade span. However, around age 11, most children are able to grasp the concept of estimation. Encouraging students to mentally approximate the numbers involved in the calculation and write them down before calculating makes this more accessible (Sowder, 2006). Students in grades 3–5 should develop computational-estimation strategies for situations that call for an estimate or as a tool for judging the reasonableness of solutions. These skills, which greatly increase math capacity and number sense, need to be practiced regularly.

Sample Problems for Computational Fluency

As students begin the process of developing their own algorithms, they will likely arrive at one of the many common intermediate algorithms, such as partial sums, partial differences, partial products, and partial quotients.

[19] Accessible algorithms have easy-to-follow steps: they generalize to large numbers, they allow for differences in thinking, and they are simple to carry out.

These algorithms build off of the children's understanding of number composition and decomposition[20] as well as their elementary understanding of the operations. Below are examples utilizing the intermediate algorithms.

Partial Sums: Adding by place value.
Billy had 145 baseball cards. His father gave him his set of 326 cards from when he was a boy. How many cards does Billy have now?

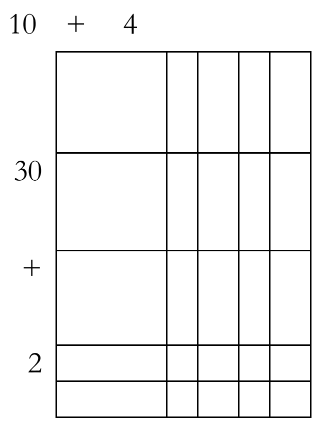
Using partial sums, the above scenario would be broken down as follows, so to add by place value: 145 + 326 = (100 + 300) + (40 + 20) + (5 + 6) = 400 + 60 + 11 = 471

Partial Differences: Subtracting by place value.
The auditorium has 280 seats. The 4th grade has sold 142 tickets to their play. How many tickets are left?

280 – 142 = (200 – 100) + (80 – 40) + (0 – 2) = 100 + 40 - 2 = 138

Partial Products: Multiplying by taking "groups of".
I would like to carpet my living room. How many square feet of carpet do I need if my room is 32 feet by 14 feet?

32 x 14 means 32 groups (or rows) of 14 that can be modeled with base ten blocks to show the "partial" products:



30 x 10 = 300
30 x 4 = 120
2 x 10 = 20
2 x 4 = 8

Add the four partial products:
300 + 120 + 20 + 8 = 448

Partial Quotients
My class made 144 cookies for our bake sale. If we put 3 cookies in a bag, how many bags of cookies will we have for the sale?

3)72
-30 10
42
-30 10
12
-12 4
0

[20]Base ten blocks can be used to concretely model composition and decomposition. As students move to less concrete representations, they can draw a simplified model of base ten blocks, such as a square to represent a flat, a vertical line segment for a rod, and a small circle for the unit cube.

To illustrate how the partial quotient algorithm works, estimate how many groups of 3 cookies can be made from 144 to determine how many cookies have been bagged. A possible estimate could be 40; however, other estimates such as 30 or 45 could be used as well. Subtract to see how many cookies are still left to be bagged after creating 40 bags. If there are enough cookies left to create another bag (3 or more), then repeat the process until the remainder is less than 3. The sum (*40 + 8 = 48*) of the number of bags made is the quotient, and the leftovers are the remainder.

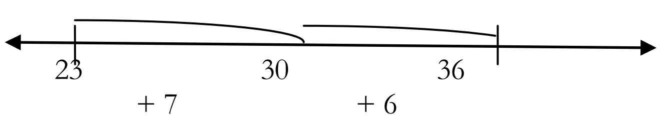
The standard U.S. algorithm, "long division," creates two difficulties for students. "First, it requires them to determine exactly the maximum copies of the divisor that they can take from the dividend." Students have difficulty estimating this, often resulting in multiplications on the side. "Second, the traditional algorithm creates no sense of the size of the answers that students are writing; in fact, they are always multiplying by single digits. Thus, students have difficulty gaining experience with estimating the correct order of magnitude of answers in division" (NCTM, 2003, pp. 85-86). The partial quotients algorithm builds experience with estimating and allows students to pick the number to multiply by, allowing them to underestimate and to use friendly numbers.

In addition to practicing the intermediate algorithms, children can reflect upon what they already know and transfer that knowledge into developing a stronger understanding of operation facts and procedures. The following problems highlight the connections between the operations using open-ended activities:

> **If You Didn't Know:** Pose the following task to the class: "If you didn't know the answer to 6 × 8 [or any fact that you want students to think about], how could you figure it out by using something that you do know?" Explain to students that their method should be something that they can do in their heads and should not rely on counting. Encourage students to come up with more than one way. Use a think-pair-share approach in which students discuss their ideas with a partner before they share them with the class (Van de Walle & Lovin, 2006, p. 92).

Subtraction by "adding on": Uses the inverse relationship between addition and subtraction.
I did 36 jumping jacks. My friend stopped after 23. How many more jumping jacks did I do than my friend?

36 – 23 becomes *23 + what number = 36*. A number grid, number line, or an open number line using "jumps" (illustrated below) can be used to model the thinking.



23 + (6 + 7) = 36, so 36 – 23 = 13

Most students who learn the "traditional" standard algorithms find subtraction more difficult than addition. Research has shown that students who are exposed to "adding (or counting) on" find subtraction easier to do and to understand than addition (NCTM, 2003).

The next problems demonstrate how to help students mentally solve problems and make accurate estimations and comparisons, thus developing their fluency and numeracy.

> *Try solving the following problem using some method other than starting with the 9 and the 4. Can you do it mentally? Can you do it in more than one way?*

Mary has 114 spaces in her photo album. So far she has 89 photos in the album. How many more photos can she put in before the album is full? (Van de Walle & Lovin, 2006, p. 101).

Some examples of strategies children may use: *114 – 90 + 1*, *119 – 89 – 5*, *(110 – 80) + (4 – 9)*

Will 718 be smaller or larger than 100? (NCTM, 2000)

If $^3/_8$ *of a cup of sugar is needed for a recipe and the recipe is doubled, will more than or less than one cup of sugar be needed?* (NCTM, 2000)

The following examples are simple ways to help children develop estimation skills. These problems ask students to think about the size of numbers and the reasonableness of their answers in the context of real-life situations involving measurement. They can be easily adapted to the needs of the class by changing the numbers, the units of measurement, or both:

Is it Reasonable? Select a number and a unit (e.g., 15 feet). Could a teacher be 15 feet tall? Could your living room be 15 feet wide? Can a man jump 15 feet high? Could the school building be 15 feet tall? Could three students stretch out their arms 15 feet? Pick any number, large or small, and a unit with which students are familiar. Then make up a series of questions (Van de Walle & Lovin, 2006, p. 46).

10,000 Collections. As a class or grade-level project, collect some type of object with the objective of reaching some specific quantity (e.g., 1,000 or 10,000 buttons, old pencils, jar lids, pieces of junk mail, soup labels, or cereal box tops). If you begin aiming for 100,000 or 1 million, be sure to think it through. One teacher spent nearly 10 years with her classes before amassing one million bottle caps. It takes a small dump truck to hold that many! (Van de Walle & Lovin, 2006, p. 50).

Students should also be able to estimate and compute with rational numbers, which often causes much more difficulty. As stated in PSSM, students must “develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience” and “use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals. The focus should be on developing students' conceptual understanding of fractions and decimals (i.e., what they are, how they are represented, and how they are related to whole numbers) rather than on developing computational fluency with rational numbers” (NCTM, 2000). The following examples can be flashed on an overhead, and students can be asked to estimate an answer:

Jack and Jill ordered two identically-sized pizzas, one cheese and one pepperoni. Jack ate $^5/_6$ of a pizza and Jill ate $^1/_2$ of a pizza. How much pizza did they eat together? (Van de Walle & Lovin, 2006, p. 162)

You are going to a birthday party. From Ben and Jerry’s ice cream factory, you order 6 pints of ice cream. If you serve $^3/_4$ of a pint of ice cream to each guest, how many guests can be served? (Van de Walle & Lovin, 2006, p. 175)

Jamal invited seven of his friends to lunch on Saturday. He thinks that each of the eight people (his seven guests and himself) will eat one and a half sandwiches. How many sandwiches should he make? (NCTM, 2000)

Equivalence

Equivalence is a broad, often poorly understood[21], idea that involves all strands of mathematics, not just the number and operations strand. In geometry and measurement, equivalence can be demonstrated in a number of ways. For example, a parallelogram can be transformed into a rectangle of equal area by cutting and pasting parts together, or students can recognize that three feet is equivalent to thirty-six inches, or one yard. In the functions and algebra strand, equivalence is shown by the proper use of the equal sign in algebraic equations and number sentences, indicating that the expression on one side has the same value as the expression on the other side. In the data, statistics, and probability strand, students face problems involving "equally likely outcomes." And in the number and operations strand, equivalence is defined as equivalent representations of numbers (including fractions, decimals, and percents). Elementary students need to be introduced to the different contexts for equivalence on their journey toward becoming quantitatively literate.

Students' ability to recognize, create, and use equivalent representations of numbers and geometric objects should expand during grades 3–5 (PSSM). Students who have a strong understanding of equivalence

- understand the place value structure of the base ten number system and are able to represent whole numbers and decimals;
- recognize that different representations of numbers are helpful for different purposes and generate them by decomposing and composing numbers;
- expand their knowledge of computing numbers mentally;
- know when and how shapes can be decomposed and reassembled and what features of the shapes remain unchanged;
- understand the relationship between fractions, decimals, and percents and the information each type of representation conveys;
- explore algebraic concepts and properties; and
- understand measurement.

Equivalence can be explored across operations and number types. By composing and decomposing numbers, students begin to see that values can be equivalent even if they are in different formats. Particularly useful, intermediate algorithms at this grade level use decomposition such as partial sums [$24 + 67 = (20 + 60) + (4 + 7)$], partial differences [$67 - 26 = (60 - 20) + (7 - 6)$], partial products [$32 \times 7 = (30 \times 7) + (2 \times 7)$], and partial quotient [*69 ÷ 3 would be (60 ÷ 3) + (9 ÷ 3)*]. A multiplication example, such as 8×25, can be thought of as $8 \times 5 \times 5$ or as 4×50. These intermediate algorithms are discussed in greater length in **Section VI E**, Computational Frequency. Equivalence is also used to determine conversions between types of number such as fractions, decimals, and percents. For example, $3/4$ can be thought of as a half and a fourth, as $6/8$, 0.75, or as 75%. Students will strengthen their number and operational sense if they have a strong understanding of numerical equivalence.

With respect to spatial relationships, students who are able to describe and analyze properties of two- and three-dimensional shapes can advance their understanding of geometric equivalence to composition and decomposition of shapes. For example, a square would be equivalent to two congruent triangles, or two squares could equal one rectangle. By doing so, they can determine the areas of two-dimensional shapes and the volume and surface area of three-dimensional shapes by decomposing complicated figures into more standard, familiar ones.

As elementary students prepare to transition to middle school, understanding equivalence is the impetus toward the thinking required for solving equations, which will help them explore algebraic ideas and properties, including commutativity and associativity.

[21]Many students incorrectly assume that the equal sign simply implies that "the answer is . . ." or that the answer is on the right-hand side of the equal sign.

Sample Problems for Equivalence

In "A Brownie Bake," students are exposed to equivalence by determining equal measurements as well as the concept of an equal share. Students are asked to determine the amount of each ingredient needed to make brownies and to then ascertain how to divide the brownies evenly among their classmates. With this hands-on problem, students will use problem solving skills, measuring techniques, and food preparation to practice these math concepts.

By using a standard commercial brownie mix, the variety of answers is limited. For a more advanced activity, students can be asked to bring in a family recipe instead. Then, a variety of answers can be produced.

To demonstrate their understanding of equivalence, students should also be given more investigative questions, such as following examples.

> If 100 $1 bills are stacked, the height is approximately 0.5". Each bill measures approximately 2.5" wide and 6" long. How big a box would you need to hold $1,000,000 in $1 bills? Show all of your work. Explain what you did to solve the problem.
>
> Sarah and her dad made a batch of cookies. Sarah's dad ate $1/2$ of the batch of cookies. Sarah and her friend ate $1/3$ of the batch. Can her brother Jim have $1/3$ of the batch also? Explain.

Data Analysis

In order to acquire the ability to think quantitatively, students of all levels must develop a good foundation in data analysis. At the grade 3–5 span, it is extremely important because students begin to understand statistical trends and to accurately read statistical figures, skills that will be crucial in middle and high school. Students are first exposed to data collection and display methods in the early elementary grades. Therefore, a considerable part of the later elementary curriculum should build upon this previous experience:

> Data especially in the form of various types of graphs play a significant role in the information we receive every day in newspapers, magazines, and on television. . . .A focus in grades 3–5 should be to add to and refine the various forms of data representations that students have likely been exposed to in the early grades (Van de Walle & Lovin, 2006, p. 320).

At this point, students will also make the connection between data analysis and other disciplines. They will begin to see that graphs are not only for math, but are also used to convey information in countless other fields. They can be given a newspaper and asked to find a table or graph and to analyze it, regardless of the topic. Additionally, teachers at this level will involve more analysis and discussion regarding data, and students will be asked to make hypotheses and form conclusions based on the data.

To begin the process of analyzing data, students can formulate an engaging, real-world question and then collect, organize, and display their data in an attempt to answer their original question. Through these exercises, students can begin to generate their own data analysis ideas and methods. These ideas can include which statistical methods are most appropriate for a given set of data or how to develop and evaluate inferences and predictions. Students should be exposed to and understand a number of ways to properly display and interpret data. At this level, students will also learn about the common measures of central tendency: the mean, median, and mode.

With respect to numeracy, a considerable portion of everyday, real-world mathematics focuses on interpreting trends and patterns in data. For students to be capable of interpreting the world around them, they need to develop these crucial data analysis skills. Furthermore, students should also be exposed to invalid statistics, so they can learn how to read data with a critical eye and how to avoid being tricked by data. Whereas probability is not a large component of this grade span, exposure to data and statistics will help lay the foundation for future knowledge acquisition of probability and chance.

Sample Problems for Data Analysis

The activity "What's the Meaning of This?" (Van de Walle & Lovin, 2006, p. 324) illustrates a way to engage students in analyzing data and exploring measures of central tendency and dispersion. The exercise provides several data sets and their corresponding mean, median, mode, and range. Students are asked to examine the data sets and their corresponding statistics to make conjectures about what each statistic describes about each set. The class can then discuss the definitions they have developed for the various statistics and how those definitions evolved as they examined each new set of data. In addition to learning about various useful statistics, students also employ problem-solving and conjecture skills.

Once students have discovered the meaning of basic statistics, they can gather real-world data from their classmates, such as quiz grades, heights, or age. They can use this data to calculate maximum, minimum, range, mode, median, and mean.

An important aspect of data analysis is the ability to estimate, experiment, and display real-life data. In the activity "Every Breath You Take," students are asked to make estimates about how many breaths they will take in a given time period; to collect and analyze data, namely, the number of breaths taken during certain time periods; and to represent data physically and graphically.

This exercise incorporates many important elements in the data analysis strand for the later elementary grades. Through estimation, students develop their problem-solving and analytical skills. They must critically analyze the problem to determine what factors or variables can affect the output or their estimate, and how to account for the given parameters (e.g., a specified period of time). Students must also determine if their estimate is reasonable, given the context. Next, by having students collect their own data, they are given the latitude to determine the best, most accurate collection method. This activity provides a real-world opportunity, rather than simply providing unrelated data. Finally, once they have gathered their data, the students can choose the best method to display their findings.

Many curricula use spinner activities to help students gather data and explore experimental probability. A problem titled "Checking the Theory" (Van de Walle & Lovin, 2006, p. 324) has students spin the spinner, record the results, and calculate probabilities:

- Provide pairs of students with a spinner face that is half red and half blue. Students should agree that the chance of blue is one-half.
- Make a tally chart for red and blue results. Collect the results of 20 spins.
- Calculate the experimental probability of getting blue. If it is not 10/20 or 1/2, discuss possible reasons why this may be so.
- Continue spinning in increments of 10 spins. Find the probability of spinning blue for the total number of spins after each increment of 10.

This problem allows students to explore equivalent fractions as well as experimental probability. Students will have the opportunity to discuss what they expect to occur (e.g., getting blue half of the time) and why there might be a discrepancy between what they expect and what actually occurs. Finally, as the students increase the number of spins, they may begin to understand how the experimental probability of an event approaches the theoretical probability as the number of trials increases.

Algebraic Reasoning

As students in the grade 3–5 span begin to reason algebraically, they explore variables, expressions, and equations. They also use their number sense, logical reasoning, and problem solving skills. On the path toward developing quantitative literacy, algebraic reasoning is "an important precursor to the more formalized study of algebra in the middle and secondary grades" (NCTM, 2000, p. 159). Algebra, in many respects, is the generalization of arithmetic and as students become more fluent with both addition and multiplication, they begin to generalize by thinking algebraically. Using algebraic reasoning, students can investigate the commutative, associative, and distributive properties. Moving gradually from the concrete to the pictorial to the abstract will provide students with the developmentally appropriate structure they need to grasp the "big ideas" of algebra.

At this grade span, the idea and usefulness of a variable as an unknown (represented by a box, letter, or symbol) should also be emerging and developing more fully. As students represent and analyze mathematical situations using algebraic symbols, they come to understand the basic notions of equality and equivalent expressions. The idea that the same variable represents the same quantity in a given equation or set of equations is a fundamental algebraic concept students will use throughout their mathematical learning and beyond.

As students develop algebraic thinking, they should be able to identify numerical and geometric patterns and sequences. Once a student determines the pattern, it can be extended using additional numbers or shapes. Identifying and extending patterns is a significant precursor to generalization and formal algebraic reasoning. Students can use tables, graphs, or charts to help organize their work and to determine the mathematical rule for a given pattern. As students explore patterns and note relationships, they should be encouraged to represent their thinking using words, expressions, and equations.

Sample Problems for Algebraic Reasoning

Understanding equivalence is an important precursor to developing algebraic thinking. The idea of equality can be initially explored using a balance scale. Students can visually and concretely see what is happening with the scale as different components are added to each side. In the beginning, the teacher should use an actual balance scale instead of a poster or picture of one. Gradually, students can be weaned to a simplified drawing of the balance scale, as illustrated in the following example.

> **Tilt or Balance:** On the board, draw a simple two–pan balance. In each pan, write a numeric expression, and ask which pan will go down or whether the two will balance. Challenge students to write expressions for each side of the scale to make it balance. For each, write a corresponding equation to illustrate the meaning of =. Note that when the scale "tilts," either a "greater than" or "less than" symbol is used (Van de Walle & Lovin, 2006, p. 311).

The "Variable Machine" lesson provides an introduction to the use of variables. Students create variable machines to discover the value of words, exploring the idea of a variable as a symbol that can stand for a number or value, and substituting numbers for variables to discover values of their names and words. For example, the letter S has a value of 19 based on its position in the alphabet.

Using "magic tricks" is another way to encourage students to think about variables. Each student is asked to think of a one-digit number. Then they are asked to (silently) manipulate the number through three or more simple operations such as add 5, subtract 3, and then multiply by 2. Based on their final number, the teacher or other classmates can "guess" their initial number by working backward from the answer. The operations can be modeled using manipulatives. By using the concrete representations, the students can see how their numbers can be so readily "guessed." These problems intrigue students so that they want to uncover the mathematics behind the number trick.

Through problem-solving, students can see how variables can represent a specific unknown in an equation. The real-world aspect of the following problem helps answer the often-asked student question, "When am I ever going to use this?" The problem has a real-life application and students at this level can relate to the practical components dealing with the bike and mowing the lawn:

> Brian is starting a summer lawn-mowing business to earn money to buy a new bike. He borrowed $225 from his dad to buy a lawn mower. He charges $35 to mow the average lawn. It costs about $1.50 for gasoline for each lawn. That means that his profit from each lawn is $33.50. How many lawns must Brian mow to pay his father back? If he wants to buy a bike for $500, how many lawns must he mow? (Van de Walle & Lovin, 2006, p. 313).

Students learn how variables are used to represent changes in quantities through observing patterns. The next problem gives students the opportunity to observe and record growth and then use tables, equations, and graphs to represent the data. Students should have similar problems in which multiple representations are used to investigate change and for which they pose and answer questions based on the data.

Real-World Functions: Discuss with your class a real situation in which the value of one measure or count will be related to another measure or count (e.g., the weight of jellybeans in increments of 10 jellybeans or height of bean plants compared to the days since they sprouted). The task is to create a table and a graph based on at least seven different values. Students should also find an equation that relates the two values. Once the graph is completed, they should use it to determine another value that's not on the table. In a similar manner, they should use their equation to find values and check to see if those values agree with the graphical representation (Van de Walle & Lovin, 2006, p. 315).

F. Numeracy in Grades 6–8: Quantitative Literacy for Middle School

Middle school is a time of considerable transition and growth. Many students struggle with the social, physical, emotional, and cognitive changes they face. The Northeast Foundation for Children (2005) produces pamphlets that outline common characteristics for children at various ages. Some characteristics typical of early adolescence include seeking peer approval, distancing themselves from adults, hesitating to take risks, beginning to think abstractly, and enjoying a challenge[22].

At this age, cognitive executive functions, such as reflection and analysis, are developing. Students will now be exposed to problems that can be solved in multiple ways and with multiple answers. For some, these concepts will be within their grasp, whereas others will need more support to scaffold the development of multiple paths and solutions to real-world contextual problems. Furthermore, children at this age can be impulsive when faced with more complex tasks, causing them to jump to conclusions or incorrect answers. Additionally, early adolescence is a time when students begin to have an increased interest in real-world experiences and a decreased interest in conventional learning. This shift leads to opportunities to teach quantitative literacy using real-life contextual problems. Recognizing the impact of these unique developmental changes (both physical and cognitive) on the students' learning is a primary task of the middle school educator. Due to the significant variance in developmental and cognitive abilities of their students, teachers need to adjust their lesson planning accordingly: The abstract student requires challenging activities while the concrete student needs more modeling and representations of mathematical ideas and concepts.

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[22]The entire list of common characteristics for adolescents aged twelve to fourteen can be found at www.responsiveclassroom.org/pdf_files/pamphlets/rc_pamphlet_cc8.pdf . Also, the Department of Education (Labrador & Newfoundland, CAN) has complied a useful resource Teaching and Learning with Young Adolescents (Sept. 2001) www.ed.gov.nl.ca/edu/k12/curriculum/documents/adolescents/index.html).

As with any transition, there are differences in how mathematics is presented at the elementary and middle levels for students. During middle school, students will encounter abstract topics for which they may not have the necessary background knowledge. In addition, middle school students now have a dedicated "math" teacher who they may only see for 45 minutes a day. Many middle school teachers notice that incoming students are proficient with many algorithms. While there is power in having students explore the multitude of algorithms for a variety of procedures, an emphasis in the middle grades is to explore the efficiency and advantages of the various algorithms so connections can be made to algebraic concepts. Because of the increased transition to abstract concepts during middle school, many students develop a dislike of math and consequently perform poorly. Thus, a middle school curriculum that embeds aspects of quantitative literacy can assist in providing students with connections and meaning between their reality and the usefulness of mathematical ideas.

In order for students to successfully transition from elementary school to high school, they need to master the following topics while in grades 6–8: geometric reasoning, ratios and extending multiplicative reasoning to proportionality, and statistics and probability.

Geometric Reasoning

Throughout elementary school, students build their fundamental knowledge of geometry to include basic vocabulary and shape recognition. While in middle school, the geometry strand provides the bridge between this fundamental knowledge from elementary school and the abstract knowledge gained in a high school geometry class. During middle school, students learn to use geometric reasoning by learning how to make generalizations and conjectures based on both inductive and deductive reasoning. However, in order to do so, students must broaden and strengthen their geometric vocabulary and their ability to visualize spatial representations.

A child needs to have an adequate and accurate geometric vocabulary in order to facilitate the development of geometric reasoning. According to the NCTM standards (2000), the vocabulary of two- and three-dimensional objects is an integral part of the geometry and measurement strand at the 6–8 grade level. Being quantitatively literate with respect to geometry and measurement refers to the ability to understand the vocabulary and to use it correctly. Without this knowledge base, middle school students are unable to appropriately compare and contrast objects, an important step in geometric reasoning. Furthermore, many standardized test questions, including those on the NECAP exams, require a student to know geometric vocabulary in order to answer correctly. By understanding the vocabulary, middle school students are able to transition from the specific characteristic of a geometric shape to a generalization about a group of geometric shapes. These generalizations are a critical step in formal reasoning and proof.

