ELL CO-OP PLC WIDA/GSE Alignment Project

Mathematics

Number and Operations

Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation(s) to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

M(N&O):7:1	**Demonstrates conceptual understanding of rational numbers with respect to** percents as a means of comparing the same or different parts of the whole when the wholes vary in magnitude (e.g., 8 girls in a classroom of 20 students, or 20% of 400 compared to 50% of 200); and percents as a way of expressing multiples of a number (e.g., 200% of 50) **using models, explanations, or other representations.**
Level 1 Entering	Identifies the vocabulary of rational numbers, such as *part, whole,* and *percent,* given oral instructions and visual models (e.g., shade in percentages of rectangles that represent different total values).
Level 2 Beginning	Matches and reads percents as parts of a whole in multiple ways, including word sentences, proportions, and decimal multipliers (e.g., match *0.25 x 200 = 50* with *25% of 200 is 50).*
Level 3 Developing	Orders different percent phrases (e.g., *20% of 400),* using symbols and vocabulary such as *greater than, less than* or *equal to.*
Level 4 Expanding	Translates word phrases that are based on real-world situations into mathematical symbols, and solves the problem (e.g., *30% of the 250 cars in the parking lot are gray. Write the mathematical phrase and find how many cars are gray.).*
Level 5 Bridging	Selects percent phrases from within the context of a written problem, translates the phrases into mathematical symbols, solves the problem, and explains reasoning.

M(N&O):7:2	**Demonstrates understanding of the relative magnitude of real numbers** by ordering, comparing, or identifying equivalent rational numbers across number

	formats, numbers with whole number bases and whole number exponents (e.g., 3^3, 4^3), integers, absolute values, or numbers represented in scientific notation using number lines or equality and inequality symbols.
Level 1 Entering	Identifies and names real numbers and basic mathematical symbols (e.g., $\leq \geq \neq \times \div + = -$) by matching symbols with spoken or written words.
Level 2 Beginning	Orders and compares numbers using terms such as *more, less, smaller, bigger, equal,* given models or illustrations (e.g., base ten blocks, number lines, pictures).
Level 3 Developing	Describes and/or discusses the change in the magnitude of a number after performing a given operation, using terms such as *greater than, less than, increases, decreases, percent, whole number, fraction, decimal,* with visual support (e.g., fraction bars, number lines, graphs, or other models).
Level 4 Expanding	Orders and compares numbers using number lines, charts, and graphs, and explains reasoning using terms such as *equality, inequality, exponent, base, positive, negative* (e.g., *2^3 is less than 2^5 because the exponents tell how many times you use the base of 2 as a factor and 2 multiplied together 5 times is more than 2 multiplied together 3 times).*
Level 5 Bridging	Evaluates and justifies conclusions about the relative magnitude of numbers, using terms such as *absolute value, powers,* and decimal place value vocabulary, with the support of word banks, charts, and tables (e.g., *The absolute value of negative three is greater than the absolute value of negative one because negative three is farther from zero, but negative one is greater than negative three).*

M (N&O) 7:4	**Accurately solves problems involving** proportional reasoning; percents involving discounts, tax, or tips; and rates. *(IMPORTANT: Applies the conventions of order of operations including parentheses, brackets, or exponents.)*
Level 1 Entering	Identifies whether percentages used in everyday situations mean that you pay more or less, using general terms such as *25% off, pay less, 7% tax, pay more.*
Level 2 Beginning	Identifies and rephrases key words in problem-solving situations, given word banks and visual support (e.g., a discount problem with a picture of the discounted item, original price tag, and percent off).
Level 3 Developing	Sequences a given set of steps in solving a problem involving percents or rates, orally and/or with pictures or manipulatives (e.g., order the number sentences that you would follow to discount or tax an item).
Level 4 Expanding	Follows oral and/or written directions that include real-world representations of proportions, such as *discounts, tax, tips* or *rates,* and performs steps needed in order to solve problems, using word banks, manipulatives, visual representation (including graphs and/or ratio tables), or graphic organizers.
Level 5 Bridging	Explains steps to solve a proportional reasoning problem and justifies the answer in written or oral discourse, using word banks, a dictionary and examples.

Geometry and Measurement

Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the

development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

M (G&M) 7-1	**Uses properties of angle** relationships resulting from two or three intersecting lines (adjacent angles, vertical angles, straight angles, or angle relationships formed by two non-parallel lines cut by a transversal), or two parallel lines cut by a transversal to solve problems.
Level 1 Entering	Identifies parallel lines, non-parallel lines, intersecting lines, and angles, from pictures of everyday objects, using a word bank with visual representations.
Level 2 Beginning	Defines or describes line and angle relationships, from pictures or real world examples, using a word bank with visual representations (e.g., *Put two angles together; they make a straight line, so they are straight angles.)*
Level 3 Developing	Compares and contrasts, orally, angle relationships from diagrams, using models and word banks.
Level 4 Expanding	Summarizes and explains how to solve problems using different types of angle relationships, from diagrams, models, and word banks.
Level 5 Bridging	Explains orally or in writing, with details, ways to solve grade-appropriate problems using different line and angle relationships.