To help form generalizations, a teacher might ask students to draw several parallelograms on a coordinate grid or with dynamic geometry software, such as Geometer's Sketchpad®. Students can then make and record measurements of the sides and angles in order to observe some of the characteristic features of each type of parallelogram. Next, they can generate definitions for these shapes that are correct and consistent with commonly accepted definitions. They can also recognize special relationships among the corresponding parts of the parallelograms. A Venn diagram (see Figure 3) may be used to summarize the observation that a square is a special case of a rhombus and a rectangle, each of which is a special case of a parallelogram.

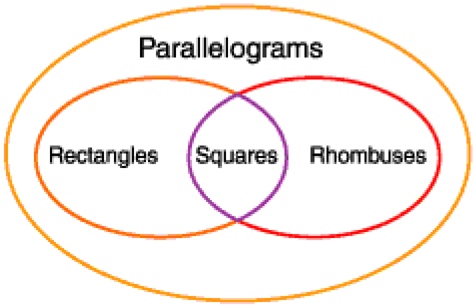


Figure 3: A Venn diagram showing the relationship among types of parallelograms (NCTM Standards, Grades 6–8 Geometry)

In addition to developing their vocabulary, proficiency in visualizing and reasoning about spatial relationships is essential for geometric modeling, reasoning, and problem solving. Many students struggle with spatial problems because they are unable to visualize a shape and its characteristics accurately. For example, determining surface area of a three-dimensional solid from its two-dimensional representation shown in a textbook is not easy for this age group. One of the main reasons students are unable to do this is because of their difficulty in visualizing the unseen faces of the three-dimensional solid. To help students develop their spatial visualization and reasoning skills, teachers should supply students with concrete, hands-on manipulatives of three-dimensional solids, such as a cube or a can of soup. Students should handle these solids and discuss how many sides they see from a given perspective and how many sides they know the shape actually has, seen or unseen, from their vantage point.
After experiencing the concrete, students can move to the two-dimensional pictorial representation and then to abstract visualization. They can also construct and deconstruct solids from their two-dimensional nets[23]. Through hands-on examination, building, and decomposing of complex two-and three-dimensional objects, students will develop their understanding of three-dimensional objects and their corresponding two-dimensional representations.

To enhance the numeracy aspect of these topics, middle school geometry can be connected to many other school subjects and real-world applications, including science, geography, writing, and art. Students can study scale drawings or models of a building from different views, or they can consider the inflexibility of triangles and their use in construction. They can also look at the building's corresponding floor plans in order to experience what a cross-section of the building may look like. Teachers can bring together vocabulary and spatial visualization by having students draw geometric figures upon hearing a written or verbal description. Conversely, students can be shown a geometric figure and be asked to compose a geometrically accurate description. Students can also observe and explore naturally occurring geometric patterns, such as naturally faceted gemstones or golden rectangles (i.e., rectangles in which the ratio of the lengths is the golden ratio, $(1+\sqrt{5})/2$.

As they promote the development of quantitative literacy in their students, middle school teachers face an additional hurdle--mathematics textbooks generally only devote a chapter or two to geometry. Often this topic is taught at the end of the school year, leading some students to become "geometry deprived." To help alleviate the "end of year crunch," teachers can teach from a quantitative literacy standpoint year-round by integrating several topics. For example, they can introduce geometric concepts as they are exploring ratios and proportions, which is a major theme of middle school mathematics.

Sample Problems for Geometric Reasoning

[23] A net is a two-dimensional figure that can be cut-out and folded up to create a three-dimensional solid.

The problem below illustrates using a geometric network to solve a real-world efficiency problem:

Caroline's job is to collect money from parking meters. She wants to find an efficient route that starts and ends at the same place and travels on each street only once.

A. The streets she has to cover are shown in map A. Find and trace such a route for her.

B. A new street, shown in map B, may be added to her route. Can you find an efficient route that includes the new street?

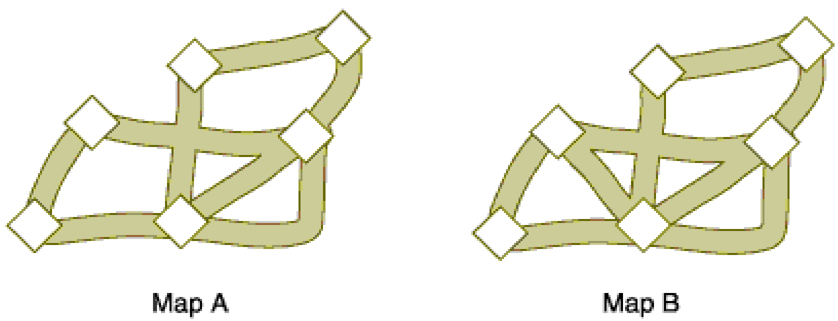


Figure 4: **Networks used to solve efficiency problems. Adapted from Roberts (1991), pp. 101–7.**

A teacher may ask students to determine one or several efficient routes that Caroline might use for the streets on Map A, share their solutions with the class, and describe how they decided on those solutions. Students should note the start and end point of each route and the number of different routes that they find. Students could then try to find an efficient route for Map B. They should eventually conclude that no routes in Map B satisfy the conditions of the problem and discuss why no such route can be found. The teacher might suggest that students count the number of paths attached to each node[24] (in Figure 4, each node is represented by a white square) and determine where they "get stuck" in order to understand better why they reach an impasse. To extend this investigation, students could look for efficient paths in other situations, or they might change the conditions of the Map B problem to find the pathway with the least backtracking. Such an investigation in the middle grades is a precursor of later work with Hamiltonian circuits, a foundation for work with sophisticated networks. Visual demonstrations, such as the networks above, are often a helpful tool for the middle-graders so they can analyze and explain mathematical relationships and events.

The kite problem (see Figure 5) can help students use their vocabulary[25], take measurements, make generalizations about congruence and similarity, and work with ratios. Middle school students can study these concepts using numerous everyday objects and diagrams, such as a kite comprised of overlapping triangles. The kite can be disassembled into four individual triangles. Through investigation, students can measure the angles of the individual triangles to determine whether the corresponding angles are congruent. They can also measure the lengths of the sides of the triangles to determine if a constant scale factor exists. Through this process, students can develop a more formal, generalized definition of congruence and similarity with respect to the sides and angles of similar triangles.

[24] In a network, a node is the intersection of two or more pathways.

[25] Teachers can extend this activity by encouraging a discussion relating the everyday usage of the term "kite" to the actual geometric definition of a kite. For example, the "kite" shown in Figure 5 is actually not a kite in the geometric sense. The definitions of some words differ greatly when used in a math-specific context versus in everyday language. Another example is the use of the word "combination." Many things (such as phone numbers) are described as a combination of numbers, when described mathematically, a phone number is actually a permutation. Understanding the subtle yet important distinctions and connections among mathematical terms and their broad use in everyday contexts illustrates core aspects of quantitative literacy.

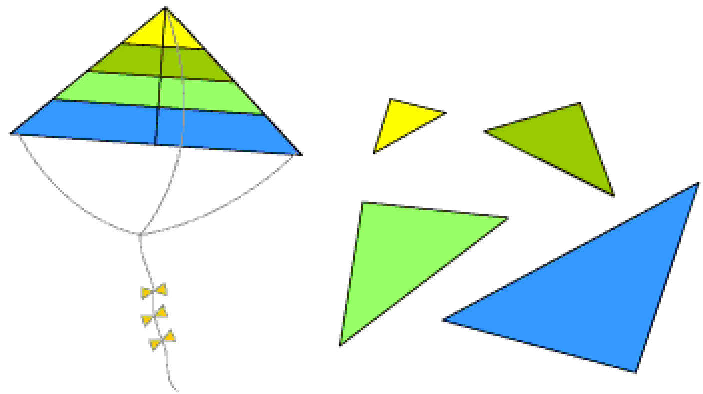


Figure 5: **Kite formed by overlapping, similar triangles (NCTM Standards, Grades 6–8 Geometry)**

Similar shapes allow middle-graders the opportunity to evaluate the relationships among different shapes. "For example, an investigation of the perimeters, areas, and side lengths of the similar and congruent triangles in the kite example could reveal relationships and lead to generalizations" (NCTM, 2000, p. 234). From this, the teacher may lead the class to discover the connection between the ratios of the side lengths, of the perimeters, and of the areas of the similar shapes. Students at this age may determine that the ratio of the perimeters is the same as the scale factor while the ratio of the areas is the scale factor squared. To enhance this activity, the teacher can ask students to conjecture why these relationships may be true. With dynamic geometry software, such as Geometer's Sketchpad®, students can test the conjectures using other examples. This guided activity is included on the NCTM Web site, along with many other useful examples.

Extending Multiplicative Reasoning to Proportionality:

In its *Principles and Standards for School Mathematics* (2000), the National Council of Teachers of Mathematics proposes an emphasis on proportionality in the middle grades. Proportional reasoning is more than just two equal ratios. It is evident through many areas of the middle level mathematics curriculum, such as percents, similarity, scaling, linear equations, slope, and probability. Several of the grade level expectations (GLEs) found in the *K-12 NH Curriculum Framework* have connections to proportionality. Many traditional textbooks devote a few lessons to solving proportions using the cross-product algorithm. However, with a numeracy emphasis, proportionality is one form of mathematical reasoning that involves covariation, multiple comparisons, and the ability to remember and process several pieces of information. It is concerned with inference and prediction, and it involves both qualitative and quantitative methods of thought.

A deep understanding of the multiplicative relationship of proportions is critical to being quantitatively literate. According to Cramer, Post, and Currier (1993), the "critical component of proportional situations is the multiplicative relationship that exists among the quantities that represent the situation." Unfortunately, though, many students can perform the procedures often associated with proportional type problems without appreciating this relationship. Hoffer (1988) stated that "being able to perform mechanical operations with proportions does not necessarily mean students understand the underlying ideas of proportional reasoning and the ability to firmly understand proportionality is a turning point in mental development." Likewise, the ability to reason proportionally develops throughout the middle grades slowly over a number of years; and problem situations involving proportional reasoning should be based on representational contexts. One of the importance factors to consider when selecting contexts to support the development of students' proportional reasoning is how well the context lends itself to meaningful representations. As a result, a middle school mathematics curriculum that develops proportional reasoning is preparing students to be quantitatively literate adults.

According to Lappan (2000), "as students encounter mathematics in the upper elementary grades, the emphasis changes from a focus on the additive structure of numbers and relationships to the multiplicative structure of numbers and relationships. This means that the students are faced with new kinds of numbers, fractions and decimals that rely on multiplication for their underlying structure" (Mathematics Education in the Middle Grades, p. 24). With regards to fractions, multiplicative reasoning is necessary to understand the concept of an equal-share. For example, if a student is told to share ¾ of his 60 jelly beans, he must determine how many jelly beans would be in each of four equal groups to determine how many beans constitute one-fourth. This can be accomplished by thinking of a missing factor or using division, the inverse of multiplication. He can then use multiplication (or repeated-addition) to calculate how many beans would equal three one-fourths. The multiplicative structure of decimals is two-fold. First, decimals are a special representation of rational numbers; therefore, the concept of an equal-share is still paramount. Second, decimals are based on an equal share which is a power of 10, where powers indicate repeated multiplication. This multiplicative structure is directly related to proportionality, a topic involved in countless real-world applications and, thus, crucial to numeracy.

When entering middle school, children have a rather concrete understanding of fractions, decimals, and percents. They can recognize the different formats and the equivalent representations of commonly used fractions, but beyond that, deep understanding is scattered and inconsistent. In order to develop a deeper understanding of the connection between the different representations of rational numbers, a student must understand proportionality. For example, ¾ and 75/100 are proportional; thus, ¾ is equivalent to 0.75. In addition, many students incorrectly assume that all fractions represent a part-to-whole ratio, and they do not understand the relationship between fractions, decimals, and percents. Furthermore, they have difficulty processing the connection between a remainder, a decimal, and the corresponding fractional part. The learning objective in middle school is to ensure that students become proficient with respect to rational numbers and to take this knowledge a step further as students begin to transition to multiplicative reasoning and understanding proportionality.

Many students, without realizing it, have probably solved everyday problems using rational numbers concepts and proportionality. For example, a student can calculate the cost of 8 limes, if they are priced as 5 limes per dollar. These prior experiences can be useful in moving students toward a more formal understanding of fractions and their various roles. Mack (1990) concluded that students can best learn fractions if they build upon their prior, informal knowledge:

> Students possess a rich store of informal knowledge of fractions that was based on partitioning units and treating the parts as whole numbers. Students' informal knowledge was initially disconnected from their knowledge of fraction symbols and procedures. Students related fraction symbols and procedures to their informal knowledge in ways that were meaningful to them; however, knowledge of rote procedures frequently interfered with students' attempts to build on their formal knowledge (p. 16).

Even though students have unknowingly solved problems involving fractions outside their mathematics classrooms, they often have entrenched misconceptions which impede the transfer of knowledge to more complex concepts. To encourage numeracy, a teacher should uncover, understand, and correct these misconceptions. Also, to enhance long-term learning, teachers should use ratios and proportions as a "unifying thread" and "cohesive theme" tying many topics together, rather than as a distinct topic (Lanius & Williams, 2003).

Understanding ratios and proportions is critically important, as they are the foundation for secondary mathematics topics, including the trigonometric ratios. Students begin working with functions, rates, and slope toward the end of middle school and will continue working with these concepts throughout high school and beyond. To build a solid foundation for algebra, middle school students need to develop a more integrated understanding of how a ratio relates to the slope of a line. Students should understand that ratios of the vertical distance to the horizontal distance between any two points on a line are proportional. For example, a vertical distance of 5 compared to a horizontal distance of 2 is proportional to a vertical distance of 2.5 compared to a horizontal distance of 1. Students will also explore the similarities and differences between direct variations (also known as directly proportional relationships) and linear functions that are not directly proportional. According to the National Mathematics Advisory Panel Final Report (2008), "A major goal of K–8 mathematics education should be proficiency with fractions (including decimals, percent, and negative fractions), for such proficiency is foundational for algebra and, at the present time, seems to be severely underdeveloped" (p. *xvii)*. If this foundation is not secure, students will struggle with much of high school algebra and many situations they may encounter in adulthood.

A strong foundation in proportional reasoning is critical to algebraic thinking. However, regardless of their preparation and performance with proportions in elementary mathematics, many students enter middle school with a pre-conceived idea that algebra will be difficult. They may have heard their parents or older siblings remarking, "I was never good at algebra in high school." Before even putting pencil to paper, students fear algebra and can have an inhibitory mental block. In order for students to be successful in middle school math and beyond, teachers should explicitly connect algebra to their prior experience with proportions. They can mitigate potential negative attitudes toward algebra by using prior knowledge and everyday language, rather than "slope" and "function," as well as a concrete approach utilizing manipulatives, graphing calculators, and web-based applets. The GLE F&A:7:2 even addresses the validity of using less formal approaches to gaining and demonstrating understanding: a student can demonstrate their conceptual understanding of linear functions as constant rate of change by "informally determining the slope of a line from a table or graph" (*NH Curriculum Framework*, p. 28). Once students become more comfortable with algebraic concepts, the teacher can move them through pictorial representations and then onto more abstract ones. This is also an opportune time to begin using proper, more formal vocabulary and terminology as the accepted norm.

The Rhode Island Department of Education has some helpful units of study for "Counting Techniques and Probability" and "Proportional Reasoning" that are specifically targeted to the middle school teacher Retrieved 01/06/2010 http://www.ride.ri.gov/instruction/curriculum/RhodeIsland/resources/units.aspx.

Another useful resource for teaching proportional reasoning is the Rational Number Project (RNP). The RNP, funded by the National Science Foundation since 1979, is an on-going research project investigating student learning and teacher enhancement. A primary focus of the project is enhancing students' proportional reasoning abilities and disseminating research findings regarding the teaching and learning of proportional reasoning that have the potential to impact classroom instruction. As of the writing of this document, the project has produced 86 research publications. Retrieved 02/06/2010 http://education.umn.edu/rationalnumberproject.

Sample Problems for Proportional Reasoning

Proportional reasoning is a topic that easily lends itself to real-life problems, as students can develop several methods to solve these problems with little outside instruction. Their answers can then be used to make generalizations. The valuable learning occurs when a student makes a plausible generalization, tests it, realizes that the generalization is true and why, and then solves a subsequent problem using the generalization, rather than revisit the initial method. If students learn multiple approaches to solving proportional reasoning problems, the following three things can be accomplished:

- Focus attention on connecting mathematics with the real world;
- Focus attention on connecting different concepts and skills within mathematics; and
- Reinforce the themes of problem solving, communication, and reasoning by identifying the strengths and weaknesses of different strategies.

These are all elements of quantitative literacy. The following are problems that illustrate extending multiplicative reasoning which can be used to make connections to proportional reasoning. These connections are appropriate for developing quantitative literacy related to proportionality in the middle school grades:

The Flu Problem

The doctor says that you have a new strain of flu virus that doubles every 1 hour. If you start with 1 flu cell, how long would it take you to have 1,000 cells?

Two methods students may use to solve this problem are drawing a tree diagram or generating a table, such as those shown below:

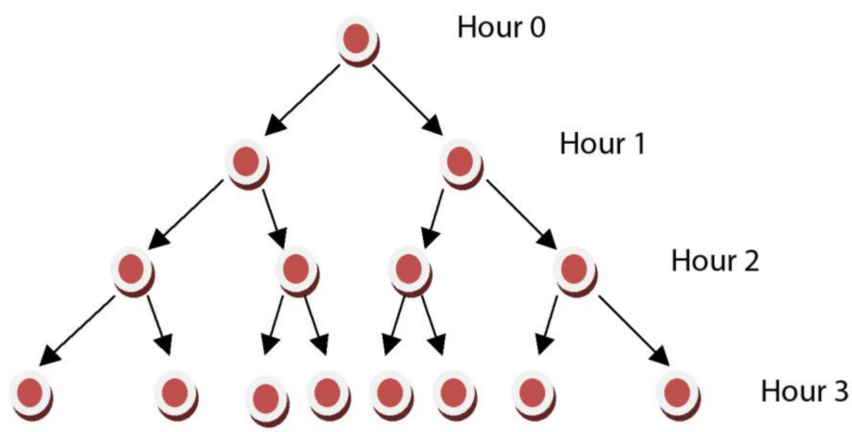


Table of Flu Cell Division

Hours	*New Flu Cells Produced*	*Total Number of Flu Cells*
0	*1*	*1*
1	*2*	*3*
2	*4*	*7*
3	*8*	*15*
4		

As students develop their multiplicative reasoning by working on the "Flu Problem," they will begin to see multiplicative patterns. Several students will be able to make a conjecture without completing the entire tree diagram or table of values. To make the exercise more challenging, students can be asked to determine how long it would take until they had 100,000 or one million cells. Teachers can also require students to find as many numerical patterns as possible and express their conjectures orally. This can be an opportunity to explore whether proportional reasoning can be applied in this problem or not. An instructional implication to consider when teaching proportional reasoning is to present counterexamples of tasks that do not represent proportionality. One way to approach this discussion would be to have students create graphs of the data presented through this situation. Through careful study of proportionality, students should come to generalize that proportional situations produce linear graphs through the origin, also known as direct variation. Thus, through a graphical representation, students can compare the characteristics of these graphs to those depicting proportionality and develop conclusions based on those comparisons. It is this type of reasoning that promotes the development of quantitative litearacy.

The next problem promotes numeracy through understanding proportions involving fractions and percents:

Recipe Conversion

You will be converting a favorite home recipe into various portion sizes. You will need a recipe with at least 6 ingredients that require 6 different quantities. At least two quantities should be two proper fractions and two should be mixed numbers. Based on the original recipe, determine the quantity of each ingredient you will need if you were going to make a half portion; a quarter portion; and a double portion.

This problem is a good example of applied, contextual fractional work that involves a familiar activity. Furthermore, it addresses both increasing and decreasing the size of the recipe and allows for the discussion of how multiplying by $\frac{1}{2}$ is the same as dividing by 2. The activity can be easily differentiated by reducing the number of ingredients or the number of different denominators within the recipe. For a student who is struggling with fractions, the teacher can use actual measuring cups and a box of rice so the student can have a hands-on experience and see what happens when a recipe is doubled. The activity can be interdisciplinary by incorporating cooking or computer skills by having students prepare a spreadsheet. The impact of this activity is that the context allows multiple representations that are accessible to most middle school students.

As students become more quantitatively literate, they can start to extrapolate from an experimental ratio to a theoretical proportion. The "M&M Math" activity (below) can be used to develop proportional reasoning in countless ways:

M & M Math

Using a 1.67 ounce bag of M&Ms, fill in the following table for each color relative to the entire bag of M&Ms:

Color	***Amount***	***Fraction***	***Decimal***	***Percent***	***Degrees***[26]
Brown					
Blue					
Yellow					
Orange					
Green					
Red					
Total					

A teacher can introduce this activity by asking students to predict the values they will fill in for each color and why or how they made those predictions. As students are physically involved in the activity, they can begin to transition between the use of fractions, decimals, percents, or degrees to represent a portion of a whole. Following the activity, students can discuss or display their completed charts, and explore why their answers may be different. Students may then be asked, "Based on the ratios for the 1.67 ounce bag, how many of each color would you expect in a 14, 32, or 64 ounce bag? Do you expect the ratios to be consistent for all bags? If not, why?" Finally, a class may decide to repeat the experiment using a larger bag of M&Ms to determine how close their estimates, based on the ratios from the 1.67 ounce bag, are to the actual ratios. To incorporate additional quantitative skills, students can display the data using other representations such as a bar graph or pie chart, or they can discuss which rational number format they find the most useful in this situation. An extension may involve part-to-part ratios, such as the ratio of yellow to red.

[26] "Degrees" refers to the number of degrees of a circle if the data were displayed on a circle graph, with the circle representing the bag of M&Ms.

When teaching numeracy, money-related problems such as the Credit Card problem below are probably the most relevant, real-world applications:

> **Credit Card**
> *You've been charging your school expenses to a credit card and have acquired a balance of $5,000. Your credit card charges an annual interest rate of 18%. Assuming you charge nothing more to your credit card, at this rate, what is your monthly payment for interest only? Suppose the credit card requires that you make minimum monthly payments of $70. How long will it take you to pay off the balance?* (Retrieved July 15, 2009, from www.math.cudenver.edu/~wbriggs/qr/news_problems.html#anchor7069).

This problem promotes valuable class discussion and critical thinking as it lends itself to numerous follow-up questions: "Do you want to get yourself into this situation?"; "What does an annual interest rate of 18% actually mean?"; "What happens when your minimum monthly payment does not even cover your monthly interest – as in this particular situation?"; "If you could pay $100 per month, how would that affect the length of time it takes to pay off the $5,000?"; "If you had a choice, would you prefer your interest payment be calculated each month or at the end of the year?"; "What is *compounding* and how does it work?" Teenagers will be bombarded with credit card applications before they leave high school; it is an important life skill to truly understand the risk and consequences of acquiring credit card debt.

Academically, most students are first exposed to the concept of unit rates as in "how many miles per hour" or "cost per pound" in middle school. However, they have most likely dealt with this concept several times before outside the classroom. The "how many per. . . ."concept is undoubtedly one of the most applicable to real-life, non-math related pursuits such as currency conversions, grocery shopping, travel, and measurement. Given that unit rates are per one unit (such as 50 miles per hour), students can solve these problems by focusing on one quantity and essentially ignoring the other. For example, if asked to calculate the distance traveled in 3 hours, the student would likely just multiply 50 by 3. Students have a general understanding how to solve these problems, but they need to become proficient with conceptual knowledge so that they can transfer the knowledge to more complex topics (such as rates of change and slope, to be discussed in the following section). Students can develop a stronger understanding of proportionality using rates if they can see the connection to (50 miles / 1 hour) = (how many miles / 3 hours). In addition, this is an activity that lends itself to other representations such as a table or graph. By using the unit rate, students can begin to generalize the multiplicative pattern that is found in the table or graph by applying the given unit rate to various number of hours in order to determine total miles traveled. For example, if x represents the number of hours traveled and y represents the total distance in miles traveled, then the relationship can be expressed by the linear equation $50x = y$. Students can begin to identify patterns in the coordinates listed in the table and how that pattern translates to a graph of this direct variation. Thus, using a word description, algebraic notation, table, and graph students can develop their understanding of proportionality through a variety of representations.

The Italian Market problem is a good example of how to teach numeracy using both price per unit and unit conversions:

> **The Italian Market**
> *Suppose you are visiting an Italian market and see tomatoes priced at 3.20 Euros per kilogram. Assume that 1 kilogram = 2.2 pounds and that the current exchange rate is $1 = 0.9 Euros. What is the price of the tomatoes in dollars per pound?* (Retrieved July 15, 2009, from www.math.cudenver.edu/~wbriggs/qr/news_problems.html#anchor7069).

The Italian Market problem provides the opportunity to discuss different currencies and various units of measurement, including the metric system. To make the problem less challenging, teachers can use whole numbers and require only a single conversion. It could be made more challenging by using several currencies or units, by including a currency exchange fee, or by asking students to answer a question such as, "Would you prefer to buy the tomatoes using dollars or yen?" There are several aspects which make this a more difficult proportion problem for students. First, the use of non-integral relationships among the quantities may cause confusion or frustration for students, especially if their proportional reasoning is in its early development. However, these "nasty" numbers represent real-world situations; and middle school students need to encounter them as often as possible. Also, the context of this problem is difficult to represent through physical models, and students of this age may not understand the concept of different currencies and exchange rates. Thus, the suggestion to use whole numbers is a way to highlight the proportional thinking needed to solve problems like this and to enhance transferability to non-integer data. Finally, multiple comparisons are occurring in this problem, so students may need organizational tools and systematic forms of record keeping-such as ratio tables-to enhance their ability to solve similar problems.

Using the "How many per..." concept below is also a great way to segue into more direct algebraic problems involving rates:

Fast, Normal, Slow Walk
Students will walk a pre-determined distance (e.g., 25 ft). Each student will take turns walking the distance at a normal, fast, and slow pace while another student keeps the time. The times are recorded and the students calculate each rate in feet per second.

Additional task: the students can graph the distance over time (rate) for each speed.

Speed	***Student _______***	***Student _______***	***Student________***
Fast			
Normal			
Slow			

Students are kinesthetic and like to be active. This activity allows them to move around. Given the various roles involved (e.g., walker, timekeeper, data recorder), many students can participate, regardless of math or language ability.

Prior to beginning the activity, students can hypothesize and estimate about the rates at various speeds. Once completed, the class can make generalizations about the relationship between rate and speed, such as a faster speed yields a higher rate (i.e., more feet covered per second). The teacher can introduce the different written formats for rate. For instance, 25 feet per 5 seconds could be written as the fraction $25/5$. Students can then discuss whether the rate, like a fraction, could be reduced to $5/1$. Finally, they could test if walking at a rate of 25 feet per 5 seconds is the same as walking at a rate of 5 feet per second.

This type of problem offers a wonderful opportunity for students to experience the connection between a chart or a table and the corresponding graph and equation—all ways of displaying proportional relationships[27]. Oftentimes, students learn these various representations individually. Seeing them simultaneously helps students realize the connections among all three forms representing the same set of data (Donovan & Bransford, 2005). By creating all three representations, the class can have the chance to discuss the benefits and disadvantages of each in any given situation. For example, would it be more useful to display this data and convey its relationship using a graph, a table, or an equation? Why or why not? Again, this activity is presented in a context that is easily represented in forms that students will have knowledge of or experience with. By analyzing the characteristics of

[27] *Functions Modeling Change: A Preparation for Calculus* (Connally, Hughes-Hallett, Gleason, et al, 2007) describes a "Rule of Four" which promotes multiple representations for functions. Functions should be represented symbolically, numerically, graphically, and verbally.

each representation, students will develop a deeper understanding of the characteristics needed to define a proportional situation, thus allowing them to know when to use proportions or not. A further extension of this activity could be to ask students how they could change the situation so it would not be proportional. The use of examples and counterexamples to develop proportional reasoning will prepare students for mathematical decision making which is at the heart of quantitative literacy.

By moving just beyond their prior knowledge, teachers can informally introduce students to algebraic thinking using problems similar to "A Lemonade Stand":

A Lemonade Stand

Scenario: You have a lemonade stand. You spent $5 on supplies. You sell a cup of lemonade for $.50. How many cups of lemonade do you need to sell to reach a "break even point" (cover all of your expenses and make no money)? What if you wanted to make $2, or $4, or $6… ?

Equation	***Cups Lemonade***	***Money Made***
		0
		$2
		$4

At this age, few students will use an equation to solve this problem. However, they will do the same calculations that would be required if one were using an algebraic equation. The class can discuss how they arrived at their values, what the output value depends on, and what happens when they change the input values. The teacher can use this opportunity to revisit the concept of a variable. This guided discussion can lead students to the equation $0 = \$5 - .5x$. Conversely, the problem could ask students to determine how much money they would make for given quantities of cups of lemonade. This question can serve as a counterexample to proportional reasoning, helping to strengthen deep understanding of the concept. Again, it also can be connected to a graphical representation of the situation to solidify when one would want to apply proportional reasoning or not.

To integrate more skills into this exercise, students can look at the various representations of the data, similar to the "Fast, Normal, Slow Walk" problem. However, when graphing the data, many students will be compelled to "connect the dots." Although the equation is that of a line, cups of lemonade are discrete data and cannot be represented as points on a continuous line. The teacher will need to make a pedagogical decision regarding this distinction since much of what students will face in high school algebra involves the equations and graphs of lines rather than of discrete points.

All of the problems previously presented are precursors to formal algebraic thinking. It shows students that they can already think algebraically without having written a standard, formal equation. They simply needed to understand the quantitative relationship and the multiplicative patterns involved. According to NCTM Principles and Standards, "students in the middle grades should learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationship and as a style of mathematical thinking for formalizing patterns, functions and generalizations" (p. 223). The NCTM Navigation Series includes problems for helping students develop algebraic thinking.

In conclusion, multiplicative reasoning and its extension to proportionality is at its critical understanding throughout the middle grades. Proportional reasoning is connected to the algebraic thinking used in most high school mathematics courses. To summarize, proportional reasoning involves:

- Multiplicative relationships
- Direct variation and $y=mx$

- Linear Relationships
- Constant rate of change
- Similarity
- Scaling
- Percentages
- Probability
- Data Collection
- Graphing
- Comparing Quantities
- Inferences
- Predictions
- Qualitative and quantitative data
- Connecting mathematical experiences
- Mathematical reasoning and problem solving

With so many connections and applications, it is imperative proportional reasoning is a focal point of middle school mathematics and the development of quantitative literacy.

Data, Probability, and Statistics

As Scheaffer (2003) highlights, "there are strong ties between statistical thinking, data analysis, and quantitative literacy in terms of historical developments, current emphases, and prospects for the future" (p. 146). In order for students to learn from the past and progress in the future, they will need to have a very strong background in statistics and data analysis. By definition, a huge component of what makes one numerate is their ability to interpret data. To that end, mathematics should be taught in a way so as to utilize tools that aid in the understanding of a given topic, including how to best display data for a given audience or means of communication and how to interpret data.

While elementary students spend valuable time gathering, displaying, and interpreting data, most students are first exposed to statistics as a formal branch of mathematics in middle school. Teachers need to build upon this introductory knowledge and begin to expand their understanding of measures of central tendency. In middle school, students use problem-solving to determine how data can affect statistics; how the composition of a sample can lead to bias; and how outliers effect the mean, median, and mode (GLE M:DSP:7:2). Students should discuss the best ways to display data in a meaningful way as well as learn how to read and interpret graphs, tables, and other statistical data. Furthermore, students can investigate how the way in which data is displayed can actually be misleading or in accurately represent the sample. For example, a line graph might show a strong positive relationship. Yet, upon closer inspection, the increments on the y-axis are unreasonably small, making the slope be over-exaggerated. While generating their own data and statistics in class is extremely worthwhile, middle school students should also spend time looking at data and statistics outside their classroom. Students can be asked to find statistics in newspaper articles and then interpret the relevance of these statistics to the underlying, newsworthy story. They can also be asked to formulate an argument and find data and statistics to either support or refute their hypothesis.

Middle school is a critical time when students begin to be exposed to the meaning of statistics and the variety of representations of data. They begin to encounter the issues of the world around them in a quantifiable way and then attempt to form quality opinion about these issues. Data and statistics are of the utmost importance to becoming quantitatively literate and becoming a productive, informed citizen.

Sample Problems for Data, Probability, and Statistics

By using a statistical data project (like the one outlined below), teachers can have students delve into many aspects of data collection, representation, and interpretation. This project is a simple, representative example of how teachers can begin to approach the statistical process to meet GLE DSP 7:6 and 8:6. Teachers can use their creativity to enhance and extend this sample project to better fit their classes' needs:

Statistical Data Project

Objective: *Demonstrate your understanding of percent equations, proportion equations, and pictorial representations of data. You will utilize classroom-taught tools in a real-word situation (data analysis). Finally, you will need to communicate accurately your findings clearly and neatly through both formula solutions and graphs.*

Procedure: *Collect data from at least 75 people on a survey question of your choice. Your survey question needs to be approved by the teacher. Your survey question needs to have at least three possible responses but no more than six responses.*

Data Presentation includes:

1. *Percent that each response is of the total population*
2. *Mean, median, and mode*
   **Include a statement of which "Tendency of Central Measurement" best describes the data and why. Note: the why needs to be specific and related to the reasons for collecting and analyzing the data.*
3. *Range, Upper Quartile, Lower Quartile, Interquartile Range*
   **Given the rule that any data outside 1.5 times the Interquartile Range are outliers, do you have any outliers?*

Data Presentation (Pictorial):

1. *Scatterplot*
2. *Histogram*
3. *Box and Whiskers Diagram*
4. *Frequency table*

This project is extensive, as it requires students to collect data, produce several statistics, and generate many pictorial presentations for a statistical project from beginning to end. The project asks students to explain which measure of central tendency describes the data most accurately. Students answers to this question are important in assessing how well they understand the different measures and how they are calculated, as well as where they are in their trajectory to becoming numerate.

Teachers need to be careful that a project such as this does not simply become a data gathering exercise. Many students and, often, many teachers, enjoy the process of formulating the question and gathering the data. Yet the real deep quantitative learning comes from interpreting and communicating the data. For example, students should be asked which pictorial representation is the most useful, given their data, or what difficulties they had completing this project. Additionally, students can be asked if they feel their statistics are an accurate snapshot of their data or the population as a whole or if biased data may have resulted.

Often textbook examples provide printed, pre-determined data. While it is a beneficial exercise for students to learn how to collect their own data, it is also an important skill for them to be able to analyze provided data. Teachers should not shy away from giving students data or statistics from outside sources and asking them to answer corresponding questions.

The "Double Dilemma" problem introduces students to probability involved in a non-math-related setting:

> ***Double Dilemma***
> *Mrs. Callens and I were talking about board games the other day. She said she and her daughter and son were playing Monopoly® over the weekend. In Monopoly, you get an extra turn when you throw a double. She said she was really lucky in those types of games because she rolled doubles about 1/3 of the time.*
>
> *Show Mrs. Callens how much you know about probability and either have evidence that proves she was lucky that day or that it was not luck— that anyone would expect to throw doubles 1/3 of the time.*

Even if a student has not been exposed to many probability problems, this problem can help develop a better understanding of chance. Working in groups, children can use several methods to help them answer the question. They can toss a pair of dice and try to generate a plausible experimental probability, or they could create a tree diagram and find all the possible combinations when tossing two die. The teacher could facilitate an extended numeracy discussion by asking students whether they think having an outcome occur one out of three times is frequent or infrequent, and what are some common occurrences that might happen one-third of the time. Through middle school, students need to move beyond simply calculating probabilities. They need to compare and contrast theoretical and experimental probabilities, create simulations, and design fair games (GLE M:DSP:6:5, 7:5, & 8:5). With this foundation, they can begin exploring odds in high school math.

G. Numeracy in Grades 9–12: Quantitative Literacy for High School

By the time students reach high school, they have been exposed to most of the math concepts usually covered in a general secondary curriculum. Therefore, the "big ideas" for the high school grade span are topics that have already been introduced, used, and explored in the lower grades. A good secondary mathematics program should focus on the processes and applications associated with these topics. High school is a period when students should apply what they have already learned in order to employ higher-order problem-solving. Interestingly, these "big ideas" are very similar to the "Process Standards" in many of the published mathematics standards (NCTM, 2000; NH Frameworks, 2006).

<u>Number Sense and Operations</u>
Number sense continues to be an important mathematics topic even in high school instruction. Success in algebra, geometry, and other topical areas hinges on having a deep understanding of numbers. NAEP (National Assessment of Education Progress) discusses number sense as "comfort in dealing with numbers—and addresses students' understanding of what numbers tell us, equivalent ways to represent numbers, and the use of numbers to represent attributes of real-world objects and quantities." This is particularly true with rational numbers and proportional reasoning, especially at the high school level. Furthermore, "[t]he curriculum should afford sufficient time on task to ensure acquisition of conceptual and procedural knowledge of fractions and of proportional reasoning" (National Mathematics Advisory Panel, 2008, p. xix).

As students progress through various curricula, ratios and proportions are used in all areas of mathematics (e.g., algebra, geometry, trigonometry, statistics, and calculus). With increased content and procedural knowledge regarding all types of numbers and operations, high school students become more creative, efficacious mathematicians. They begin to use ratios and proportions to manipulate equations and facilitate solving problems. Additionally, most real-world applications and post-schooling vocations involve ratios or proportional thinking, such as cooking, architecture, and engineering.

High school mathematics curricula devote much time to learning various special ratios, such as the trigonometric ratios. Students should understand when an expression is "undefined," what "undefined" means, and how to determine when or where an expression is "undefined." As students move toward calculus, they begin to realize how changing the magnitude of either a numerator or a denominator impacts the value of the rational number.

During high school, students begin to work more with irrational and imaginary numbers. They must be able to compare and contrast the different types of numbers as well as understand their uses and limitations. Students should be able to put various numbers into the context of the complex number system and interpret meaning when a solution is a certain type of number. For example, when the solutions to a quadratic equation are imaginary, a student should be able to interpret what those types of solutions mean.

Secondary math teachers should spend sufficient time helping students develop number sense. Number sense is largely absent from high school content due to a number of potential factors: teachers feel that number sense is or should be mastered in the elementary and middle school grades; the current high school curriculum is already full with high school specific content, not allowing time to focus on number sense; or secondary education instruction and pedagogy is not rich in number sense content, leaving many high school teachers without the proper training or confidence to teach it.

<u>Sample Problems for Number Sense and Operations</u>

In order to be quantitatively literate, students must have a solid grasp of the properties of rational numbers. As evidenced by the NECAP (2007) released items, many problems focused on ratios and proportions:

Renata is a sales representative for a printer company. She sells two models of printers: Model P and Model Q.

- *Last month she sold a total of 120 printers.*
- *The ratio of Model P printers sold to Model Q printers sold was 3:5.*

If Renata is paid a $25 commission for every Model P printer sold and a $20 commission for every Model Q printer sold, what was her total commission last month?

A. $1,480
B. $2,475
C. $2,625
D. $2,760

In this problem, students must truly understand ratios and proportions in order to manipulate the numbers and expressions. It is a real-world example that could be found in the workplace or in everyday life. Students can solve the problem by first understanding that 3:5 represents that for every 3 Model P printers and there are 5 Model Q printers. Next, they must recognize that for every 8 printers, 3 of them are Model P, which establishes a ratio of 3:8. The information can be represented as an equation: $\frac{x}{120} = \frac{3}{8}$. Solving the equation tells students that there are 45 Model P printers if there are 120 printers total. Students can either set up another proportion or subtract 45 from 120 to find that there are 75 Model Q printers. Finally, arithmetic yields a $1,125 commission on Model P Printers, a $1,500 commission on Model Q printers, and a total commission of $2,625.

The maple syrup problem below (NECAP, 2007) is another example that makes the connection between percentages and ratios or proportions:

The Doucettes produce and sell maple syrup.

- *Each year they sell all the maple syrup they produce.*
- *Last year they sold 640 gallons of maple syrup.*
- *This year they will sell maple syrup at a price that is 20% lower than it was last year.*

- *How many gallons of maple syrup must the Doucettes sell this year so their income from maple syrup sales stays the same as it was last year? Show your work or explain how you know.*

Students may recognize the inverse relationship between quantity and price in this situation (i.e., as price decreases, the number of gallons the Doucettes need to sell increases). As for solving, one method requires the students to understand that if the price is to be 20% lower, then the number of gallons sold in the previous year is 80% of the number of gallons sold in the current year. To recognize this, and to create the proper proportion in this situation demonstrates a deep understanding on the part of the student. High school math teachers expect students to be algorithmically fluent and conversant in basic math concepts. "A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation" (Grade-Span Expectations).

Once in high school, students are required to use their conceptual and procedural math knowledge to solve real-world problems. They must be capable of transferring knowledge from one context to another, of being able to use their math skills in many different situations, and of synthesizing several math concepts to solve one problem. The K–8 math curriculum is often compartmentalized: students spend a unit on fractions, then a unit on percents, then a unit on decimals, and so on.

In the executive summary of the Final Report of the National Mathematics Advisory Panel (2008) (retrieved March 10, 2009 from http://my.uen.org/mydocuments/downloadfile?userid=vdahn&documentid=5148837), the following is stated:

> Teachers and developers of instructional materials sometimes assume that students need to be a certain age to learn certain mathematical ideas. However, a major research finding is that what is developmentally appropriate is largely contingent on prior opportunities to learn. Claims based on theories that children of particular ages cannot learn certain content because they are "too young," "not in the appropriate stage," or "not ready" have consistently been shown to be wrong. Nor are claims justified that children cannot learn particular ideas because their brains are insufficiently developed, even if they possess the prerequisite knowledge for learning the ideas.

The structuring of U.S. elementary mathematics curricula based on the idea of age appropriateness has led to the organization of mathematics in discrete, unit by unit topics separated from each other, instead of promoting the connections among the skill topics. With standards-based elementary mathematics curricula such as Investigations, Everyday Math, and Trailblazers, attempts are currently being made to spiral mathematical concepts and skills through elementary grades to promote the connections needed to truly develop quantitative literacy within all students.

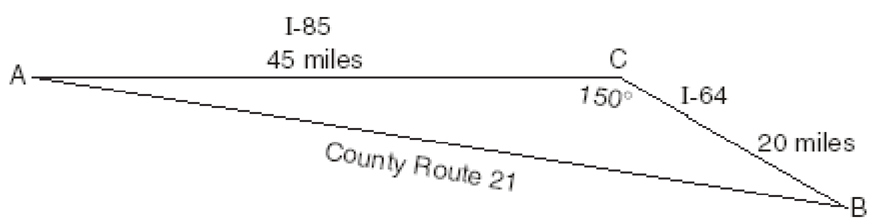
However, current trends indicate a discord between the organization of elementary and high school mathematics curricula with the latter having a more integrated organizational structure to how mathematics is taught at the secondary level. While a class might be working on a unit calculating trigonometric ratios, they must also incorporate dozens of other math skills to solve the problems. These skills may include right triangle geometry and the Pythagorean Theorem, simplifying radicals, or converting between degrees and radians. Students often do not realize that most problems are multi-step and require them to use concepts and skills that are not the direct focus of the current lesson. They quickly realize that they will need to use mathematical concepts that they learned in the past without being reminded to do so. Students also learn that they are responsible for determining the methods and concepts to use in order to find a solution and that there are often several different ways to solve a problem.

During high school, most students are exposed to math as a means of problem-solving, as opposed to solving problems using rote procedures. They are given the opportunity to make estimates, try various solution methods, and determine if their answers make sense and are reasonable. They are also given the opportunity to put together all of the math skills they have learned. Finally, high school students are exposed to finding math solutions to larger, real-world problems in a controlled environment: "Problem solving is the process through which students discover and apply the power and utility of mathematics. Skills in problem solving is essential to productive citizenship" (NCTM, 1989).

<u>Sample Problems for Problem-Solving/Applying Math Skills</u>

The problem below illustrates the number of math concepts necessary to solve a single problem at the 11th grade level (NYS Regents B Exam, 2002):

> *Kieran is traveling from City A to City B. As the accompanying map indicates, Kieran could drive directly from A to B along County Route 21 at an average speed of 55 miles per hour or travel on the interstates, 45 miles along I-85 and 20 miles along I-64. The two interstates intersect at an angle of 150° at C and have a speed limit of 65 miles per hour. How much time will Kieran save by traveling along the interstates at an average speed of 65 miles per hour?*



To solve the problem, students can calculate the distance from City A to City B using the Law of Cosines. They could also use the Law of Sines; however, this method, while leading to a correct answer, would be less efficient and more time-consuming. Regardless, using either method would also require them to use the proper order of operations. Students would then need to use the rates of speed and distances of the two possible routes to determine the time lapsed. Finally, they need to determine the difference between the times. Many students are daunted by such problems since there is no clear-cut plan or procedure dictating their solution method; they are responsible for determining what concepts must be used. Problems like the one above help develop numeracy in a number of ways. First, students are given a real-world problem to solve that does not have a single clearly defined solution method (i.e., students need to determine how they want to solve the problem). Second, the problem incorporates many mathematical content and process topics; and third, students must integrate and synthesize their mathematics knowledge.

The following problem illustrates a real-world problem which is relevant to biologists, ecologists, and administrators in the local government and tourism industry, among others:

> **Units at Glen Canyon Dam**
>
> *During the last week of March 1996, the US government announced that a spike flood would be released through the Grand Canyon from Glen Canyon Dam. The flood had been suggested by ecological and biological groups for many years as a way of restoring the Colorado River and the habitats along its banks. The release of water averaged 25,800 cubic feet per second and lasted a week. The total volume of the reservoir behind the 710-foot high Glen Canyon Dam is estimated to be 27 million acre-feet or 33 billion cubic meters. What fraction of the total water supply was released during this spike flood? Comment on whether this release would compromise the other uses of Glen Canyon water in the near future (agriculture and hydroelectric pow-*

er). (Retrieved August 18, 2009, from
www.math.cudenver.edu/~wbriggs/qr/news_problems.html#anchor149419)

From a numeracy standpoint, this interdisciplinary problem utilizes large numbers and asks for the answer in fractional form. Again, students are asked to find an answer, and their solution methodology is not dictated—they can use any method they choose. Once they have answered the question, they are asked to comment on the effects of the water release: Quantitative literacy often involves a qualitative component where students can discuss and make judgments or suggestions.

Communication and Representation

Another "big" idea in high school math is developing mathematical communication skills and being able to present, understand, and interpret mathematical data in various representations and formats. "Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations" (Grade-Span Expectations).

By the time students enter high school, they have been exposed to countless math terms and definitions. They have also been shown various representations, such as graphs, tables, and equations. In high school the opportunity to use all the terms, definitions, and representations they have learned to solve mathematical problems should become the focus of the mathematics curricula.

Oftentimes, methods of solving a problem and displaying their data are prescribed by the teacher or by the lesson. However in high school, students should be given more freedom to solve problems how they see fit, using the tools and knowledge they have acquired throughout their education. For example, a certain problem may be solved by one student algebraically, and by another using a geometric technique.

Unfortunately, many students may not have developed a comprehension of the interconnectedness of mathematical representations. For example, if one student shares a graphical or geometric solution, a student who used an algebraic solution may not see the similarities, the differences, or the connections between their two methods. This could lead to the student with the algebraic solution feeling that his or her solution is incorrect, when it was only presented in a different format.

With any given problem solving situation, students may need the guidance and the expertise of the teacher to help them see the connections between solution strategies. By doing this, students will begin to understand that a specific solution or representation method may be preferable based on the given situation. Once students begin to understand the connections within mathematics, they will begin to develop what NCTM refers to as "Mathematical Power":

> This term denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems. This notion is based on the recognition that mathematics as more than a collection of concepts and skills to be mastered; it includes the methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual, mathematical power involves the development of personal self-confidence (NCTM Curriculum and Evaluation Standards for School Mathematics, 1989, p. 5).

This is the "power" that we wish to develop in all of our students; it is the "power" that being quantitatively literate will help enforce.

Students also should to be able to communicate mathematically in multiple ways, be it verbally, pictorially, graphically, algebraically, or geometrically. They may choose to use a table, illustrate a pattern, or provide a proof as part of their explanation. In addition, they should to be able to understand, analyze, and critique data represented in any of the formats previously listed. Lastly, students should be able to defend their solutions and thought process or be able to refute an incorrect solution.

Sample Problems for Communication and Representation

The math problem below may be approached in several different ways. A class discussion following such an activity will allow students to communicate their approach and understand that there are various ways to generate a solution. Actual answers may vary, but each solution must be reasonable and show appropriate representations.

Some problems are better than others at promoting quantitative literacy. The problems below are ones that involve a real-life, contextual situation, inquiry and problem-solving, and the synthesis of several skills. Furthermore, problems that involve the opportunity to reflect, estimate, represent, discuss, or defend a solution are valuable as teachers help their students become quantitatively literate. Discussed below are problems that teachers can use to develop greater numeracy.

Design a Tent

(Retrieved August 18, 2009 from www.nottingham.ac.uk/education/MARS/tasks/g10_2/fall.html)

Your task is to design a tent. Your design must satisfy these conditions:

- *It must be big enough for two adults to sleep in (with their baggage).*
- *It must be big enough for someone to move around in while kneeling down.*
- *The bottom of the tent will be made from a thick rectangle of plastic.*
- *The sloping sides and the two ends will be made from a single, large sheet of canvas. (It should be possible to cut the canvas so that the two ends do not need sewing onto the sloping sides. It should be possible to zip up the ends at night.)*
- *Two vertical tent poles will hold the whole tent up.*

1. Estimate the dimensions of a typical adult and write these down.
2. Estimate the dimensions you will need for the rectangular plastic base. Estimate the length of the vertical tent poles you will need. Explain how you get these measurements.
3. Draw a sketch to show how you will cut the canvas from a single piece. Show all the measurements clearly, calculate any lengths or angles you don't know, and explain how you figured out these lengths and angles.

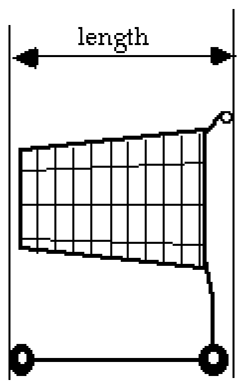
In this problem, students can apply several different skills, including estimation. In addition, visualizing and diagramming the tent may be helpful with the problem-solving process. They will also have the chance to consider the more complex relationships between two-dimensional and three-dimensional figures as they sketch how to cut the tent from a piece of canvas. Some students may choose to make a model to assist them in their task. Through calculations with measurements and angles, they may consider applying the Pythagorean Theorem and trigonometric ratios. When communicating their results to other students, they will need to defend why their design is reasonable. This can be done verbally or using pictures or models. As they discuss and analyze each other's solutions, they may provide feedback for those whose results are not mathematically correct.

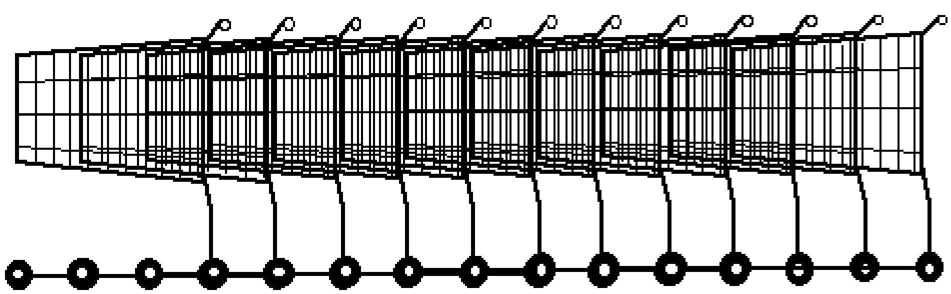
Oftentimes, the best, most engaging quantitative reasoning problems involve something so commonplace that students are surprised mathematics can even be involved. The problem below involves shopping carts and requires students to calculate how much space they take up (Retrieved August 18, 2009, from www.nottingham.ac.uk/education/MARS/tasks/g10_1). This problem can illustrate what an architect designing a supermarket needs to consider or how a design or production engineer needs to approach a new shopping cart design.

Supermarket Carts
The aim of this assessment is to think mathematically about supermarket carts and create a rule that can be used to predict the length of storage space needed, given the number of carts.

The diagram below shows a drawing of a single supermarket cart.
It also shows a drawing of 12 supermarket carts that have been "nested" together.

The drawings are **one twenty-fourth (1/24th) real size**.





1. Create a rule that will tell you the length of storage space (S) needed when all you know is the number of supermarket carts to be stored. You will need to show *HOW* you built your rule; that is, we will need to know what data you drew upon and how you used it.
2. Now show how you can figure out the number of carts that can fit in a space S meters long.

One of this problem's many great attributes is that it can be solved and communicated using a table, graph, or equation. Although the use of all of these tools is not essential, each has its particular strength. For instance, a table can be used to organize information and to look for patterns. A graph can help to display the data, look for patterns, and aid the visual learner. An equation will generalize the observations made by the student, giving a rule as asked for in the problem, and making it easier to make predictions. Furthermore, the problem does not dictate how the student should solve the problem—it leaves the method open-ended.

As mentioned previously, students are fascinated with money. Therefore, the problem below involving winning the lottery may be especially useful in engaging students:

Getting Rich With Lotteries?
(Retrieved August 18, 2009, from www.math.cudenver.edu/~wbriggs/qr/qrtop.html)
A survey sponsored by the Consumer Federation of America and Primerica (telephone survey of 1,010 adults 18 and older) revealed that 27% of those polled believe that their best chance of building wealth for retirement is playing lotteries. Among households with annual incomes of $35,000 or less, the percentage is 40%. People were also asked how much they could save by investing $25 per week for 40 years at an annual percentage rate of 7%. Fewer than one-third responded that more than $150,000 could be saved.

1. What are the chances of winning the grand prize in a "typical" lottery that requires matching 6 numbers drawn from a barrel of 40 numbers?
2. How much could you save by investing $25 per week for 40 years at an annual percentage rate of 7%?
3. Discuss your interpretation of these two calculations.

"Getting Rich With Lotteries?" involves many aspects of quantitative literacy that are particularly important to the high school student, especially when considering what the "big ideas" are for that grade span. By involving percents and probability, students have the chance to showcase their number sense. Even if their procedures or calculations are incorrect, this problem gives them the opportunity to make estimates and to think about a reasonable answer. As the problem does not prescribe a solution method, the students can think creatively and try different problem-solving techniques. Finally, they must interpret and discuss their calculations.

H. Numeracy for Institutions of Higher Education

Part I: Quantitative Literacy in Higher Education

i. The Importance of Numeracy in Higher Education

The case for quantitative literacy is well documented throughout **Section III**, which highlights the importance of a quantitatively literate workforce and quantitatively literate public for productive citizenship. Scientists, business leaders, politicians, and the mathematical community have noted that there is a need for all citizens to understand, evaluate, and apply data. The availability and proliferation of technology shifted the mathematical needs from rote computational tasks to complex problem-solving, modeling, and reasoning. Departments in the mathematical sciences need to take a leadership role by developing quantitative literacy in their students, collaborating with their colleagues in other disciplines, and promoting quantitative reasoning in K–12 schools.

This recognition of the need for a quantitatively literate public resulted in the response from several educational organizations and government committees, such as the National Council of Teachers of Mathematics (NCTM), the National Mathematics Advisory Panel, the Mathematical Association of America (MAA), and the American Statistical Association. The highly publicized debates between those who believe that mathematics should be taught as a set of skills and concepts and those who favor an inquiry and problem-solving approach greatly altered mathematics curricula and pedagogy over the last two decades (see **Section III**). Colleges and universities have seen their share of curricula changes in mathematics as well—from the reform calculus movement to the emphasis of what mathematical and statistical education should look like for all students in introductory college mathematics courses (e.g., the Crossroads and Beyond Crossroads report from the American Mathematical Association of Two-Year Colleges (AMATYC), along with the Mathematical Association of America (MAA) Committee on Curriculum Renewal Across the First Two Years of College (CRAFTY) and the Guidelines for Assessment and Instruction in Statistical Education (GAISE)). While these reform curricula emphasize and integrate many elements of mathematical literacy and numeracy, there still remains no clearly established consensus of what quantitative literacy entails, along with where and how it should be integrated into the curricula. However, there is a consensus on the importance of developing quantitative literacy and the need for efforts to improve students' abilities in this area. This consensus is documented in the findings of the NCED/MAA/MSEB Forum on Quantitative Literacy (Steen, 2004):

- Preparation: Most students finish their schooling ill-prepared for the quantitative demands of college, the workforce, and productive citizenship
- Awareness: The increasing importance of quantitative literacy is not sufficiently recognized by the public (in particular, high school students and their parents), government agencies, policy makers, and educational institutions
- Benchmarks: The lack of agreement of quantitative literacy expectations makes it difficult to establish clear benchmarks
- Assessment: Quantitative literacy is largely absent from current accountability and assessment systems
- Professional support: Faculty in all disciplines needs adequate professional support for the development of quantitative literacy across the curriculum, especially since many faculty members may not feel competent to promote quantitative literacy in their courses

A selected list of quantitative literacy programs adopted by colleges can be found in Steen's *Achieving Quantitative Literacy: An Urgent Challenge for Higher Education* or online (www.stolaf.edu/people/steen/Papers/qlprogs.pdf). This list continues to grow; local academic quantitative literacy programs are gaining momentum. Below is a list of such programs:

- Dartmouth College: Mathematics Across the Curriculum
  www.math.dartmouth.edu/~matc/
- Colby Sawyer
  www.colby-sawyer.edu/academics/experience/quantitative/index.html
- Keene State College: Integrated Studies Program Design
  http://keenestateinfo.com/changes/wp-content/uploads/2007/01/isp.doc

Membership in the Northeast Consortium on Quantitative Literacy (NECQL) is free by attending the annual conference. Organizations such as the National Numeracy Network (NNN) and the Special Interest Group of the Mathematical Association of America on Quantitative Literacy (SIGMAA-QL) seek to strengthen the quantitative literacy movement. See the following links for more information:

- www.trincoll.edu/depts/mcenter/NEConsortium.html
- http://serc.carleton.edu/nnn/

Findings from the National Adult Literacy Survey indicate that 50% of adults in the United States were unable to make low-level inferences using printed materials and unable to perform quantitative tasks involving single operations using numbers that could be found in the text or were explicitly stated. Furthermore, the Program for International Student Assessment (PISA), which measures the application of mathematical literacy in real-world contexts, found that 15-year olds in the U.S. perform below average compared to students in most of the Organization for Economic Cooperation and Development countries (Estry & Ferrini-Mundy, 2005). A 2004 study, conducted by the US Department of Education, found that nationally 34% of undergraduates report taking at least one remedial level mathematics course, though the rate is higher for two-year public colleges than for four-year public colleges. Furthermore, an earlier study, in 1988, found that the graduation rate among students who take no remedial level courses was 57%, compared to 29% for students who took one or two remedial level courses and 19% for students who took four remedial level courses (Kansky, 2008). Locally, about 36% of all mathematics student registrations throughout the Community College System of New Hampshire are in developmental, non-degree credit courses and less than 30% of all students across the system score above the newly adopted common cutscore[28] on the Accuplacer assessment to enroll in a threshold credit-bearing course (New Hampshire Community Technical College System, 2007). College professors and prospective employers agree that far too many high school students are not prepared either to enter credit-bearing courses or to enter the workforce—many do not have the technical and quantitative skills needed for advancement.

Secondary school mathematics relies on a canon that developed from nineteenth century European mathematics and spread across the world through colonialism, becoming the curriculum that is still prevalent today (Steen, 2001). This curriculum generally begins with arithmetic in the elementary grades and develops into algebra, geometry, functions and analysis, trigonometry, and calculus in secondary school. Within the organization of a school structure, this course of study is reasonable as it moves from simple to advanced, builds on previously learned concepts, and transitions from the concrete, to the symbolic, to the abstract. However, outside the school organization, this course sequencing makes less sense (Steen, 2001). Furthermore, the calculus-driven high school curriculum is unlikely to produce students who are quantitatively literate (Steen, 2004). This is perhaps due to how the curriculum is delivered and the priorities are set.

In addition to the possible problematic course sequencing, students may not be well-prepared because their secondary mathematics courses tend to focus on procedures and memorization. Conversely, quantitative literacy requires knowledge beyond formulas, equations, and procedures and requires approaching complex problems with careful reasoning, the ability to think mathematically, and the ability to ask intelligent questions of experts (Steen, 2001). The National Commission on Mathematics and Science Teaching for the 21st Century, in its report *Before It's Too Late*, reports that 60% of all new jobs in the early part of the twenty-first century will require skills that are possessed by only 20% of the current workforce (National Commission on Mathematics and Science Teaching for the 21st Century, 2000).

With the growth in technology, data and statistics have become regular tools for informed decision-making. The global workforce and changing economy require the ability to solve complex problems in highly specialized areas and the capacity to adapt to new situations and tools. Colleges and universities must assume an increasingly important responsibility in preparing students for the quantitative challenges of today and the future.

ii. Quantitative Literacy in Higher Education: The Challenge

Quantitative literacy at the college level involves building not only numerical and statistical literacy, but also general mathematical literacy that specifically meets the needs of a diverse student body. This complex objective implies that quantitative literacy is a campus-wide responsibility and is not achieved through a single course such as Liberal Arts Mathematics, but rather built throughout students' undergraduate careers. Furthermore, while it seems logical for quantitative literacy to be housed with mathematical sciences and for mathematics departments to take sole ownership for the development of quantitative literacy, students will come to view quantitative literacy as something that only occurs in mathematics classes (Steen, 2004). In doing so, they are forgoing rich opportunities to integrate numeracy, mathematical thinking, and quantitative analysis in contexts in diverse disciplines.

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[28] The common cutscore is for the Elementary Algebra Portion of the Accuplacer exam, which assesses mainly Algebra I skills.

Quantitative literacy and mathematics are closely related, but they are not the same. Process standards (e.g., problem-solving, reasoning, proof, connections, and communication), critical thinking, and "Habits of Mind" (e.g., perseverance, risk-taking, and conserving memory) are gaining momentum throughout the mathematics curriculum (and are heavily emphasized in the New Hampshire K–12 Mathematics Curriculum Framework) and are key elements of quantitative literacy, but are largely absent in the traditional high school curricula (e.g., algebra, geometry, pre-calculus, calculus sequence) and the traditional college algebra course. College algebra tends to be the gateway course for further mathematical study or a popular choice as a terminal mathematics course. And, for the most part, college algebra, if taught in its traditional sense, does little to develop quantitative literacy and will be mostly useless for students in their adult lives.

Many aspects of quantitative literacy are not sufficiently dealt with in mathematics courses. As students progress in their mathematics education, the level of abstraction increases and often connections grounded in meaningful context are limited. Conversely, quantitative literacy is usually grounded in such contexts; furthermore, it often involves sophisticated reasoning (e.g., inference) with what could be classified as elementary mathematical concepts (e.g., arithmetic). Often these situations rely on the ability to analyze and interpret data and make inferences. Hence, the connection between statistics and quantitative literacy is probably stronger than the connection between typical mathematics courses, like college algebra, and quantitative literacy (Steen, 2004). Since all statistical models have limitations in accurately representing real-world situations, quantitatively literate coursework needs to address how to recognize and account for such limitations.

A complication of dealing with elementary mathematical concepts (even in sophisticated ways) is that many college mathematics educators believe quantitative literacy should be developed solely in the K–12 arena; higher-level concepts should be dealt with in post-secondary education (Steen, 2004). It may be the case that many college mathematics professors are more interested in teaching mathematics rather than meeting quantitative literacy goals. This limited view of quantitative literacy as "watered down" mathematics fails to recognize the full complexity of developing numeracy. Quantitative literacy is a demanding, college-level learning expectation and continues to develop throughout one's life. Furthermore, the development of quantitative literacy competencies is a challenge; experience with struggling students shows that quantitative literacy tasks are no less challenging than tasks that students may encounter in calculus courses (Steen, 2004).

Quantitative literacy at college is not easily definable, but involves elements of analytical arguments, process standards, "Habits of Mind," fluency with data-driven decisions, and sophisticated ideas like inference, along with the ability to solve increasingly complex tasks unique to one's environment—all common elements in many disciplines. Clearly, the development of quantitative literacy requires a campus-wide, cross-disciplinary effort.

Unfortunately, the message high schools students often receive from colleges is that quantitative reasoning is not valued. Course placements and admissions are typically based upon a set of algebraic skills that involve little to no quantitative literacy. High schools will not believe that colleges and universities value quantitative literacy when entrance requirements and course placement exams ignore it.

iii. Quantitative Literacy in Higher Education: The Goals

Establishing clear institutional quantitative literacy goals, along with how to meet and assess those goals, is a key step in developing a quantitative literacy plan in higher education. Some general goals for any quantitative literacy program in higher education are outlined and discussed below, but it is important for each institution to formally establish its own set of goals based on its focus and mission.

As previously discussed, quantitative literacy is an important cross-disciplinary institutional goal. This requires institutions of higher education to have campus-wide discussions about developing quantitative literacy for all students, along with discussions regarding curricular changes to meet these goals. The following goals are at the heart of a quantitative literacy program for all students:

- Prepare students for productive citizenship by reasoning quantitatively and solving sophisticated problems unique to their lives across varying mathematical contexts (e.g., political polls, voting methods, sampling, tax policy, medical studies, and financial decisions)
- Prepare students to deal with the technological demands of a changing society
- Prepare students to ask and explore questions relevant to their lives that require quantitative reasoning or data analysis
- Increase students' positive dispositions toward mathematical sciences and develop confidence in their abilities to engage in problem-solving and quantitative analysis
- Develop students' "Habits of Mind," such as perseverance, risk-taking, and challenging solutions
- Develop students' abilities to read and comprehend mathematical and statistical information and represent and understand different representations of quantitative ideas

Part II: Developing a Quantitative Literacy Plan at the College Level

i. Developing a Vision

An important first task of a college-level quantitative literacy committee is developing a vision for quantitative literacy that aligns with the college's mission statement, goals, and objectives. This vision must also fit into the college's strategic plan and clearly articulate the overarching quantitative literacy goals of the institution. The goals should be flexible and adaptable in order to meet the needs of a rapidly changing technological society and the changing demands in the workforce. Since quantitative literacy should be built throughout a student's academic, professional, and personal life, quantitative literacy is not achieved through a single course. Hence, some institutional goals may include the following:

- Deepen students' abilities to deal with quantitative information and to solve problems that involve data analysis and statistical techniques
- Develop an appreciation for the role quantitative literacy has in everyday life
- Establish the foundation for building numeracy throughout one's life
- Establish clear prerequisite competencies for entering students, sample core competencies for all students during the first two years (Table II.1), and competencies within the major program (Table II.3)
- Engage in cross-disciplinary, institution-wide, on-going discussions about what the college values in a quantitatively literate student and the role that quantitative literacy should play in each major and department
- Increase awareness in the college community (particularly among faculty and administration) of quantitative literacy goals, and its value and role in everyday life and in being a knowledgeable and productive citizen[29]
- Develop a clear assessment plan that measures the degree to which students are meeting the competencies and that provides steps for improvement
- Establish clear oversight of the quantitative literacy plan and how the plan will be evaluated

ii. Quantitative Literacy Goals and Expectations

Examining the quantitative literacy goals of various colleges and universities reveals many differences in how those goals are achieved. For example, some institutions reach their goals by offering a specific course while others use an integrated approach throughout the program. Regardless, there are many common themes among those goals. In particular, the overarching goal is to provide students with a quantitative foundation they can apply in ways meaningful to their lives. This quantitative foundation is developed not only throughout students' PreK–

[29] Increasing awareness will require workshops and seminars emphasizing both the connections between mathematics and quantitative literacy and the differences. Also, a sustained professional development program should be implemented to educate cross-disciplinary instructors on quantitative literacy, its goals and values, and appreciation and understanding of quantitative literacy in their disciplines.

16 career, but also continues after formal education as adults grapple with sophisticated problems unique to their careers and lives. Furthermore, it is important for continued college and university involvement in the articulation of quantitative literacy prerequisites for students entering college and universities.

Prerequisite quantitative literacy goals are articulated throughout this plan in the PreK–12 sections (**Sections VI D, E, F, and G**); high school mathematical expectations are articulated in the New Hampshire K–12 Mathematics Curriculum Framework. These prerequisites and high school mathematics expectations include the following "big ideas":

- **Number Sense:** including procedural and conceptual understanding of ratios, decimals, percents, operations, and properties
- **Geometry and Measurement:** including angle relationships, properties of two- and three-dimensional figures, congruency, similarity, area and volume, and spatial reasoning
- **Functions and Algebra:** including linear and nonlinear patterns and rates of change, expressions, and equations
- **Data, Statistics, and Probability:** including understanding of various distributions, measures of center and spread, and general descriptive statistics
- **Process Standards:** including problem-solving, communication, connections, reasoning, representations, and applying skills

Process standards are particularly important as they lie at the heart of quantitative literacy. The ability to connect ideas is also central to all disciplines. Colleges and universities will have to assess whether or not these prerequisites have been met and how students achieve these competencies if they have not been met prior to admission (e.g., through a series of courses).

It is important to note that the NH K–12 Mathematics Curriculum Framework was developed with collaboration between PreK–12 and college and university partners. Furthermore, as outlined in the standards, the recommendation is that all students engage in four years of high school mathematics completing at a minimum Algebra I and II and Geometry, or their integrated equivalents, along with a fourth year of mathematics at a level equivalent to or beyond Algebra II. These expectations are articulated in the high school column and advanced mathematics column of the NH K–12 Mathematics Curriculum Framework. These recommendations are consistent with the research conducted by Achieve, Inc., which was founded by the nation's governors and business leaders to increase academic standards and in conjunction with the Pew Charitable Trust and the Association of American Universities. For important research related to the preparation of students in mathematics and science and the transition from high school to college, see *Research Relating to Making the Transition from High School to College and the Workforce*, at www.plymouth.edu/graduate/nhimpact/resources.html .

It is also important to recall the distinction between mathematics and quantitative literacy (Part VI B), and the description of quantitative literacy as dealing with elementary mathematics in sophisticated ways. According to Hughes-Hallett (2003):

> A mathematically literate person grasps a large number of mathematical concepts and can use them in mathematical contexts, but may or may not be able to apply them in a wide range of everyday contexts. A quantitatively literate person may know many fewer mathematical concepts, but can apply them widely (p. 92).

When examining various college and university quantitative literacy models, programs that articulate various competencies at different stages in students' undergraduate careers are consistent with the goal that quantitative literacy is campus-wide, cross-disciplinary, and develops over time (Estry & Ferrini-Mundy, 2005). Before outlining sample core foundational quantitative literacy competencies that should be developed at a community college or prior to a student's junior year (Table II.1), along with those competencies that are developed in major-specific work (Table II.3), it is important to note that many of the core competencies build upon the prerequisites and seek to strengthen them through modeling and use in everyday contexts. Students in every discipline should be able to apply problem-solving techniques, "Habits of Mind," and use quantitative arguments in sophisticated ways. Moreover, the competencies do not necessarily involve complex mathematics or symbolic manipulation. These competencies are philosophically different from the "plug-and-chug" approach of solving hundreds of linear equations or teaching a set of problems each of which is similar to sample problems done in the text. The "plug-and-chug" approach is far from the development of the heart of quantitative literacy, which is grounded in the ability to apply quantitative reasoning in unfamiliar contexts.

Students who are taught that mathematics is a series of steps to be mastered are being misled. Unfortunately, students often view mathematical problem solving as being routine mimicry, not open-ended, ambiguous, or relevant. For example, students who enter introductory statistics courses in college often have little difficulty computing slope, but they often struggle to interpret the slope within the context of the problem. In trying to understand students' attitudes toward mathematics, Hughes-Hallett (2003) asked Harvard's pre-calculus and calculus students to rank their agreement with the following two statements on a scale of 1 to 5, with 5 indicating strong agreement:

- A well-written problem makes it clear what method should be used to solve it
- If you can't do a homework problem, you should be able to find a worked example in the text to show you how

Both sets of students had average rankings higher than 4, indicating that these students perceived mathematics to be a set of procedures. For much of their formal education, students have become accustomed to believing that mathematics avoids ambiguity and the solution methods are clear. In order to effectively teach quantitative literacy, educators need to challenge these ingrained beliefs. Since quantitative literacy does not focus on procedures and clearly defined solution methods, colleges and universities have much work to do to help such students achieve numeracy standards.

Currently, colleges and universities in New Hampshire are far from adopting a common set of quantitative literacy competencies and a common model, but the list below (Table 11.1) attempts to begin the discussion around some common college-level quantitative literacy expectations that align to the view of quantitative literacy that has been described. Furthermore, requirements across colleges vary widely, especially for quantitative literacy requirements, causing transferability issues. These competencies capture the overarching "big ideas" that most programs had in common, but it is not an exhaustive list of the subsequent skills and concepts that might be addressed to achieve these competencies. Colleges and universities should address these skills and concepts but be careful to avoid teaching competencies, such as a checklist through particular skills and concepts.

Table II.1* Sample Core Quantitative Literacy Foundational Competencies

Important Note: Students will appreciate and value the use of these competencies in their everyday lives (e.g., in the algebraic competencies–sine functions and sound waves, logarithmic functions and the Richter Scale, exponential functions and interest). Courses should be student-centered and focus on activity-based instruction that integrates technology (e.g., dynamic statistical packages, calculator-based labs, spreadsheets, and online virtual manipulatives) and emphasizes conceptual understanding of topics studied while building procedural fluency and highlighting applications and connections to various disciplines.

Type of Competency	Competencies
Statistical	• Students will understand the four components of the statistical process: formulating questions, collecting data, analyzing data, and interpreting results. • Students will understand that probability is a tool for statistics (e.g., using the probability to determine the likelihood of a particular outcome happening just by chance.) • Students will understand the nature of variability in each of the four components of the statistical process (i.e., anticipating variability when formulating questions, designing data collection methods that acknowledge variability, using distributions to analyze data, and allowing for variability when interpreting results).
Algebraic	• Students will understand the language of functions and graphs, the concept of rate of change for both linear and non-linear functions and use these concepts in varying contexts. • Students will recognize, describe, and work with a wide variety of patterns (e.g., linear, exponential, trigonometric) and mathematical representations (e.g., symbolic, graphic, numeric, verbal) to analyze, interpret, and model real-world situations. • Students will recognize the value of and apply common mathematical properties (e.g., commutative and associative properties for addition and multiplication, distributive property of multiplication over addition or subtraction, and the identity elements for addition and multiplication) in simplifying algebraic expressions and in doing mental mathematics in everyday life.
Numerical	• Students will estimate and assess the reasonableness of answers and use a variety of numerical methods to solve problems. • Students will understand the fundamental role that numbers play within the realm of quantitative reasoning and their lives, and the relationships among them. • Students will develop an understanding and an appreciation of very small and very large numbers and their use in real-world situations.
Process and Habits of Mind	• Students will understand the problem-solving process and build a positive disposition toward problem-solving and mathematics, as well as develop the confidence and perseverance needed to solve sophisticated problems unique to their environments. • Students will effectively communicate mathematical ideas in both written and oral form and be able to critically read mathematics with understanding.

	• Students will use sound logical reasoning to analyze and interpret arguments and to make and logically defend conjectures. • Students will develop a curiosity about mathematics, recognize the value of exploration and investigation, and understand that mathematics is the language of nature and science.
Technology	• Students will use 21st century tools to manage, create, integrate, evaluate, and access information across disciplines. • Students will develop knowledge of the ethical and responsible use of 21st century tools in solving real-world problems. • Students will develop cognitive and technical proficiency in using 21st century tools to solve problems within and across disciplines and contexts.

*These were adapted from the GAISE framework, CRAFTY report, various syllabi at River Valley Community College, and NH's Information and Communication Technologies (ICT) standards. These adaptations summarize commonalities across various quantitative literacy plans.

In addition to these competencies, some colleges and universities (e.g., Keene State College) have developed outcomes for any course that falls within an integrated studies program which help capture the essence of quantitative literacy being cross-disciplinary and imbedded in sophisticated contexts. The integrated study outcomes at Keene State College fall within the following areas: diversity, ethics, global issues, or social and environmental engagement. Moreover, all courses that fall under integrated studies must meet at least one of these areas. For the specific competencies, see Keene State College Integrated Studies Program Design.

The following competency is an example of how the competencies in Table II.1 appear different than traditional mathematics competencies:

> Students will understand the language of functions and graphs, the concept of rate of change for both linear and nonlinear functions, and use the concepts in varying contexts.

The focus of this competency is not solely computational, such as how to use function notation or how to calculate slope. While these may be a piece of the competency, the focus is on understanding the language of functions and the ability to interpret rates of change in context, with the eventual goal of having students solve problems unique to their lives. Many students struggle to meet this goal because they are plagued by often- undiagnosed misconceptions. Thus, these misconceptions must be addressed in order for the competencies to be met. Some common misconceptions students have for the competency outlined above are given below.

Table II.2 – Typical Student Difficulties and Misconceptions with Graphs

(American Association for the Advancement of Science, 1993)*

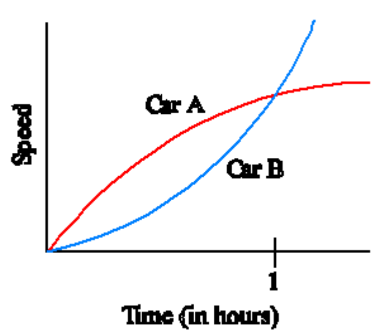
Students' Difficulties	Description
Graphs as Literal Pictures	Students at all levels tend to believe that graphs should look like their mental images of a situation rather than a symbolic representation of the situation. For example, many students will interpret distance-time graphs as paths of the actual journeys.

Pointwise vs. Global Understanding	Students typically develop a pointwise (local) understanding of graphs but have difficulty with across time situations (global) (Ferrini-Mundy& Lauten, 1993). This is attributed to the fact that we often ask pointwise questions rather than questions about maximum or minimum values, or intervals over which the function is increasing or decreasing, for example.
Height Slope Confusion	Students often confuse the slope of a graph with its maximum or minimum value and have difficulty recognizing the slope as representing a rate of change.
Scaling	Students have difficulty choosing appropriate scales when constructing graphs (e.g., they often believe that the scales on the *x*- and *y*-axes need to be the same even if this obscures the relationship). Additionally, even when scales are clearly marked, students tend to read each tick as representing one unit in situations where it may represent more than one unit.
Translating Between Representations	Students have difficulty translating between algebraic representations of functions and graphical representations of functions, particularly, going from the graphical form to the algebraic form.

*Unless otherwise noted.

The following is an example of a question geared toward the Pointwise vs. Global Understanding misconception (Andrusiak, 2007).

Pointwise vs. Global Understandings
Look at the graph below. Assume Car A and Car B start from the same position and are traveling in the same direction.



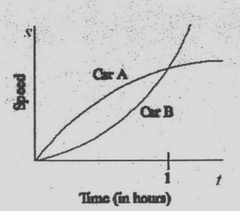
a) What is the relationship between the position of Car A and Car B at 1 hour (i.e., is one car further ahead or are they at the same spot)? Explain your answer.
b) What is the relationship between the speed of Car A and Car B at 1 hour? Explain your answer.

c) Between 0 hours and 1 hour inclusive, when are the cars the furthest apart? Explain your answer.

(Adapted from National Council of Teachers of Mathematics, Principles and Standards for School Mathematics, 2000).

Part a and *Part b* represent pointwise questions since they each ask about what happens at a particular time (i.e., $t = 1$ hour). However, notice that the vertical axis represents speed and not distance traveled. Students who hold a strong conviction that a graph must look like their mental image of the situation are likely to reason incorrectly in *Part a* that the cars are at the same spot since the graphs cross at 1 hour. Asking students to describe the situation can help alleviate the misconception. Once students realize that Car A has been traveling faster than Car B the entire time in the interval from $t = 0$ to $t = 1$, they realize that Car A must be ahead of Car B. These same students are likely to answer *Part b* correctly since the speeds are the same at 1 hour since the graphs cross. *Part c* represents an across-time question since it asks about a relationship over an interval of time requiring students to describe a pattern of change in one variable given a change in another variable. Again, students who harbor the mental image misconception incorrectly answer that the cars are farthest apart at about 1/2 hour since the graphs are the farthest apart there, once again failing to see that the vertical axis represents speed and that Car A has been traveling faster over the entire interval and hence the cars must be farthest apart at $t = 1$ hour. These misconceptions are illustrated in the response below.

2) The graph below shows the relationship between the speed, *s*, and time, *t*, of two cars that start from the same position and are traveling in the same direction.



a) Which car is further ahead at $t = 1$ hour? (circle one)

Car A　　Car B　　Both cars are at the position.

b) Explain your answer.

They both cross eachotter at the 1 hour mark

c) Which car is traveling faster at $t = 1$ hour? (circle one)

Car A　　Car B　　Both cars are traveling at the same speed.

d) Explain your answer.

Both cars cross the 1 hour mark at the same time

e) Between $t = 0$ hour and $t = 1$ hour, at what time are the cars the furthest apart? Explain your answer.

30 minutes. Common Knowledge

While functions could have been used to model the situation above and various calculations could have been made, including the area under the curves and above the *x*-axis to find distances, the focus was on the conceptual understanding of the situation.

As an important note, translating between multiple representations is often a successful strategy to help students deal with misconceptions like those shown above. It is helpful to move between verbal descriptions, symbolic representations, tables, and graphs. Students are quite successful on the above problem when provided the opportunity to discuss the context or given a verbal description of the context (e.g., Two cars leave from the same spot and travel in the same direction. One car travels faster than the other for one hour. At one hour, the cars are traveling the same speed. Which car is farther ahead after one hour, or are they at the same distance? When will they be the farthest apart during the first hour?). Further examples of quantitative literacy are discussed in Part III.

Table II.3 shows sample quantitative literacy competencies developed during work in one's major. These competencies are intended to build on students' previous work with the core competencies, which focus on applied knowledge.

Table 11.3 - Sample Quantitative Literacy Competencies Developed During Work in Major

Important Note: Students are expected to independently pose questions and actively seek solutions (i.e., the work done in the major should not be limited to scripted tasks/projects). These competencies can be viewed as applied competencies in major work and embedded in contexts that are meaningful to the students.

Type of Competency	Competency
Critical Analysis	• Students will develop and analyze quantitative and mathematical arguments, recognize their limitations and potential sources of error, and determine whether or not conclusions are valid.
Modeling	• Students will develop quantitative and mathematical models to represent real-world situations and realize the limitations of the models, the use of technology in creating the models, potential errors, and misleading representations.
Data-Driven Decisions	• Students will identify problems and recognize the need for data-driven and evidence-based decisions. • Students will investigate real-world problems, implement statistical design including collecting and analyzing appropriate data, recognize potential sources of bias and measurement error, understand the role of data in establishing cause and effect relationships, and use appropriate technology to solve quantitative problems.

iii. Assessment and Evaluation of Quantitative Literacy Goals

Many issues surround the assessment and evaluation of quantitative literacy goals. First, colleges and universities need to clearly establish competencies upon which they are going to assess students' progress toward meeting goals. Guidelines need to be set for these assessments, and many questions need to be investigated. Should the assessments be embedded throughout students' programs (e.g., exams, portfolios, journals, and labs)? How will the college determine student progress toward meeting these goals (e.g., campus-wide assessments and longitudinal studies)? How will this affect students who transfer into the college? Will there be any follow-up after graduation? If so, how will it be done? Clearly, in order for a valid assessment and evaluation program to be in place, the faculty will need professional development and support in this area.

Furthermore, current assessments used for college entrance exams, as well as for college placement, do not value quantitative literacy. Such assessments often de-contextualize problems. For example, course placement at most of the community colleges in New Hampshire is based on scores from the Accuplacer exam. For many students, their placement into initial credit-bearing mathematics courses is based on an elementary algebra exam, which is devoid of meaningful context. Additionally, standardized tests such as the SAT and ACT seem to involve very few problems embedded in authentic, real-world contexts. The state assessment in mathematics is not able to assess students on tasks that would be coded as Level IV on Webb's Depth of Knowledge (see alignment at http://facstaff.wcer.wisc.edu/normw/). Level IV tasks involve extended reasoning, analysis, and synthesis within and among content areas. These types of tasks are left to be assessed at the local level. Hence, local school districts need to pay attention to these tasks and see how they align with quantitative literacy goals. Until colleges and universities make some changes in these areas, high schools have little reason to pay attention to quantitative literacy. According to Steen (2004):

> The chief policy goal for assessing QL should be to move in the direction suggested by NSF Director Colwell: establish proficiency levels for QL that relate to students progress through K–16 education. Ideally, these proficiency levels would be tied to existing and emerging state standards in mathematics and science, to college expectations for entrance and placement, and to criteria for "rising junior" status and graduation. The goal would be a common QL yardstick benchmarked to the several stages of education, including preparation of prospective teachers in all fields (p. 59).

iv. Implementation Challenges

There are many implementation challenges associated with developing a local quantitative literacy plan, challenges for both policy and practice. The following is a partial list of challenges that need to be addressed:

- **Transferability:** Currently quantitative literacy requirements vary widely across New Hampshire colleges. These differences pose a challenge for transferal of credits, particularly for students who exit the community college system since it bears the burden of preparing students for transferability according to all the various requirements.
- **Faculty development and support:** Faculty members are not necessarily ready to teach quantitative literacy nor integrate it into their classrooms. In addition, since quantitative literacy cuts across disciplines, adequate time and funds need to be devoted to faculty professional development, support, release time, and so on. Furthermore, many faculty members do not value quantitative literacy and avoid teaching anything quantitative in their courses. The attitudes of faculty, staff, and students concerning quantitative reasoning need to be addressed.
- **Testing to meet quantitative literacy competencies and placement:** Tests that measure quantitative literacy need to be developed and implemented. Course placement and college entrance exams should include specific content to encourage high schools to place more emphasis on quantitative literacy. College courses and programs should be assessed to determine whether students are meeting quantitative literacy competencies.
- **Course load and course development:** Colleges need to make decisions as to how the competencies are integrated into their programs. Will the competencies be integrated into the current courses or will new courses be created? Will students need to take additional courses to meet requirements? What courses satisfy the competencies or some of the competencies?

- **Integration into statewide standards and teacher certification:** Careful analysis needs to be done to determine how quantitative literacy fits into the current New Hampshire K–12 Mathematics Curriculum Framework, teacher certification standards, and teacher preparation programs.
- **Oversight:** Colleges need to develop oversight to ensure that quantitative literacy goals are being met and assessed.
- **Traditional course tracking:** Traditional mathematics sequences dominate most high schools and colleges. College algebra is largely a failure for preparing students to be quantitatively literate, but it remains a very popular terminal course. Flexible ways to meet competencies need to be addressed and mathematics needs to be imbedded in meaningful contexts. Additionally, traditional preparation in a mathematical science does not necessarily result in an increase in quantitative literacy.
- **Quantitative literacy in general education:** Colleges will need to discuss and reflect on the role of quantitative literacy in the general education program. Schools that currently have a quantitative course requirement in their general education programs have made a step in the right direction, but one course is not consistent with the message that quantitative literacy is a cross-disciplinary responsibility developed over time.

Quantitative literacy is an area that is dynamic, and quantitative literacy plans should be revisited and assessed frequently. To better understand what types of pedagogical practices best promote quantitative literacy, institutions of higher education need to lead the charge and continue to build awareness of the importance of quantitative literacy.

Part III: Fostering the Development of Quantitative Literacy in Higher Education

i. Recommendations and Examples

The table below (Table III.1) briefly synthesizes some recommended responses to the findings from the national forum on quantitative literacy (see Part 1) as they appear in "Achieving Quantitative Literacy–An Urgent Challenge for Higher Education" (Steen, 2004).

Table III.1 – Findings and Recommendations from the National Forum on Quantitative Literacy

Findings	Recommendations
Preparation	• Embed quantitative literacy in courses across the curriculum • Ensure students' numeracy is reinforced and developed throughout PreK–16 education • Emphasize "Habits of Mind" in mathematics courses and teach students to think about mathematics in context • Create alternative routes to advanced mathematics
Awareness	• Publicize examples of quantitative literacy that cut across multiple contexts • Align quantitative literacy initiatives with other educational efforts • Make quantitative literacy a cross-disciplinary discussion and effort • Emphasize the role of inference and quantitative analysis in public policy-making • Seek the attention of media, politicians, and public funding agencies
Benchmarks	• Establish benchmarks for quantitative literacy for PreK–16 • Expect graduates to be quantitatively literate
Assessment	• Increase the role of quantitative literacy on tests such as college admissions tests and placement exams • Minimize the reliance of standardized tests that deemphasize meaningful contexts • Provide resources for instructors to develop ways to accurately assess quantitative literacy • Create an online resource of quantitative literacy assessment tasks for faculty to draw on (e.g., see the examples in the Colby-Sawyer online bank under academic departments and natural sciences)
Professional Support	• Create a structural home for quantitative literacy • Develop the necessary technical infrastructure to support the teaching and learning of quantitative literacy • Develop web-based inventory of quantitative literacy resources • Support professional development opportunities and networks • Infuse quantitative literacy into teacher preparation programs and standards

The remainder of Part III offers examples of tasks that are related to quantitative literacy and the competencies presented in the previous section. By no means do these examples attempt to cover all of the competencies, nor are they necessarily formatted in a way that they would be presented to students. These examples are chosen to illustrate the nature of quantitative literacy, regardless of pedagogical delivery. Many of these examples cut across multiple competencies and involve the use of process standards and "Habits of Mind." Students are asked to explore quantitative questions related to other fields, such as medicine, economics, and public policy. Furthermore, a key element of students' development of quantitative literacy is their ability to pose their own questions and independently investigate solutions. There is no unique, or necessarily best, way to attack these problems. Students must apply their mathematical skills, but they also need to interpret the situation, determine a strategy, solve the problem, and express it in terms that make sense given the particular situation.

Statistical Literacy Examples

Statistical Significance and Experiments

From 1986 to 1988, a group of researchers from eight European countries and Australia conducted a study to determine whether or not there was any difference in the proportion of AIDS-related complex (ARC) patients who developed AIDS under two different treatments. Each patient in the first treatment group was given the drug zidovudine (commonly called AZT), while each person in the second treatment group was given a combination of AZT and acyclovir (commonly called ACV). A total of 134 patients agreed to participate in a double-blind study[30] where the treatments were randomly assigned. The following table shows the number of ARC patients who developed AIDS during the study.

	Treatment			
		AZT	**AZT w/ ACV**	**Total**
Developed AIDS?	**Yes**	12	10	22
	No	55	57	112
	Total	67	67	134

Design and conduct a simulation to determine the chance of obtaining a difference in proportions as large as or larger than that observed in the experiment, by chance alone, assuming that the treatments make no difference. What questions does this experiment raise? Discuss whether or not the treatments seem to make a difference (adapted from Watkins, Scheaffer, and Cobb, 2004).

Data and Distributions in the Media

Find some examples of data and distributions used in the media (e.g., newspapers, online reports, magazines, radio shows). Do you believe the conclusions that the author has made? Why or why not? Are the data accurate? Are the distributions accurate or misleading? What additional information do you need?

Risk and Public Policy

Should our tax dollars support increased airline safety? What are the costs and risks? Should we allocate our resources elsewhere? Suppose we made airline travel 10 times safer. How many lives would be saved per year? How much would this cost per life saved? If the costs of airline tickets increase, what will be the impact on travel by automobile and the number of lives lost in car crashes? Include a detailed analysis. [Further exploration: examine the cost per life saved for various initiatives supported by your federal tax dollars and discuss your conclusions] (Adapted from Burger & Starbird, 2005).

Using Probability as a Tool for Statistics

(from www.ed.state.nh.us/education/doe/organization/curriculum/Math/documents/MathematicalInvestigationsCompetencies41608V2.pdf)

[30] In a double blind study, neither the patient nor the researcher is aware of which group the patient is in.

Quinn's dog day care has been accused of rejecting dogs with long tails. The most recent data show that out of ten dogs considered for the day care service, two were rejected. The two dogs rejected had an average tail length of 42 cm. The tail lengths of the 10 dogs that were being considered for the day care are: 12 cm, 10 cm, 14 cm, 24 cm, 44 cm, 48 cm, 36 cm, 18 cm, 22 cm, and 34 cm.

1) Do you think that the dog day care service is discriminating against dogs with long tails? Explain.
2) Design a simulation to determine how likely it is by chance alone to select two dogs for rejection whose average tail length is at least 42 cm. Display the sampling distribution of the sample averages.
3) Based on your results from problem two, determine if the day care center has some explaining to do. Support your conclusions.

Algebraic Examples

Distribution of Resources

It is often the case that the distribution of resources across a population is not equitable. That is, the poorest p percent of the population does not own p percent of the resource. If the distribution of resources were equitable, then any 20% of the population would have 20% of the resource, any 30% of the population would own 30% of the resource, and so on. How can we measure whether or not the distribution of resources is becoming more or less equitable over time? How can we measure which county has the most equitable resource distribution? Suppose $F(x)$ represents the fraction of the resource owned by the poorest x percent of the population. For example, $F(0.20) = 0.05$ indicates that the poorest 20% of the population owns 5% of the resource. What would F look like if the resources were distributed evenly? What is $F(0)$ and $F(1)$, and what are their meanings in the context of the problem? Is F an increasing or decreasing function? Is F concave up or concave down? What is the meaning of the concavity? The Gini index of inequality, or Gini coefficient, is a measure of the inequality of wealth distribution and can be found by looking at twice the difference in area between the line of equality and F. Create a scatterplot of the Gini coefficients for various countries against their per capita gross domestic products. Describe any trends or patterns that you notice. What clusters do you notice? What do these clusters mean? What is your interpretation of where the United States lies in the scatterplot? (Adapted from Hughes-Hallet et al., 2005 and Steen, 2004).

Numerical Examples

False Positives and the Probability of Having a Disease

The population of the United States is about 300,000,000 people. There are approximately 500,000 people in the U.S. living with AIDS (i.e., they know that they have the disease) and estimates are as high as 1,200,000 people in the U.S. who are HIV positive (i.e., they are living with the virus but many of them do not know they have it). HIV testing is approximately 95% accurate for those that actually have the disease and 99% accurate for those that do not have the disease. Suppose you go in for HIV testing and receive a positive result. What is the probability that you actually have the disease? Should you be concerned, and what are your courses of action? What implications do these data have for public policy and testing?

Fermi Problems

Fermi problems are problems that require students to estimate an answer to a problem that seems beyond their reach.

Example: How many hot dogs will be eaten by fans attending American and National League baseball games during the course of a year?

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VII. CREATING A NUMERACY ACTION PLAN

In order to improve numeracy of all students, a school must embark on a school-wide numeracy focus that is strategic and intentional. The entire faculty and staff must work collaboratively to ensure that all students will have the opportunity to use and develop quantitative literacy skills, such as solving problems and interpreting data in all content areas. In order for a Numeracy Action Plan to be successfully launched, the school must have the district's support. The critical steps needed to implement a numeracy action plan are outlined and described below.

The Appendix includes a variety of tools to help create an action plan, specifically **Appendices O through U.** In addition, **Section V H** includes many "Guiding Questions" that may provide an opportunity to reflect upon a school or district's numeracy goals and outcomes.

Step 1: Form a numeracy/instructional team.

The numeracy team should represent the entire school and consist of administrators, numeracy/instructional leaders, faculty of all content areas including ESL and special education, and other staff. The purpose is to guide the development and implementation of the school-wide numeracy plan. A school may also want to include parents and/or higher education faculty on the team. Parents are often a driving force in any educational program, and their early involvement could be critical to the numeracy program's success. College and university faculty are not only at the forefront in quantitative literacy research but they also are knowledgeable about which skills students need to be best prepared for college-level work. These are issues that may be addressed as a school is adopting a numeracy plan.

Step 2: Conduct a numeracy needs assessment.

- Gather or review data to identify specific numeracy needs for all students and for categories of students
- Analyze existing curricula and programs to determine if they meet the needs identified by the data
- If the existing curricula and programs do meet the needs, determine whether the students are improving while using the curricula and programs that are currently in place
- If the existing curricula and programs do not meet the needs, determine how the curricula and programs can be supplemented to fill the gaps
- Establish which components of the existing curricula and programs are working well toward students becoming quantitatively literate
- Establish which components of the existing curricula and programs are not working well or are weak

Step 3: Conduct an analysis of infrastructural components to identify missing and/or other elements.

- Review schedule to identify the actual amount of time spent on the following activities: teacher collaboration both within and across departments, professional development, problem-solving and data interpretation activities, and extended learning opportunities
- Review professional development opportunities related to quantitative literacy
- Examine the roles and responsibilities of the numeracy/instructional coach or the mathematics coach (if applicable)
- Inventory useful and accessible technology

Step 4: Conduct an analysis of instructional components.

- Review the types of assessments that are used to assess students' mathematics and numeracy performance. Do these assessments provide the information needed to guide instruction?

- Review the interventions for struggling students, English Language Learners, and special education students. Do these assessments target students' needs?
- Review extended learning opportunities. How are they tied to academic learning? How do they enrich student learning?
- Check alignment of curriculum components district-wide to ensure systematic spiraling of numeracy acquisition.

<u>Step 5: Share findings with the entire faculty and other stakeholders including parents and students. Ask for their input and involvement.</u>

<u>Step 6: Create a three-year plan for improving students' numeracy.</u>

- Set clear objectives and goals
- Organize sub-committees that are tied to the objectives
- Engage the entire faculty and community by inviting them to participate
- Set measurable benchmarks throughout each year and at the end of each year
- Provide periodic opportunities for formal and informal reflection

<u>Step 7: Review the data and refine the plan for Year 2.</u>

VIII. APPENDIX

Appendix A: Definitions and Descriptions of Numeracy and Quantitative Literacy

Posters
- Appendix B: "What is Quantitative Literacy?" Poster
- Appendix C: "Quantitative Literacy Involves. . ." Poster
- Appendix D: Conceptual Framework for 21st Century Numeracy
- Appendix E: "Habits of Mind" Poster – Elementary Version
- Appendix F: "Habits of Mind" Poster – Secondary Version

Parent and Community Resources
- Appendix G: Framework for Leveraging Parent Involvement
- Appendix H: Resources for Parents
- Appendix I: Supplemental Community Programs and Resources

Teacher Resources
- Appendix J: Improving Numeracy Outcomes for All Students
- Appendix K: Cognitive and Metacognitive Behaviors in Non-Routine Problem-Solving
- Appendix L: Flow Chart of the Cognitive Processes During Problem-Solving
- Appendix M: Examples for Revising Mathematics Problems to Emphasize Numeracy

School Leadership Resources
- Appendix N: Factors that Contribute to School Effectiveness
- Appendix O: Numeracy Capacity Survey
- Appendix P: Implementation Strategies
- Appendix Q: Evaluating a Rigorous Mathematics Curriculum with an Emphasis on Numeracy Across the Curriculum
- Appendix R: Numeracy Goals Classroom Observation Form
- Appendix S: How School Leaders Can Enhance Numeracy Instruction
- Appendix T: What Administrators Should See Students Doing in the Classroom
- Appendix U: NH Mathematics Alignment Protocol and Gap Analysis Tool

Appendix V: Connections between NECAP Test Items and Grade Level and Grade Span Expectations

Appendix W: Numeracy Book and Resource List

Appendix X: Numeracy Task Force Members, Contributors, and Reviewers

Appendix A: Definitions and Descriptions of Numeracy and Quantitative Literacy from Various Sources

Michigan State University
www.msu.edu/~acadgov/documents/QLFinalReport-1.pdf

Quantitative literacy is the ability to formulate, evaluate, and communicate conclusions and inferences from quantitative information. *Quantitative literacy* employs analytical arguments and reasoning built upon fundamental concepts and skills of mathematics, statistics, and computing. Quantitatively literate MSU students will be more empowered members of a global society through their ability to represent and critique their world.

Hope College
http://pewscimath.hope.edu/HollinsQRDefinition.htm

Quantitative reasoning is the application of mathematical concepts and skills to solve real-world problems. In order to perform effectively as professionals and citizens, students must become competent in reading and using quantitative data, in understanding quantitative evidence and in applying basic quantitative skills to the solution of real-life problems.

University of Virginia www.web.virginia.edu/IAAS/reports/subject/competencies/quantitative.htm#def

Quantitative reasoning is correctly using numbers and symbols, studying measurement, properties, and the relationships of quantities, or formally reasoning within abstract systems of thought to make decisions, judgments, and predictions.

Mount St. Mary's College
www.msmc.la.edu/pages/2480.asp

Quantitative literacy is knowledge of and confidence with basic mathematical/analytical concepts and operations required for problem-solving, decision-making, economic productivity and real-world applications; this entails the ability to:

- Competently perform basic computational/arithmetic operations;
- Demonstrate skills at estimating and approximating results;
- Perform basic algebraic and/or logical operations that involve levels of abstraction;
- Demonstrate basic problem-solving skills; and
- Show competence in applied analytical skills.

The following definitions were aggregated by William Briggs at the University of Colorado at Denver (retrieved December 10, 2008, www.math.cudenver.edu/~wbriggs/qr/qrtop.html):

"Quantitative reasoning" or sometimes "Numeracy" is not just the ability to do mathematics although some basic skills in mathematics are a pre-requisite. However, it extends to the ability to understand quantitative arguments and reason about quantitative concepts in all areas of the curriculum, the arts and humanities, social sciences, and the natural and life sciences (Hainline, 2001).

Quantitative literacy is "[t]he knowledge and skills required to apply arithmetic operations, either alone or sequentially, using numbers embedded in printed material (for example, balancing a checkbook or completing an order form" (*National Adult Literacy Survey*, National Center for Education Statistics, 1993).

Quantitative literacy is "an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work" (*International Life Skills Survey*, Policy Research Initiative, Statistics Canada, 2000).

Mathematics literacy is "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen" (*Programme for International Assessment,* Organization for Economic Cooperation and Development, 2000).

Quantitative literacy involves "confidence in mathematics, cultural appreciation, interpreting data, logical thinking, making decisions, mathematics in context, number sense, practical skills, prerequisite knowledge, symbol sense" (Steen, L. A., The Case of Quantitative Literacy; Mathematics and Democracy: The Case for Quantitative Literacy).

Quantitative *literacy* involves understanding the role of numbers in the world. It provides the ability to see below the surface and to demand enough information to get at the real issues. – Ted Porter, historian

Beyond arithmetic and geometry, *quantitative literacy* also requires logic, data analysis, and probability. . . . It enables individuals to analyze evidence, to read graphs, to understand logical arguments, to detect logical fallacies, to understand evidence, and to evaluate risks. *Quantitative literacy* means knowing how to reason and how to think. – Gina Kolata, journalist

Quantitative literacy can be defined as the level of mathematical knowledge and skills required of all citizens. It includes the ability to apply aspects of mathematics (including measurement, data representation, number sense, variables geometric shapes, spatial visualization, and chance) to understand, predict, and control routine events in people's lives. – John Dossey, mathematics educator

Quantitative literacy requires one to understand the nature of mathematics and its role in scientific inquiry and technological progress; to grasp sufficient mathematics to understand important scientific and engineering concepts; and to possess quantitative skills sufficient for responding critically to scientific issues in the media and public life. – F. James Rutherford, physics educator

The heart of *quantitative literacy* is real world problem solving—the use of mathematics in everyday life, on the job, and as an intelligent citizen. Problem solving must be both mathematically defensible and useful in the real world. – Henry Pollak, applied mathematician

Numeracy is not the same as mathematics. It is an aggregation of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively and autonomously in quantitative situations arising in life and work. – Iddo Gal, cognitive scientist

Quantitative reasoning as an interpretive activity that takes place within a deductively structured framework. It involves a tapestry of meaning provided by a warp of abstract patterns and a weft of context and story line. In *quantitative reasoning*, context provides meaning. – George Cobb, statistician

Important *quantitative competencies* are those that can be used to solve problems people would frequently encounter on the job or in their roles as citizens or parents. Quantitative competencies require identifying and solving problems not in algebra and geometry, but in the five SCANS competency domains such as planning, information, and systems analysis. – Arnold Packer, economist

Quantitative literacy involves understanding the mathematical concepts and skills that are necessary for everyday life. It includes computation, interpretation, inquiry, and application of mathematical concepts that are critical for life in the contemporary world. – Glenda Price, college provost

Appendix B: "What Is Quantitative Literacy?" Poster

What is Quantitative Literacy?

- The mathematical knowledge and skills required to function in today's society
- Using mathematics to explain everyday life situations
- Using mathematics to solve problems
- A Habit of Mind
- Communicating effectively using numbers
- Thinking mathematically about real-life situations involving numbers
- Applied number sense
- Understanding real-life probability and statistics
- Reading and interpreting tables and graphs
- Mathematics in context
- The ability to construct logical arguments using numbers and quantities
- The way math can be used to simplify and make sense of many complex situations that occur in life

"The expectation that ordinary citizens be quantitatively literate is primarily a phenomenon of the late 20th century... If individuals lack the ability to think numerically, they cannot participate fully in civic life…Today's students need both mathematics and numeracy. Whereas mathematics asks students to rise above context, quantitative literacy is anchored in real data that reflect engagement with life's diverse contexts and situations."

--Lynn Arthur Steen, professor of mathematics at St. Olaf College, and executive editor of *Mathematics and Democracy: The Case for Quantitative Literacy*

Appendix C: "Quantitative Literacy Involves . . ." Poster

Quantitative Literacy Involves…

Real-life situations

To understand the mathematics that people encounter in everyday life—such as unit conversions, sizes and measurements, polling and other statistical data, probabilities of everything from disasters to winning the lottery—people must be quantitatively literate.

Using math to solve problems

Memorizing a formula, inputting values, then obtaining a solution is not problem solving. A person must be able to analyze a situation involving quantities for which a solution *may not be readily apparent,* devise a strategy to solve it, carry out the strategy, then reflect on the solution to determine if it is reasonable from a quantitative perspective.

A synthesis of mathematical skills

Several components contribute to the whole of being quantitatively literate. These components include: computational fluency, number sense, measurement skills, estimation skills, data interpretation, logical thinking and reasoning, decision making, and effective communication using quantities. Quantitative literacy is infused in several areas of math and in most areas of life.

Responsible citizenship

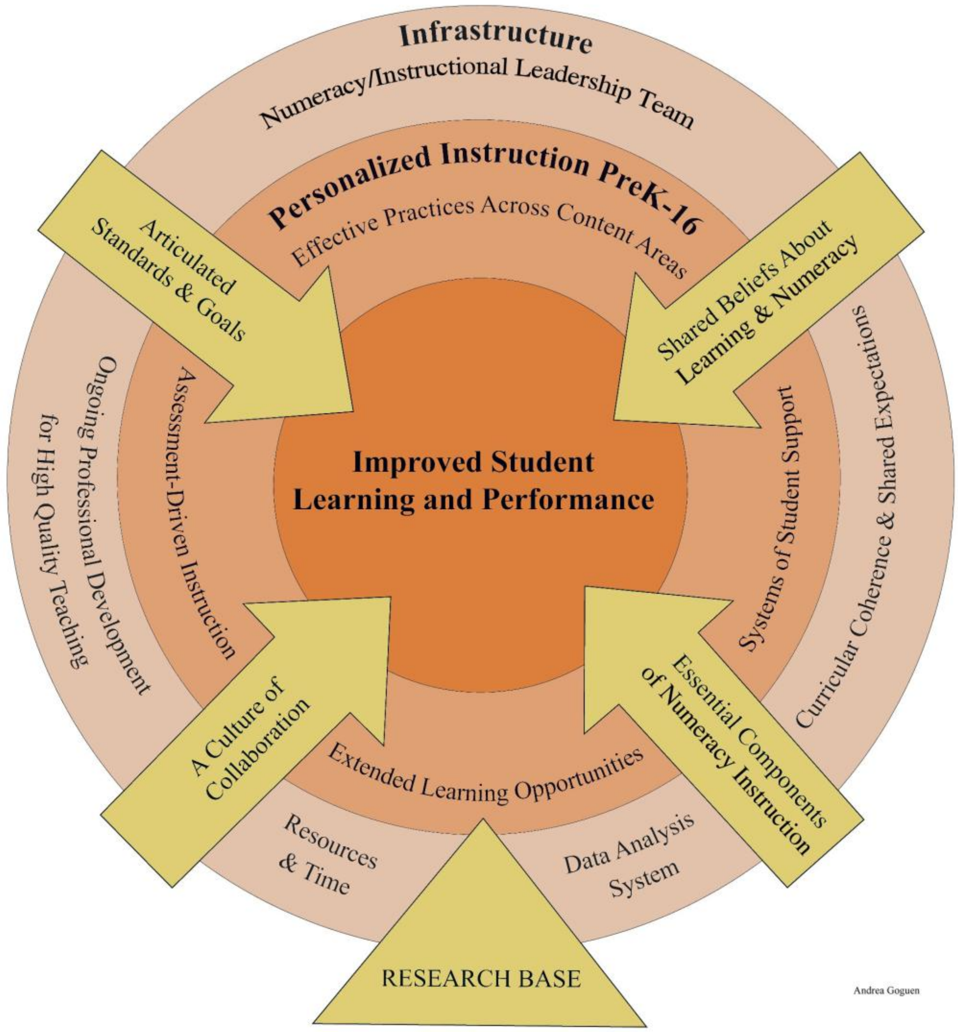
For generations, it has been expected that every citizen of this country be able to read and write proficiently. Today, being quantitatively literate has also reached this point: in order to fully participate in and contribute to modern society, one must be quantitatively literate.

"Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning. Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently. These are skills required to thrive in the modern world."

Lynn Arthur Steen, professor of mathematics at St. Olaf College and executive editor of *Mathematics and Democracy: The Case for Quantitative Literacy*

Appendix D: New Hampshire's Conceptual Framework for 21st Century Numeracy

New Hampshire's Conceptual Framework for 21st Century Numeracy



Appendix E: The Habits of Mind Poster – Elementary Version

HABITS OF MIND

We would like students to:

1. Be curious about "how" and "why" math works
2. Take risks when doing math
3. Work hard and be persistent when doing math
4. Value exploration and investigation when doing math
5. Develop confidence in solving math problems
6. Develop math intuition when solving math problems
7. Develop logical thinking skills when solving math problems
8. Write and discuss math using math terms correctly
9. Learn how to estimate effectively
10. Be able to check answers for reasonableness and accuracy
11. Know when and how to use different problem solving strategies
12. Know how to use technology appropriately and effectively
13. Appreciate that math is the language of nature and science
14. Use math to simplify and make sense of real life situations
15. Recognize that it is possible to learn from mistakes
16. Become quantitatively literate
17. Be able to work effectively as a member of a group
18. Be able to work effectively alone

"It is not enough to know something; the learner must possess the ability to do something with that knowledge, whether it is to solve a problem, reach a conclusion or present a point of view."

(Bob Kansky, *Report Summary: Understanding University Success)*

The "Habits of Mind" are adapted from Understanding University Success and A Consensus Model for Preservice Teacher Education in Mathematics and Science

Appendix F – The Habits of Mind Poster – Secondary Version

HABITS of MIND

We would like students to:

1. Develop a "curiosity" about mathematics and seek the "whys" behind the "how to's";
2. Recognize that hard work, persistence, and risk-taking are needed to do mathematics;
3. Develop and demonstrate self-confidence when doing mathematics and analytical thinking;
4. Value the process of exploration and investigation of mathematical concepts;
5. Make and test conjectures and verify or contradict those conjectures;
6. Recognize the need for logical arguments, proofs and deductive reasoning to verify conjectures and recognize that intuitive explanations are important;
7. Realize the need to communicate mathematics in writing and orally;
8. Recognize the role of estimation and the need to examine the reasonableness of results;
9. Recognize the need to employ strategies and heuristics in solving problems and when to employ them;
10. Demonstrate that solving problems in mathematics involves analyzing examples and appreciating the subtleties of an assumption or its limitations;
11. Value the use of technology, learning to apply it only when needed and appropriate, and recognize that technology does not replace knowledge of basic facts and skills;
12. Appreciate that mathematics is the language of nature and science and is a tool for quantitative reasoning;
13. Recognize that failure is a fact of life and that to be successful at challenges, one will experience failure, but will, hopefully, learn from it; and
14. Work as a member of a group, but proceed independently to draw inferences.

"Acquisition of these habits of mind supercedes mastery of content knowledge. Their achievement is necessary because 'It is not enough to know something; the learner must possess the ability to do something with that knowledge, whether it is to solve a problem, reach a conclusion or present a point of view.'"

(Bob Kansky, *Report Summary: Understanding University Success)*

The "Habits of Mind" above are adapted and compiled from
Understanding University Success and A Consensus Model for
Preservice Teacher Education in Mathematics and Science.

Appendix G - Framework for Leveraging Parent Involvement

Framework for Leveraging Parent Involvement for Increasing Numeracy Skills in Students K-12

NH State PIRC (Parent Information and Resource Center), www.nhpirc.org

Areas of Parent Involvement	Definition	Sample Numeracy Action Points	Results for Students	Results for Parents	Results for Schools
Parenting Responsibilities of families	Help all families establish home environments to support children as students.	• Parenting sessions, Numeracy Tip sheets, DVDs, etc., providing parenting skills and information to all families at convenient times and locations • Opportunities for reinforcement of the "Curriculum of the Home" (everyday activities at home that support math and learning at school) • Trips to libraries, museums, etc.	• Balance time spent on homework, chores, and other activities • Regular attendance • Awareness of importance of school, routine, and study habits	• Self-confidence about parenting as children proceed through school • Knowledge of child and adolescent development	• Understanding of families' goals and concerns for children • Respect for families' strengths and efforts
Communicating Two-way communication	Design effective forms of school-to-home and home to school communications about school programs and children's progress.	• School Community Compact (written academic and character goals including expectations for students, parents, and teachers) • Home Visits by teachers to reinforce importance of subject and parent involvement • Home Gatherings–(Math focus) • Parent to Teacher &Teacher to Parent Notes (e.g., Happy Grams) • Family Resource Library/Parent Room • E-Parenting (web-based student information reporting and sharing) • Websites, Blogs, and Newsletters • Parent Bulletin Board	• Awareness of own progress in subjects and skills • Knowledge of actions needed to maintain or improve grades • Awareness of own role as courier and communicator in partnerships	• High rating of quality of the school • Support for child progress and responses to correct problems • Ease of interactions and communications with school and teachers	• Ability to communicate clearly • Use of network of parents to communicate with all families
Volunteering Involvement at and for the school	Recruit and organize parent help and support.	• Volunteer Orientation Packet • "Parent Resource Pool" List • Volunteer File/Database • Math workshops (led by parents and/or teachers) • Family Math Nights	• Skills that are tutored or taught by volunteers • Skills in communicating with adults	• Understanding of the teacher's job • Self-confidence about ability to work in school and with children • Enrollment in programs to improve own education	• Readiness to involve all families in new ways, not only as volunteers • More individual attention to students because of help from volunteers
Learning at	Education and infor-	Parent Education and Literacy Activi-	• Skills, abilities, and test	• Discussions with	• Respect of family time

Home Involvement in academic activities	mation that provides ideas to families about how to help students at home with homework and other curriculum-related activities, decisions, and planning.	ties • *Helpful Homework Hints* course for parents • *Studying at Home* course for parents • *Raising Good Kids* (character development) course for parents • State standards written in parent/student friendly language • Courses development in concert with local schools or districts, e.g., *Everyday Math*	scores linked to class work; homework completion • View of parent as more similar to teacher, and home in sync with school • Self-confidence in ability as learner and positive attitude about school	child about school, class work, homework, and future plans • Understanding curriculum, what child is learning, and how to help each year	and satisfaction with family involvement and support • Recognition that single-parent, dual-income, and low-income families can encourage and assist student learning
Decision Making Participation and leadership	Include parents in school decisions, developing parent leaders and representatives	• School Community Council • Parent Education Committee • Continuous Improvement Team • Parent Leadership Training • School Parent Involvement Policy • Homework Policy • Parent-Teacher-Student Conferences	• Awareness that families' views are represented in school decisions • Specific benefits linked to policies enacted by parent organizations	• Awareness of and input to policies that affect children's education • Shared experiences and connections with other families	• Awareness of families' perspectives in policies and school decisions • Acceptance of equality of family representatives on school. committees
Collaboration with Community	Identify and integrate resources and services from the community to strengthen school programs, family practices, and student learning and development.	• Community-Building Ideas • Community Resource & Services Directory • Community internships, apprenticeships and mentorships supporting math knowledge, skills and practical applications	• Knowledge, skills, and talents from enriched curricular and extracurricular experiences and explorations of careers • Self-confidence and feeling valued by and belonging to the community	• Knowledge and use of local resources to increase skills and talents or to obtain needed services for family • Interactions with other families, and contributions to community	• Knowledge and use of community resources for improving curriculum and instruction • Strategies to enable students to learn about and contribute to the community

School, Family and Community Partnerships by J. L. Epstein, *Solid Foundation* – Sam Redding, ADI
For training and technical assistance in building or strengthening your school's or district's family engagement efforts, visit www.nhpirc.org or www.nhparentsmakethedifference.org , or call (603) 224-7005.

Appendix H: Resources for Parents

Prepared by the NH Parent Information and Resource Center (NHPIRC)

Web Resources for Parents
Retrieved August 10, 2009, from www.ed.gov/parents/academic/help/math

- Educational REALMS: www.stemworks.org
- Eisenhower National Clearinghouse for Mathematics and Science Education: www.enc.org
- Family Education Network: www.fen.com
- Figure This! Math Challenges for Families: www.figurethis.org/index40.htm
- KidSource: www.kidsource.com/kidsource/content/Learnmath8.html
- Links Learning: www.linkslearning.org
- The Math Forum: www.mathforum.org/parents.citizens.html
- Math in Daily Life: www.learner.org/exhibits/dailymath/
- National Council of Teachers of Mathematics: www.nctm.org/families/
- National Institute of Standards and Technology: www.nist.gov/public_affairs/kids/kidsmain.htm
- National Science Foundation: www.nsf.gov
- Newton's Window: www.suzannesutton.com/

Print Resources for Parents
Retrieved August 10, 2009, from www.ed.gov/parents/academic/help/math/

Brochures and Booklets for Parents

Counting on Excellence: How Parents Can Help Their Children Learn Mathematics. Recommendations from the National Mathematics Advisory Panel. Available for free from http://edpubs.ed.gov; see English and Spanish versions at: www.ed.gov/about/bdscomm/list/mathpanel/index.html , under "Parent Resources."

Helping Your Child Learn Math booklets. Order them in bulk for free from http://edpubs.ed.gov; see English and Spanish versions (PDF) at www.ed.gov/parents/academic/help/math/index.html

Other Publications for Parents

Apelman, M. & King, J. (1993). *Exploring everyday math: Ideas for students, teachers, and parents*. Portsmouth, New Hampshire: Heinemann.

Barber, J., Parizeau, N., Bergman, L. & Lima, P. (2002). *Spark your child's success in math and science: Practical advice for parents*. Berkeley, California: Great Explorations in Math and Science.

Dadila-Coates, G. & Thompson, V. (2003). *Family math II: Achieving success in mathematics*. Berkeley, California: Lawrence Hall of Science.

Hartog, M. D. & Brosnan, P. (2003). Doing mathematics with your child. ERIC Digest.

Kaye, P. (1988). *Games for math: Playful ways to help your child learn math from kindergarten to third grade*. New York: Pantheon.

Kulm, G. (1991). *Math power at home*. Washington, DC: American Association for the Advancement of Science.

Kulm, G. (1991). *Math power in the community*. Washington, DC: American Association for the Advancement of Science.

Mayfield-Ingram, K., Thompson, V. & Williams, A. (1998). *Family math: The middle school years algebraic reasoning and number sense*. Berkeley, California: Lawrence Hall of Science.

Milbourne, L. A. & Haury, D. L. (2003). *Helping students with homework in science and math*. ERIC Digest.

Miller, M. K. (1999). *Quick and easy learning games: Math (Grades 1-3)*. New York: Scholastic Professional Books.

National Council of Teachers of Mathematics. (1996). *Family math awareness activities*. Reston, Virginia: National Council of Teachers of Mathematics.

Polonsky, L., Freedman, D., Lesher, S. & Morrison, K. (1995). *Math for the very young: A Handbook of activities for parents and teachers*. New York: John Wiley & Sons.

Reys, B. (1999). *Elementary school mathematics: What parents should know about problem solving*. Reston, Virginia: National Council of Teachers of Mathematics.

Walthall, B. (Ed.). (1995). *IDEAAAS: Sourcebook for science, mathematics, and technology education*. Washington, DC: American Association for the Advancement of Science.

Books for Children

The following is only a sampling of the many available math-related children's books that your child might enjoy. Please ask your local or school librarian to help you find other appropriate titles. Many of books listed here are also available in languages other than English. Your librarian can help you locate books in a particular language.

This list is divided into two groups, those books most appropriate for you to read with your younger child and those that will appeal to your older child, who reads independently. However, you are the best judge of which books are appropriate for your child, regardless of age.

Preschool-Grade 2 (All titles need the publication date, city and state)

Adler, D. A. (1996). *Fun with fractions*. New York, NY: Holiday House.

Anno, M. (1997). *Anno's math games*. New York, NY: Penguin Group.

Axelrod, A. (2003). *Pigs at odds: Fun with math and games*. New York, NY: Simon and Schuster.

Brown, M. (1976). *One two three: An animal counting book*. New York, NY: Little Brown.

Burns, M. (1995). *The greedy triangle* (Brainy Day Books). New York, NY: Scholastic.

Carle, E. (1996). *1,2,3 to the zoo*. New York, NY: Penguin Group.

Dee, R. (1990). *Two ways to count to ten*. Geneva, IL: Holt.

Feelings, M. (1976). *Moja means one: Swahili counting book*. New York, NY: Dial Books.

Fox, M. (2002). *The Straight line wonder*. New York, NY: Mondo.

Greene, R. G. (2001). *When a line bends, a shape begins*. Boston, MA: Houghton Mifflin.

Hitz, D. (1986). *Demi's Count the animals 1 2 3*. New York, NY: Grosset and Dunlap.

Hoban, T. (1998). *So Many Circles, So Many Squares*. New York, NY: Harper Collins/Greenwillow.

Hopkins, L. B. (2001). *Marvelous math: A book of poems*. Madison, WI: Turtleback Books.

Hudson, C. W. (1988). *Afro-Bets 1 2 3 book*. East Orange, NJ: Just Us Productions.

Hutchins, P. (1989). *The doorbell rang*. New York, NY: Harper Collins/Greenwillow Books.

Jones, C. (1998). *This old man*. Boston, MA: Houghton Mifflin Company.

Lionni, L. (1985). *Numbers to talk about*. New York, NY: Pantheon Books.

Miller, J. (1992). *Farm counting book*. New York, NY: Little Simon.

Pinczes, E. J. (2002). *A remainder of one*. Boston, MA: Houghton Mifflin.

Pluckrose, H. (2001). *Numbers and counting: Let's explore*. Strongsville, OH: Gareth Stevens.

Schwartz, D. M. (1997). *How much is a million?* New York, NY: Scholastic.

Scieszka, J. (1995). *Math curse*. New York, NY: Viking Press.

Tafuri, N. (1993). *Who's counting?* New York, NY: Harper Collins/Harper Trophy.

Ziefert, H. (2003). *A dozen ducklings lost and found: A counting story*. Boston, MA: Houghton Mifflin/Walter Lorraine Books.

Grades 3-5

Adler, D. A. (2000). *Shape up! Fun with triangles and other polygons.* New York, NY: Holiday House.

Burns, M. (1975). *I hate mathematics!* (A Brown Paper School Book). New York, NY: Little, Brown.

Clement, R. (1994). *Counting on Frank*. Strongsville, OH: Gareth Stevens.

Garland, T. H. (1997). *Fibonacci fun: Fascinating Activities With Intriguing Numbers*. Palo, Alto, CA: Dale Seymour Publications.

Holub, J. (2003). *Riddle-Iculous math*. Morton Grove, IL: Albert Whitman.

Julius, E. K. (1995). *Arithmatricks: 50 easy ways to add, subtract, multiply and divide without a calculator*. Hoboken, NJ: John Wiley & Sons.

Lopresti, A. S. (2003). *A place for zero: A math adventure*. Watertown, MA: Charlesbridge Publishing.

Murphy, S. J. (2002). *Sluggers' car wash*. New York, NY: HarperCollins.

Neuschwander, C. (1999). *Sir Cumference and the first round table: A math adventure*. Watertown, MA: Charlesbridge Publishing.

Pappas, T. (1993). *Fractals, googols and other mathematical tales*. San Francisco, CA: Wide World Publishing.

Peterson, I. & Henderson, N. (1999). *Math trek: Adventures in the math zone*. Hoboken, NJ: John Wiley & Sons.

Schmandt-Besserat, D. (1999). *The history of counting*. New York, NY: HarperCollins.

Swartz, D. M. (2004). *G is for googol: A math alphabet book*. Harrisburg, PA: Triangle Press.

Tang, G. (2004). *The grapes of math: Mind stretching math riddles*. New York, NY: Scholastic.

Viorst, J. (2009). *Alexander who used to be rich last sunday*. New York, NY: Atheneum.

Wise, B. (2002). *Whodunit math puzzles*. New York, NY: Sterling.

Zaslavsky, C. (1998). *Math games & activities from around the world*. Chicago, IL: Chicago Review Press.

Math Software
Retrieved August 10, 2009, from www.ed.gov/parents/academic/help/math/

Many Web sites provide information and reviews that you can use to select the best mathematics software for your child. Here are just a few of those Web sites:

- Children's Math Software: www.educational-software-directory.net/children/math.html
- CODiE Awards: www.siia.net/codies/2010/
- Learning Village: http://www.learningvillage.com
- SuperKids (the educational software review page): http://www.superkids.com
- Viewz: www.viewz.com/reviews/

Appendix I: Supplemental Community Programs and Resources

21st Century Learning Centers in NH

The 21st Century Community Learning Center program offers expanded academic enrichment opportunities for children attending high poverty schools. Tutorial services and academic enrichment activities are designed to help students meet local and state academic standards in subjects such as reading and math. In addition, programs may provide youth development activities, drug and violence prevention programs, technology education programs, art, music and recreation, counseling, and character education to enhance the academic component of the program.

Both teachers and parents need to be made aware of their option to utilize these learning centers.

Contact: **New Hampshire Department of Education: 603-271-6133**

Supplemental Educational Service Providers in NH

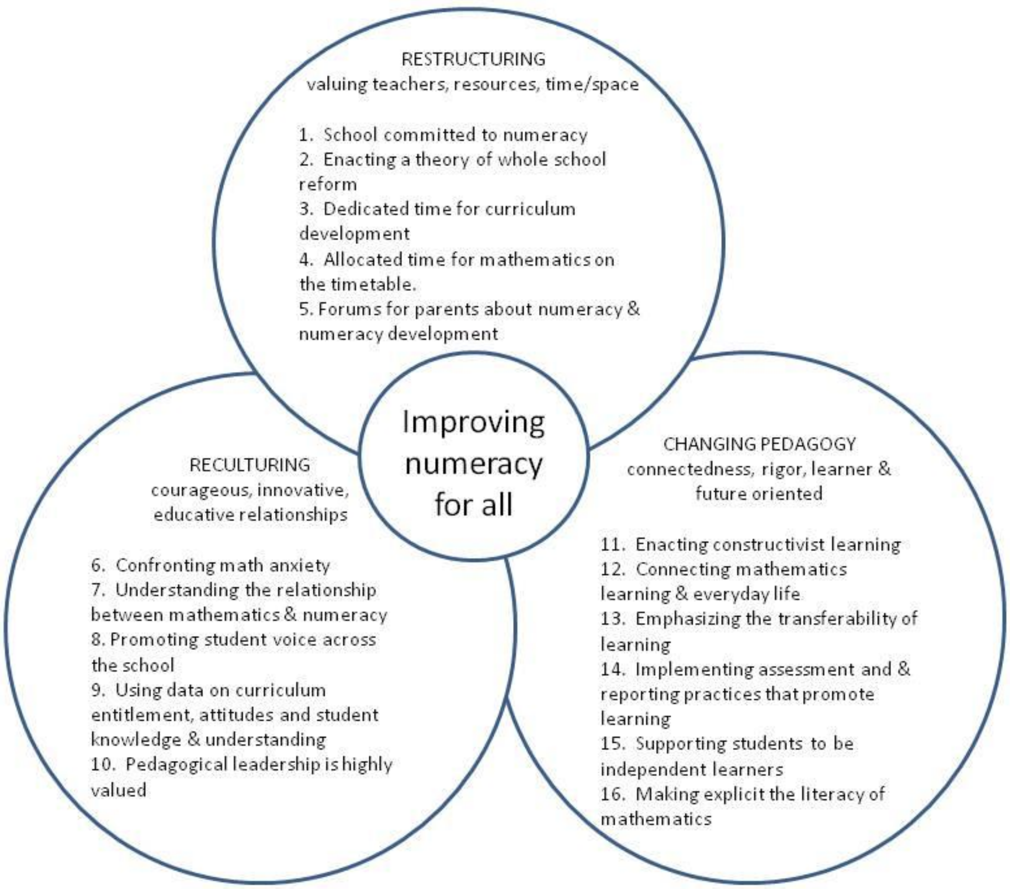
Under the No Child Left Behind law, schools that do not make adequate yearly progress (AYP) for the third year in a row must offer parents of low-income students the option of supplemental educational services (SES). SES offers additional academic assistance for low-income students who attend Title I schools. Supplemental services include tutoring or remedial services after school, on weekends, or during summers. This instruction must take place outside of the regular school day and must be provided by an approved entity.

Both teachers and parents need to be made aware of these services. Below are the DOE contacts as well as a list of approved SES providers.

Contact: New Hampshire Department of Education, Title I: 603-271-3769

Appendix J: Improving Numeracy Outcomes for All Students

Vincent, J., Stephens, M., & Steinle, V. (2004). *Numeracy research and development initiative 2001–2004: An overview of the numeracy projects.* Australian Government: Department of Education, Science and Training. p. 31



Appendix K: Cognitive and Metacognitive Behaviors in Non-Routine Problem-Solving

Yimer, A. & Ellerton, N. F. (2004). Cognitive and metacognitive aspects of mathematical problem solving: An emerging model. Illinois State University. Retrieved July 22, 2009, from www.merga.net.au/documents/RP672006.pdf

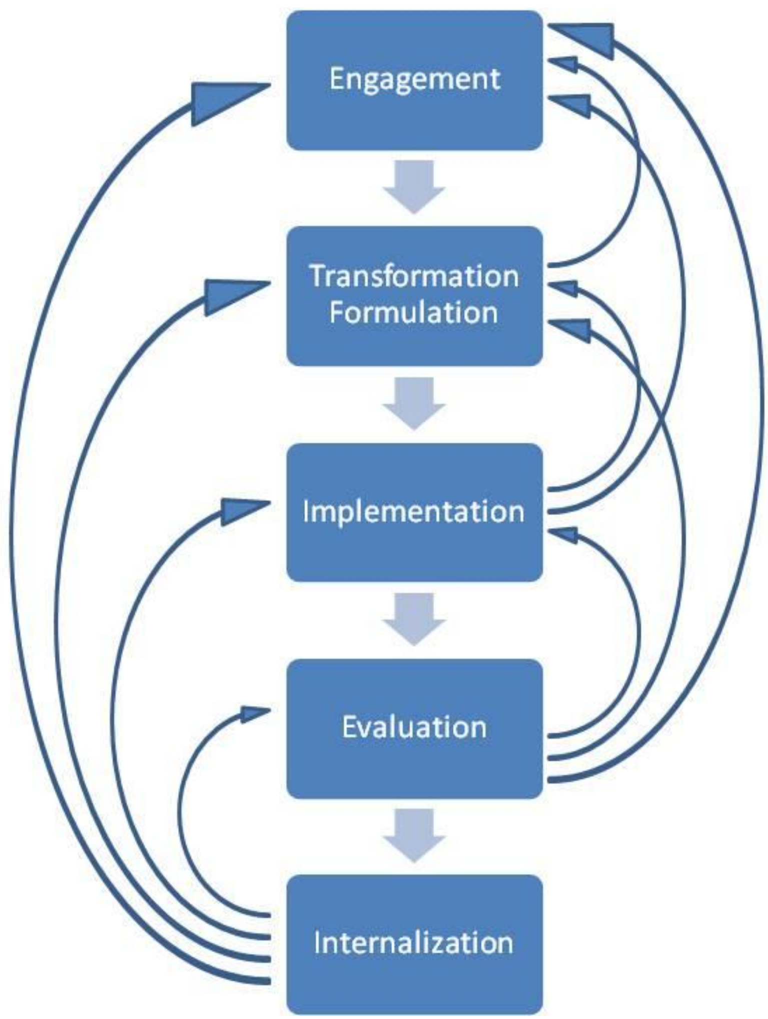
CATEGORIES OF COGNITIVE AND METACOGNITIVE BEHAVIOURS

1. **Engagement**: Initial confrontation and making sense of the problem.
   A. Initial understanding (jotting down the main ideas, making a drawing)
   B. Analysis of information (making sense of the information, identifying key ideas relevant information for solving the problem, relating it to a certain mathematical domain)
   C. Reflecting on the problem (assessing familiarity or recalling similar problems solved before, assessing degree of difficulty, assessing the necessary store of knowledge one has in relation to the problem)
2. **Transformation-Formulation**: Transformation of initial engagements to exploratory and formal plans.
   A. Exploring (using specific cases or numbers to visualize the situation in the problem)
   B. Conjecturing (based on specific observations and previous experiences)
   C. Reflecting on conjectures or explorations whether they are feasible or not.
   D. Formulating a plan (devising a strategy either to test conjectures or devising global or local plans)
   E. Reflecting on the feasibility of the plan vis-à-vis the key features of the problem
3. **Implementation**: A monitored acting on plans and explorations.
   A. Exploring key features of plan (breaking down plan into manageable sub plans where necessary)
   B. Assessing the plan with the conditions and requirements set by the problem
   C. Performing the plan (taking actions either computing or analyzing)
   D. Reflecting on the appropriateness of actions
4. **Evaluation**: Passing judgments on the appropriateness of plans, actions, and solutions to the problem
   A. Rereading the problem whether the result has answered the question in the problem or not
   B. Assessing the plan for consistency with the key features as well as for possible errors in computation or analysis
   C. Assessing for reasonableness of results
   D. Making a decision to accept or reject a solution
5. **Internalization**: Reflecting on the degree of intimacy and other qualities of the solution process.
   A. Reflecting on the entire solution process
   B. Identifying critical features in the process
   C. Evaluating the solution process for adaptability in other situations, different way of solving it, and elegance
   D. Reflecting on the mathematical rigor involved, one's confidence in handling the process, and degree of satisfaction

Appendix L: Flow Chart of the Cognitive Processes during Problem-Solving

Yimer, A. & Ellerton, N.F. (2004). Cognitive and metacognitive aspects of mathematical problem solving: An emerging model. Illinois State University. Retrieved July 22, 2009, from www.merga.net.au/documents/RP672006.pdf

The arrows in the flowchart indicate the different paths one can take when solving a problem. These paths can be cyclical as a student engages in self-assessment before they decide on another path. The path taken depends on one's background, understanding, and problem-solving ability.



Appendix M: Examples For Revising Mathematics Problems to Emphasize Numeracy

Carpenter, K. (2005). Numeracy across the curriculum. Retrieved July 31, 2009, from www.hstwohioregions.org/sitefiles/Carpenter%20Numeracy%20kickoff%20powerpoint.ppt

Traditional Mathematics	Revised to Emphasize Numeracy
We are investing $1,000 at 5% for 5 years compounded semi-annually. How much money would you make?	We are investing $1000 for 5 years. Which rate would you choose? Explain your reasoning: a) 5% compounded semi-annually b) 4.9% compounded quarterly c) 4.75% compounded continuously
A certain machine produces 300 nails per minute. At this rate, how long will it take the machine to produce enough nails to fill 5 boxes of nails if each box will contain 250 nails?	Determine the time for filling 5 boxes of nails containing 250 each, given the rate at which nails are produced. Describe the procedure you would use to determine this time.
What is the probability of drawing a blue marble from a bag containing 3 green, 5 yellow, 6 blue, and 10 red?	How many blue marbles would you need to add to the original bag of marbles to make the probability of drawing a blue marble 0.5?
Find the circumference and area of a circle with a diameter of 15 feet.	You are very hungry. Given the choice between a 12" round pizza or a 12" square pizza that would cost the same, which would you choose. Defend your choice.

Appendix N: Factors that Contribute to School Effectiveness

Five Factors of Effective Schools
Adapted from Edmonds, R.R. (1979). Effective schools for the urban poor. *Educational Leadership, 37*(1), p. 15 – 24.

1. Strong leadership and principal
2. Emphasis on basic skills mastery
3. Clean, orderly, and safe school environment
4. High expectations for student achievement
5. Frequent assessments

Eleven (11) Factors of Effective Schools
Adapted from Sammons, P., Hillman, J. & Mortimore, P. (1995). *Key characteristics of effective schools: A review of school effectiveness research.* London, England: Institute of Education for the Office for Standards in Education. Retrieved August 10, 2009, from
www.eric.ed.gov/ERICDocs/data/ericdocs2sql/content_storage_01/0000019b/80/14/49/79.pdf

1. Professional leadership
2. Shared vision & goals
3. A learning environment
4. Focus on teaching & learning
5. Purposeful teaching
6. High expectations
7. Positive reinforcement & feedback
8. Assessing progress of students & school
9. Students' rights and responsibilities
10. Parental involvement
11. Professional development

Appendix O: Numeracy Capacity Survey

Richardson, J. & Mero, D. (2007). *Making the mathematics curriculum count: A guide for middle and high school principals.* Reston, VA: National Association of Secondary School Principals.

For more information on NASSP products and services to promote excellence in middle level and high school leadership, visit www.principals.org .

The following template can be found at www.principals.org/s_nassp/sec.asp?CID=1338&DID=56265

Directions:

- Rate "Importance at Our School" for each item on a scale from 1 to 5.
- Rate "Exists at Our School" for each item on a scale from 1 to 5.
- Calculate the difference between column 1 and column 2.
- When you have rated all items in both columns and recorded the difference in the third column, return the survey for compilation with those completed by other stakeholders at your school.

Importance rating: 1 = Not important 5 = Very important

Practice rating: 1 = Infrequent or rare occurrence at this school
5 = Frequent or common practice at this school

	Importance at Our School	Exists at Our School	Difference
Collaborative Leadership and School Capacity			
1. The principal's role in improving the school's numeracy opportunities is clearly evident.			
2. The principal and school leaders encourage collegial decision-making.			
3. The principal and school leaders support integration of numeracy instruction.			
4. The principal, school leaders, and staff members believe that numeracy is their responsibility.			
5. Adequate fiscal resources are provided to support numeracy.			
6. Data-driven decision-making guides numeracy improvement planning.			
7. Scheduling structures are in place to support the implementation of numeracy initiatives.			
8. Scheduling structures are in place to support numeracy professional development.			
9. The school improvement plan includes numeracy as a major improvement goal.			
Strategic Use of Assessment			

10. School and student data sources are used to support the instructional improvement focus.			
11. Professional development to improve numeracy is based on assessment data.			
12. Standardized, formal assessments are used to assess the mathematics mastery of all students.			
13. Teachers utilize data to learn the numeracy capabilities of all students they teach.			
14. Assessment data is used in staff to plan and support student learning.			
15. The evaluation and monitoring process identifies skills mastered and not mastered by each student.			
16. Teachers use informal mathematics assessments within content classes to develop a better understanding of student numeracy instructional needs.			
Professional Development to Support Numeracy			
17. The numeracy team assesses and plans the numeracy professional development focus.			
18. Student numeracy needs drive professional development plans for teachers.			
19. Reflective teaching and self-assessment of instructional practices provide direction as to ongoing numeracy professional planning.			
20. Teachers with numeracy expertise and experience serve as models and mentors to less experienced colleagues.			
21. Data from informal/walkthrough visits provide areas of focus for numeracy professional development.			
22. Teachers participate in shared-teaching sessions to learn and refine numeracy strategies.			
23. Content area teachers receive ongoing, job-embedded professional development to learn instructional/numeracy strategies.			

Instructional Practices to Improve Student Achievement			
24. Content teachers use effective instructional activities in support of developing student numeracy.			
25. Teachers effectively use a variety of strategies to support numeracy.			
26. Teachers provide personalized support to each student based on assessed needs.			
27. Content teachers create numeracy-rich environments to support learning.			
28. Content teachers effectively use small group instructional strategies to improve student numeracy.			
29. Teachers effectively use a variety of numeracy learning strategies for all students.			
30. Teachers effectively use a variety of numeracy strategies that support problem solving in the content area for all students.			
31. Teachers use technology to support improved numeracy for all students.			
32. Teachers regularly use vocabulary development strategies and formal numeracy language to support student learning.			
33. Teachers regularly use strategies to support the reading/mathematics connection.			
Intervention to Improve Student Achievement			
34. Administrators and teachers use assessment data to develop individual numeracy plans to meet the instructional needs of all students.			
35. Intervention is highly prescriptive and supports the identified numeracy deficits of individual students.			
36. Electives are available to support improved numeracy of struggling/striving students.			
37. Ample tutoring sessions are available to support improved student numeracy.			
38. The most highly skilled teachers work with both accelerated and struggling/striving students.			
39. Teachers effectively use numeracy strategies to support all students' learning of content.			
40. The school numeracy improvement plan supports strategies ranging from intervention for struggling students to expanding the numeracy power of all students.			

Appendix P: Implementation Strategies

Richardson, J. & Mero, D. (2007). *Making the mathematics curriculum count: A guide for middle and high school principals.* Reston, VA: National Association of Secondary School Principals.

For more information on NASSP products and services to promote excellence in middle level and high school leadership, visit www.principals.org .

The following template can be found at www.principals.org/s_nassp/sec.asp?CID=1338&DID=56265

To compile planning information for your school, collect completed numeracy capacity (see *Appendix M*) surveys and follow these instructions:

- For item 1, add the numbers placed in column 1 on each of the completed surveys and find the average by dividing the total by the number of respondents.
- Using a blank survey form, place the average response for item 1 in column 1.
- Continue in this manner for all items 1 – 40 for column 1.
- Using the same method, compile average survey results for "Current Practice at Our School" (column 2).
- Compute the difference between the importance rating (column 1) and the practice rating (column 2). Record the differences for items 1 – 40 in column 3.

Now What? How can we use this information to implement an action plan?

Using the survey results, distribute and discuss the results in a faculty meeting, professional learning community gathering, or focus-group meeting.

STEP 1
Review the procedures used to compile information from the survey.

On this survey, numerical ratings of one through five have been given to each of the numeracy survey items in two categories: "How important do you think the item is to improving student achievement? (1=not at all important; 5=very important) and "To what degree is the item practiced in your school?" (1=not evident at all; 5=implemented and practiced to a high degree).

Explain that the ratings for each item were averaged across all respondents, and the difference between the importance rating and the practice rating was computed to arrive at the figure in the third column.

STEP 2
Organize participants into groups of five or six to complete the following activity. Read the directions for this activity:

1. Identify and discuss the areas where the difference between importance and practice are greatest and what the possible reasons might be.
2. Assume the role of school planning team members (teachers, parent, support staff) or technical consultants making recommendations to the principal of the school about issues needing immediate change.

Using the results, along with other available data and school resources, help the "team" decide where to begin numeracy efforts. Be prepared to defend your advice.

STEP 3
Poll the group to determine results, and record the results on chart paper.

Suggest that, depending on the situation, school leaders may want to begin with the areas where the largest amount of change is needed (because of a sense of urgency). Conversely, the most effective tactic may be to begin in areas where they are assured "quick wins" or where there is "low-hanging fruit" to motivate faculty members and gain momentum.

You will also need to pay attention to instances where an unusually low level of importance is attached to an item.

STEP 4
Facilitate a brief discussion with participants about the results.

STEP 5
Submit results to the principal and numeracy team for consideration and action.

Appendix Q: Evaluating a Rigorous Mathematics Curriculum with an Emphasis on Numeracy across the Curriculum

Adapted from Southern Regional Education Board. (2008). Establishing Benchmarks for New and Maturing HSTW sites. Atlanta, GA: SREB.

Benchmarks:

- All students take a mathematics class their senior year.
- All students take at least four full-year courses in mathematics.
- Teachers show students how mathematics concepts are used to solve problems in real-life situations.
- Students use a graphing calculator to complete mathematics assignments at least monthly.
- Students orally defend a process used to solve a mathematics problem at least monthly.
- Students work together to solve a challenging mathematics assignment and receive both a group and individual grade at least monthly.
- Students work in groups to brainstorm how to solve a mathematics problem at least monthly.
- Students solve mathematics problems other than those found in their textbook.
- Career and technology students use mathematics to complete challenging assignments.
- Students complete a written report for a major mathematics problem at least once a semester.
- Students are assigned word problems at least weekly.

Appendix R: Numeracy Goals Classroom Observation Form

Richardson, J. & Mero, D. (2007). *Making the mathematics curriculum count: A guide for middle and high school principals.* Reston, VA: National Association of Secondary School Principals.

For more information on NASSP products and services to promote excellence in middle level and high school leadership, visit www.principals.org .

A similar template can be found at www.principals.org/s_nassp/sec.asp?CID=1338&DID=56265

Date: ______________________________________
Teacher: ____________________________________
Content Area & Grade Level: ____________________________________
Current Lesson, Aim, or Objective: ____

Criteria are to be decided upon by departments/grade teams in cooperation with administrators and the numeracy/instructional leadership team. Areas to be considered may include climate, curriculum, instruction, technology, and assessment. The procedure may be similar to those used in walkthroughs or focus walks. Example criteria are included in the template below.

Criteria	Evidence	Suggestions/Questions
Uses proper vocabulary		
Uses proper notation		
Labels diagrams, graphs, and charts correctly		
Uses technology correctly and in a meaningful way		
Allows opportunities to solve contextual problems		
Utilizes open-ended, contextual problems/Provides real-world, current problems		
Does not solely focus on memo-		

rization and procedures		
Welcomes multiple methods of problem solving		
Encourages discussion and communication about problem solving		
Shares an enthusiasm for (or hides a dislike of) quantitative data		

Appendix S: How School Leaders Can Enhance Numeracy Instruction

Reynolds, B. & Walker-Glenn, M. (2007). In David, S. (Ed.), *Strategies for numeracy across the curriculum.* Southwest Ohio: High Schools That Work.

What Can School Leaders Do Now to Enhance Numeracy Instruction?

- Be a good role model. Allow others to see you using quantitative skills and data analysis for authentic audiences.
- Provide teachers with professional development opportunities to learn instructional strategies related to numeracy.
- Participate with teachers during those professional development sessions to become familiar with strategies that support numeracy across the curriculum and show your support for quantitative literacy in your school.
- Encourage teachers to engage in book studies about teaching numeracy as part of their professional development.
- Encourage the development of instructional units of study that integrate numeracy as a natural part of the presentation and outcome of the content being studied.
- Expect to see evidence of numeracy instruction in lesson/unit plans, in classroom observations, and in analysis of student products (review of working folders as well as finished products).
- Provide time for teachers to analyze student products to determine instructional implications.
- Create a risk-free environment for the staff to begin to experiment with different instructional strategies that promote numeracy. Encourage them to learn from failures and help them celebrate successes.

Appendix T: What Administrators Should See Students Doing in the Classroom

Reynolds, B. & Walker-Glenn, M. (2007). In David, S. (Ed.), *Strategies for numeracy across the curriculum.* Southwest Ohio: High Schools That Work.

What Administrators Should See Students Doing in the Classroom

- Being actively engaged in the learning process
- Using existing mathematical knowledge to make sense of a task
- Making connections among mathematical concepts
- Reasoning and making conjectures about the problem
- Communicating their mathematical thinking orally and in writing
- Listening to others' thinking and solutions to problems
- Using a variety of representations, such as pictures, tables, graphs, and words for their mathematical thinking
- Using mathematical and technological tools, such as physical materials, calculators, and computers, along with textbooks and other instructional materials
- Building new mathematical knowledge through problem solving

Appendix U: NH Mathematics Alignment Protocol and Gap Analysis Tool

NH Mathematics Alignment Protocol
Gap Analysis
Putting All the Pieces Together

1. **District Plan to Improve Student Achievement in Mathematics**
    - Our district has someone responsible for overseeing mathematics curriculum across all grade levels
    - Our district's strategic plan has long and short term goals for improving mathematics achievement with measurable outcomes
    - Our district coordinates all available resources (fiscal, personnel, and professional development) to support mathematics curriculum alignment
    - Our district plan is based on a careful evaluation of student achievement and program evaluation data

2. **Programs, Texts, and Resources**
    - Programs, texts, and resources are chosen based on their alignment to the K-12 Mathematics New Hampshire Curriculum Framework (2006) and New Hampshire PreK-16 Numeracy Action Plan for the 21st Century (2010)
    - Programs, texts, and materials are chosen with clarity around the amount and types of professional development teachers will need to implement the program with fidelity
    - Programs, texts, and resources are chosen with input and understanding in partnership with educators representing all grade levels and courses (e.g. career and technical, alternate programming, bilingual etc)
    - Programs, texts, and resources are placed within the context of the district's curriculum in mathematics so that everyone understands how the program should be implemented (e.g. pacing, sequence, assessments, pedagogy, etc.)

3. **Teacher Practice**
    - Every teacher is prepared to teach each mathematics concept in multiple ways
    - Every teacher uses high-quality research to support instructional decisions and practices relating to the teaching of mathematics.
    - Every teacher re-teaches concepts strategies, and skills based on formative assessment information
    - Every teacher has planned opportunities to discuss student work to reflect on instructional practice and student progress
    - Every new teacher (new to the grade, school or profession) is supported with a mentor
    - Every teacher is prepared to teach diverse learners (e.g. English language learners, students with disabilities)
    - Every teacher integrates explicit instruction in reading and writing into mathematics
    - Every teacher balances individual and group work with specific guidance for students to work well as part of a team
    - Every teacher is evaluated on a regular basis each year with a resulting plan for support and goals for improvement

4. **Teacher Content Knowledge**
    - Every teacher understands the mathematics behind each NH GLE/GSE in a way that allows them to teach beyond rote memory of skills to provide students with experiences that develop and enhance conceptual understanding of mathematics
    - Our district has a policy and protocol to ensure that a teacher is assigned to teach mathematics courses or grade levels only when s/he has demonstrated proficiency in the content and effective pedagogical strategies
    - Our district has a policy and protocol to ensure that every teacher is supported so that s/he gains a deeper understanding of specific mathematics topics as needed

5. **Professional Development**
   - The professional development plan is cohesive, that is, it has long term goals that extend beyond one year, specifies goals and coordinates across schools and grade levels
   - Professional development uses multiple funding sources in order to address mathematics holistically so that connections are made across topics (e.g. addressing content, diverse learners, pedagogy, assessment) rather than providing professional development by topic or funding stream
   - The district has worked in partnership with the bargaining unit (union) to address contractual barriers to planning and implementing focused, directed professional development in mathematics that balances meeting individual teacher needs with school needs
   - Professional development addresses and makes the connections between mathematics content and pedagogy
   - Professional development utilizes educational research related to the teaching and learning of mathematics.
   - There is ongoing and systematic support for teachers to transfer professional development into practice in their classrooms
   - Mathematics coaches are identified through an application process that emphasizes math content expertise and are trained in the coaching process

6. **Formative, Interim, and Summative Assessment**
   - Every teacher provides ongoing formative assessment in order to determine instructional "next steps" for individual students and the class
   - The district has developed grading policies in cooperation with teachers and the school community that articulate how grades are earned and assigned
   - The district has designed interim assessments that involve more than on demand paper and pencil methodologies; that is, students are asked to demonstrate and apply what they have learned through application and open-ended items
   - The district has identified high school course competencies, decided on appropriate competency assessment methods, and defined the necessary and sufficient evidence for students to demonstrate mastery in the area of mathematics.
   - Teachers meet on a regular basis within and across grades to calibrate how they evaluate student work including developing rubrics and benchmarks
   - Feedback to students includes more than a grade; rather, it provides information about how to improve
   - Summative assessments (e.g. comprehensive course assessments) are designed in collaboration with teachers across schools and programs
   - Teachers understand how NECAP and other standardized test results can be used in their classrooms and as program evaluation tools

7. **Supports for Students**
   - Every classroom's environment places students at the center of all decision making
   - Every student is well known so that immediate steps are taken when s/he is falling behind in mathematics
   - Every student has the materials (e.g. textbooks, manipulatives, calculators) s/he needs in order to participate in the mathematics curriculum
   - Every student's family is informed about student's progress in formal and informal ways (e.g. conferences, notes, progress reports, telephone calls)
   - Every student has the support s/he needs to be a successful mathematics student (e.g. ramp up programs, tutoring, extended mathematics classes, credit recovery)

8. **Vertical and Horizontal Alignment**
   - Every teacher understands the instructional "big ideas" of each grade level and how they build from the preceding grade and toward the next grade level

- Our district has K-12 mathematics curriculum that includes a process to annually review that mathematics curriculum and its implementation using data, teacher reflections, and current research to determine its effectiveness and make adjustments accordingly

9. Depth of Knowledge

- Every teacher has had professional development to understand and use the definitions of *Depth of Knowledge, (DOK)* to design lessons, assignments, and assessments
- Every teacher is comfortable modeling cognitive processing aloud for students as a part of instruction
- All instruction, assignments, and classroom assessments incorporate and balance the intended rigor of every NH GLE/GSE for each grade level

10. Distribution of Emphasis

- Each grade level's mathematics curriculum emphasis is aligned to the distribution of emphasis across content associated with the NECAP test
- Every teacher understands the distribution of emphasis and in their planning allocates instruction that is in alignment to this content emphasis

Gap Analysis Tool
Putting All the Pieces Together

New Hampshire has a critical need for a statewide commitment to and understanding of how action plans to improve mathematics education can be implemented with fidelity. As a result of current achievement data from NECAP and other testing, each district should complete a gap analysis of its current mathematics instructional system against the ten alignment components.

Aligned mathematics systems are defined as having these interdependent components that are further clarified by the bulleted indicators within each area. Mathematics achievement will improve only when there is a clear plan of action that is appropriately resourced and supported to address all of these components within every district's strategic plan. For each set of indicators we ask that you answer one or more of the following questions.

1. To what extent is this in place?
2. What evidence supports this judgment?
3. What resources do we have to continue work in this area?
4. What expertise do we have to support this area?
5. What barriers exist that prevent us from attending to this area?

Complete this gap analysis in partnership with representatives from your entire school and district community. More important than having a finished product are the examination and conversations that are necessary to complete the gap analysis.

Reference documents to use when completing this analysis include:

- K-12 Mathematics New Hampshire Curriculum Framework (2006)
- New Hampshire PreK-16 Numeracy Action Plan for the 21st Century (2010)

1. District Plan to Improve Student Achievement in Mathematics

Indicators:

- Our district has someone responsible for overseeing mathematics curriculum across all grade levels.
- Our district's strategic plan has long and short term goals for improving mathematics achievement with measurable outcomes.
- Our district coordinates all available resources (fiscal, personnel, and professional development) to support mathematics curriculum alignment.
- Our district plan is based on careful evaluation of student achievement and program evaluation data.

To what extent is our current plan reflective of each of these indicators?	
What evidence do we have that this plan will yield the results we hope? What are specific outcomes at the district, school, and student levels?	
What data have we used and how thoroughly have we analyzed data about each area of resources to develop our district plan?	
How have we developed system-wide and community understanding and commitment to our plan?	

2. Program, Texts, and Resources

Indicators:

- Programs, texts, and resources are chosen based on their alignment to the K-12 Mathematics New Hampshire Curriculum Framework (2006) and New Hampshire PreK-16 Numeracy Action Plan for the 21st Century (2010)
- Programs, texts, and materials are chosen with clarity around the amount and types of professional development teachers will need to implement the program with fidelity.
- Programs, texts, and resources are chosen with input and understanding in partnership with educators representing all grade levels and courses (*e.g.,* career and technical, alternate programming, bilingual, etc.).
- Programs, texts, and resources are placed within the context of the district's curriculum in mathematics so that everyone understands how the program should be implemented (*e.g.,* pacing, sequence, assessments, pedagogy, *etc*.).

At which grade levels or with which populations are we most confident that the materials are in alignment? How do we know?	
At which grade levels or with which populations are we most concerned that there is substantial alignment work to be done?	
How do we identify future opportunities to address alignment gaps related to programs, texts, or resources?	
What support do we need in order to continue this alignment work?	

3. Teacher Practice

Indicators:

- Every teacher is prepared to teach each mathematics concept in multiple ways.
- Every teacher uses high-quality research to support instructional decisions and practices relating to the teaching of mathematics.
- Every teacher re-teaches concepts, strategies, and skills based on formative assessment information.
- Every teacher has planned opportunities to discuss student work to reflect on instructional practice and student progress.
- Every new teacher, (new to grade, school, or profession) is supported with a mentor.
- Every teacher is prepared to teach diverse learners (*e.g.,* ELL, students with disabilities, *etc*).
- Every teacher integrates explicit instruction in reading and writing into mathematics.
- Every teacher balances individual and group work with specific guidance for students to work well as part of a team.
- Every teacher is evaluated on a regular basis each year with a resulting plan for support and goals for improvement.

How do we know and what evidence do we have to evaluate the extent to which each teacher in our district engages in high quality instructional practice in mathematics?	
How do we document and communicate our beliefs and expectations around best instructional practice in mathematics?	
What evidence do we have that new teachers are well supported by mentoring or other strategies?	
What help do we need to improve our school and district work in this area (*e.g.,* training, research, expertise, *etc*.)?	

4. Teacher Content Knowledge

Indicators:

5. Every teacher understands the mathematics behind each NH GLE/GSE in a way that allows them to teach beyond rote memory of skill and to provide students with experiences that develop and enhance conceptual understanding of mathematics.

6. Our district has a policy and protocol to ensure that each teacher is assigned to teach mathematics courses or grade levels only when s/he has demonstrated proficiency in the content and effective pedagogical strategies.

7. Our district has a policy and protocol to ensure that each teacher is supported so that s/he gains a deeper understanding of specific mathematics topics as needed.

How do we know and what evidence do we have to evaluate the degree to which each teacher of mathematics in our district is well grounded in mathematics content?	
How do we make decisions about the grade level, courses, or populations of students each teacher is responsible for teaching? How can we improve this process?	
What systems do we have in place to ensure that we are providing professional development and support to teachers who need to develop a deeper understanding of mathematics content?	
What are our barriers and resources with respect to making improvements in teacher content knowledge?	

5. Professional Development

Indicators:

- The professional development plan is cohesive, that is, it has long term goals that extend beyond one year, specifies goals and coordinates across schools and grade levels.
- Professional development uses multiple funding sources in order to address mathematics holistically so that connections are made across topics (*e.g.,*, addressing content, diverse learners, pedagogy, assessment) rather than providing professional development by topic or funding stream.
- The district has worked in partnership with the bargaining unit (union) to address contractual barriers to planning and implementing focused, directed professional development in mathematics that balances meeting individual teacher needs with school needs.
- Professional development addresses and makes the connections between mathematics content and pedagogy.
- Professional development utilizes educational research related to the teaching and learning of mathematics.
- There is ongoing and systematic support for teachers to transfer professional development into practice in their classrooms.
- Mathematics coaches are identified through an application process that emphasizes math content expertise and are trained in the coaching process.

How do we plan professional development within schools and across the district(s)? Who plans, with what information, and how are decisions made?	
How are professional development Providers selected? For example, if teacher groups make these decisions, how is their work focused, supported, and evaluated for effectiveness? How well do these decisions align with the district plan?	
What supports or help do we need to more effectively plan for or implement professional development?	
What resources do we have to support professional development -- both staff and financial?	

6. Formative, Interim, and Summative Assessment

Indicators:

- Every teacher provides ongoing formative assessment in order to determine instructional "next steps" for individual students and the class.
- The district has developed grading policies in cooperation with teachers and the school community that articulate how grades are earned and assigned.
- The district has designed interim assessments that involve more than on demand paper and pencil methodologies, that is, students are asked to demonstrate and apply what they have learned through application and open-ended items.
- The district has identified high school course competencies, decided on appropriate competency assessment methods, and defined the necessary and sufficient evidence for students to demonstrate mastery in the area of mathematics.
- Teachers meet on a regular basis within and across grades to calibrate how they evaluate student work. Including the development of rubrics and benchmarks.
- Feedback to students includes more than a grade; rather, it provides information about how to improve.
- Summative assessments (*e.g.,* comprehensive course assessments) are designed with teachers across schools and programs.
- Teachers understand how NECAP and other standardized test results can be used in their classrooms and as program evaluation tools.

What formative assessments do we use in our district at each grade level? How do we know that teachers are using on-going information to support students' learning?	
What interim assessments do we use in our district at each grade level? How are they developed, scored, and calibrated across teachers and schools?	
What summative assessments do we use in our district? How are they developed, scored, and calibrated across teachers and schools?	
What support do we need in order to develop a comprehensive assessment and grading system across all grade levels?	

7. Supports for Students

Indicators:

- Every classroom's environment places students at the center of all decision making.
- Every student is well known so that immediate steps are taken when s/he is falling behind in mathematics.
- Every student has the materials (*e.g.,* textbooks, manipulatives, calculators) s/he needs in order to participate in the mathematics curriculum.
- Every student's family is informed about their student's progress in formal and informal ways (*e.g.,* conferences, notes, progress reports, telephone calls).
- Every student has the support s/he needs to be a successful mathematics student (*e.g.,* ramp up programs, tutoring, extended mathematics classes, credit recovery).

What systems do we have in place to know that every student is in a supportive classroom environment?	
What is currently in place to support students who are falling behind in mathematics?	
What policies and practices are in place to ensure that families are kept informed of student progress, successes, and concerns?	
What help do we need in order to provide better supports to every student in our district?	

8. Vertical and Horizontal Alignment

Indicators:

- Every teacher understands the instructional "big ideas" of each grade level and how they build from the preceding grade and toward the next grade level.
- Our district has K-12 mathematics curriculum that includes a process to annually review that mathematics curriculum and its implementation using data, teacher reflections, and current research to determine its effectiveness and make adjustments accordingly.

What process do we use to ensure that our curriculum addresses each content strand and subpart of the NH GLE/GSE and Numeracy Action plan in order to identify the connections and sequencing of topics across grade levels?	
What process do we use to evaluate our mathematics curriculum? Who is involved, what data and other evidence do we use?	
What support do we need in order to ensure that we have an articulated alignment within and across grades?	

9. Depth of Knowledge

Indicators:

- Every teacher has had professional development to understand and use the definition of *Depth of Knowledge*, (DOK) to design lessons, assignments and assessments.
- Every teacher is comfortable modeling cognitive processing aloud for students as a part of instruction.
- All instruction, assignments, and classroom assessments incorporate and balance the intended rigor of every GLE/GSE for each grade level.

How are teachers currently using DOK as part of instruction and assessment? What training has been provided to every teacher?	
To what extent has the district reviewed its entire curriculum to ensure that there is adequate attention to DOK so that students are prepared in basic skills and higher order thinking?	
How do we know that every teacher is able to model cognitive processing in order to help students understand the conceptual underpinnings of the mathematics GLE/GSE?	
What support or assistance do we need to better understand and apply DOK as part of our district's curriculum, instruction, and assessment?	

10. Distribution of Emphasis

Indicators:

- Each grade level's mathematics curriculum emphasis is aligned to the distribution of emphasis across content associated with the NECAP tests.
- Every teacher understands the distribution of emphasis and in their planning allocates instruction that is in alignment to this content emphasis.

What process has our district used to ensure that our mathematics curricula are aligned to state's distribution of emphasis?	
How does our district balance instructional needs of students who have gaps in their mathematics content with our pacing guides?	
How does our district monitor and evaluate how each teacher makes decisions about emphasizing when and how mathematics NH GLE/GSE are introduced and taught at each grade level?	
What support or assistance does our district need in order to review and make decisions about our distribution of emphasis within and across grades?	

Appendix V: Connections between NECAP Test Items and Grade Level and Grade Span Expectations

The problems below are a representative sampling of 2007 NECAP released items along with the corresponding Grade Level (or Span) Expectation. These problems and expectations are synonymous to topics and components commonly addressed in numeracy instruction and described throughout this Numeracy Action Plan. It is important to note that, although a question is on a grade-specific test, it is testing an expectation of a previous grade level.

From the 3rd grade 2007 NECAP exam - Look at this chart.

Stars Earned

Student	**Number of Stars**
Anna	173
Carla	184
Erin	198
Holly	177
Judy	201
Susan	189

Which student earned more stars than Carla but fewer stars than Erin?

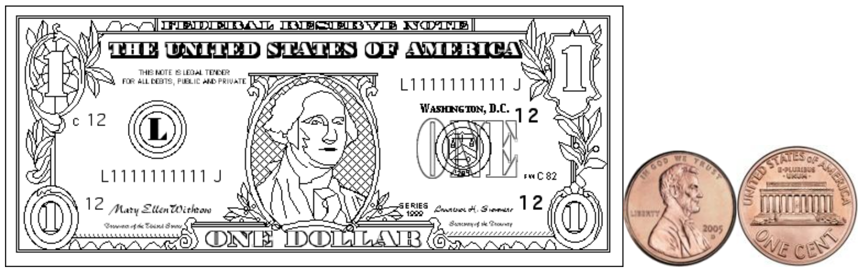
A. Anna
B. Holly
C. Judy
D. Susan

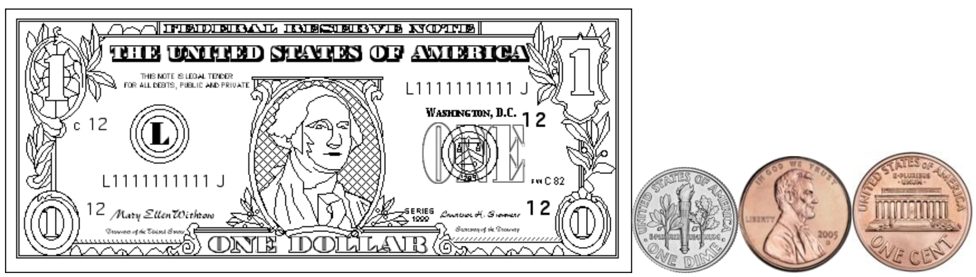
GLE: Number & Operations 2.2 - Demonstrates understanding of the relative magnitude of numbers from 0 to 199 by ordering whole numbers; by comparing whole numbers to each other or to benchmark whole numbers (10, 25, 50, 75, 100, 125, 150, or 175); by demonstrating an understanding of the relation of inequality when comparing whole numbers by using “1 more”, “1 less”, “10 more”, “10 less”, “100 more”, or “100 less”; or by connecting number words and numerals to the quantities they represent using models, number lines, or explanations.

From the 3rd grade 2007 NECAP exam - Mandy had $0.62. Then she earned 2 quarters. Which amount of money has the same value as the money Mandy has now?

A.

B. 

C. 

D. 

GLE: Number &Operations 2.5 - Demonstrates understanding of monetary value by adding coins together to a value no greater than \$1.99 and representing the result in dollar notation; making change from \$1.00 or less, or recognizing equivalent coin representations of the same value (values up to \$1.99).

From 4th Grade 2007 NECAP exam - The students in Mr. Hill's class are solving this problem. Peter had 10 pennies. Then he found more pennies. Now Peter has 16 pennies. How many pennies did Peter find? Three students wrote these number sentences to solve the problem.

16−10=□	10−□=16	10+□=16
Ella	**Connie**	**Andy**

Who wrote a correct number sentence?
A. only Connie and Andy
B. only Connie
C. only Andy and Ella
D. only Ella

GLE: Number & Operations 3:3 - Demonstrates conceptual understanding of mathematical operations by describing or illustrating the inverse relationship between addition and subtraction of whole numbers; and the relationship between repeated addition and multiplication using models, number lines, or explanations.

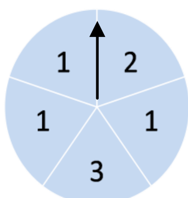
From the 6th Grade 2007 NECAP exam

Which shape is possible?

A. a rhombus with 4 acute angles
B. a parallelogram with 4 angles of equal measure
C. a rhombus with sides that measure 4 cm, 4 cm, 8 cm, and 8 cm
D. a parallelogram with sides that measure 2 cm, 4 cm, 6 cm, and 8 cm

GLE: Geometry & Measurement 5.1 - Uses properties or attributes of angles (right, acute, or obtuse) **or sides** (number of congruent sides, parallelism, or perpendicularity) **to identify, describe, classify, or distinguish among** different types of triangles (right, acute, obtuse, equiangular, or equilateral) or quadrilaterals (rectangles, squares, rhombi, trapezoids, or parallelograms).

From the 6th Grade 2007 NECAP exam

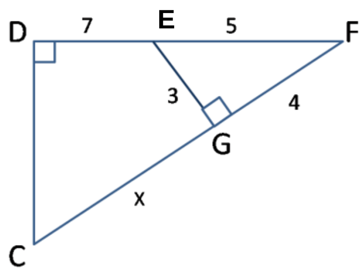


Lydia will use these spinners to form a two-digit number. She will use the number the arrow on Spinner A lands on as the tens digit and the number the arrow on Spinner B lands on as the ones digit. Lydia will spin each arrow once. What two-digit number is Lydia **most likely** to form?

GLE: Data, Statistics, & Probability - 5.5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an event and expresses the result as a fraction.

From the 8th grade 2007 NECAP exam - Keesha's computer can receive information at a rate of 52 kilobytes per second. At this rate, how many kilobytes of information can her computer receive in one minute? A. 3012 B. 3120 C. 5020 D. 5200 **GLE: Number & Operations 7.4 - Accurately solves problems involving** proportional reasoning; percents involving discounts, tax, or tips; and rates. (IMPORTANT: *Applies the conventions of order of operations including parentheses, brackets, or exponents.*)
From the 8th grade 2007 NECAP exam - Ian and Lauren are each making patterns that follow the same rule. **Ian's pattern: 1, 4, 16, 64** **Lauren's pattern: – 2, – 8, ____** What is the next number in Lauren's pattern? A. –14 B. –16 C. –24 D. –32 GLE: **Functions & Algebra 7.1 - Identifies and extends to specific cases a variety of patterns** (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; **and generalizes** a linear relationship using words and symbols; generalizes a linear relationship to find a specific case; or writes an expression or equation using words or symbols to express the **generalization** of a nonlinear relationship.

From the 11th grade 2007 NECAP exam



Triangle *CDF* is similar to triangle *EGF* ($\Delta CDF \sim \Delta EGF$). What is the value of x?

A. 15
B. 11
C. 9
D. 6

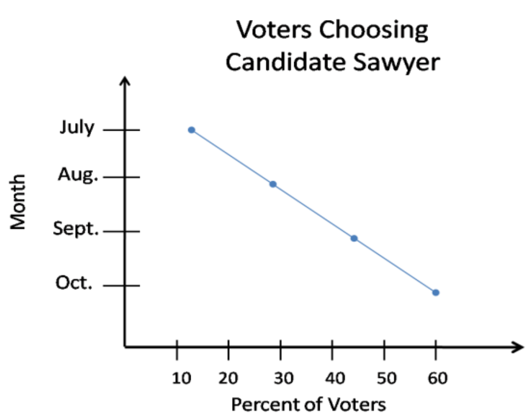
GSE: Geometry & Measurement 10.5 - Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts.

From the 11th grade 2007 NECAP exam - This table shows the results of polls taken during the four months preceding an election between two candidates—Sawyer and Hillman.

Percent of Voters Choosing Sawyer

July	Aug.	Sept.	Oct.
15%	30%	45%	60%

Hillman published this graph in a newspaper.



How could this graph be misleading about Sawyer's popularity?
A. The graph does not show the number of voters polled each month.
B. The graph does not show the percent of voters who chose Hillman.
C. The graph gives the impression that Sawyer's popularity is decreasing.
D. The graph gives the impression that Sawyer's popularity is changing at a constant rate.

GSE: Data, Statistics, & Probability 10.3 - Identifies or describes representations or elements of

representations that best display a given set of data or situation, consistent with the representations required in M(DSP)-10-1.

Appendix W: Numeracy Book and Resource List

Adapted from the following sites:

www.math.cudenver.edu/~wbriggs/qr/papers.html

www.statlit.org/NumeracyBooks.htm

www.stolaf.edu/other/ql/publ.html

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The Americans with Disabilities Act of 1990 (ADA) - disability
NH Law against discrimination (RSA 354-A)

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