M (G&M) 7-2	**Applies theorems or relationships** (triangle inequality or sum of the measures of interior angles of regular polygons) to solve problems.
Level 1 Entering	Matches different types of triangles and regular polygons with their names and their properties (e.g., using cards that include vocabulary, properties, drawings).
Level 2 Beginning	Creates models using geoboards or other manipulatives, given oral descriptions of polygons and their properties (e.g., *Make a hexagon with all sides equal* or *Make a triangle with sides of one, two, and three units).*
Level 3 Developing	Classifies polygons and discusses reasons for their classification, using cards and/or manipulatives that include vocabulary and attributes.
Level 4 Expanding	Explains orally how to solve a problem involving theorems or relationships, such as the sum of interior angles, referencing vocabulary cards, pictorial representations, and/or examples.
Level 5 Bridging	Explains orally or in writing (including use of examples) how to solve a problem requiring the application of a theorem or relationship (e.g., why three given line segments can or cannot form a triangle).

M (G&M) 7-4	**Applies the concepts of congruency** by solving problems on a coordinate plane involving reflections, translations, or rotations.
Level 1 Entering	Identifies congruent shapes from a set, and develops understanding of a line of symmetry, using manipulatives such as mirrors, geoboards, and pictorial representations.
Level 2 Beginning	Follows simple directions to manipulate shapes (e.g., flip, slide, turn), in order to demonstrate reflections, translations, or rotations, using manipulatives.
Level 3 Developing	Matches the type of shape movement with its general and technical term, using word banks and examples (e.g., match *flip* with *reflection* and model).

Level 4 Expanding	Follows written instructions to draw a transformation on a coordinate plane, given a model and a word bank (e.g., rotate triangle ABC 90° around point A).
Level 5 Bridging	Describes the type of movement using the terminology of a coordinate plane, referencing word banks and prior examples (e.g., *The shape was reflected across the* y*-axis, the* x*-coordinates changed to their opposites, the distance to the y-axis is the same).*

M (G&M) 7-5	**Applies concepts of similarity** by solving problems involving scaling up or down and their impact on angle measures, linear dimensions and areas of polygons, and circles when the linear dimensions are multiplied by a constant factor. Describes effects using models or[sc] explanations.
Level 1 Entering	Points to or names shapes and dimensions, given a picture word bank (e.g., point to the *base* of the triangle, color the *area* of the circle).
Level 2 Beginning	Compares similar shapes and their dimensions, using a key or a visual word bank (e.g., *this triangle is bigger, this side is smaller, these angles are equal).*
Level 3 Developing	Classifies visually-supported examples of similar figures in real world situations, with a partner.
Level 4 Expanding	Predicts the effects on perimeter of multiplying the sides of a polygon by a constant factor, using models and examples, in a small group.
Level 5 Bridging	Describes orally or in writing what happens to the area of a rectangle if the base and height are multiplied by a constant factor, given a model.

M (G&M) 7-6	**Demonstrates conceptual understanding of the area** of circles or the area or perimeter of composite figures (quadrilaterals, triangles, or parts of circles) and the **surface area of** rectangular prisms, **or volume of** rectangular prisms, triangular prisms, or cylinders using models, formulas, or by solving related problems. Expresses all measures using appropriate units.
Level 1 Entering	Identifies two-dimensional shapes in composite figures or nets of solids, and matches solids with nets, with a partner.
Level 2 Beginning	Creates cards including vocabulary, diagrams, formulas, attributes, and real-world examples.
Level 3 Developing	Classifies visually-supported real-world situations according to use of perimeter, area, surface area, or volume in (e.g., painting a box uses surface area).
Level 4 Expanding	Orders steps for computing perimeter, area, surface area, or volume in real-world situations, using sequential language and models.
Level 5 Bridging	Explains orally or in writing, with details, ways to solve problems involving perimeter, area, surface area, or volume.

Functions and Algebra

Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

M (F&A) 7-1	**Identifies and extends to specific cases a variety of patterns** (linear or nonlinear) represented in models, tables, sequences, graphs, or in problem situations; **and generalizes** a linear relationship using words and symbols; generalizes a linear relationship to find a specific case; or writes an expression

	or[sc] equation using words or[sc] symbols to express the **generalization** of a nonlinear relationship.
Level 1 Entering	Recognizes when patterns exist, given a series of numbers or shapes, by sorting the patterns and the non-patterns, and chooses the next terms in a sequence by using pictorial representations or symbols.
Level 2 Beginning	Classifies patterns as linear, nonlinear, or neither, using models, pictorial representations/graphs, and graphic organizers.
Level 3 Developing	Matches a pattern with the rule, expressed both symbolically and in words (e.g., *2 ,4, 6, 8,…; n+2; add two to each number),* and discusses reasoning for the match with a partner or in a small group
Level 4 Expanding	Writes the rule for a pattern, symbolically and in words (e.g., *2 ,4, 6, 8,…; n+2; add two to each number)*, using models and word banks.
Level 5 Bridging	Distinguishes between linear and nonlinear equations and explains reasoning, using a word bank and/or models and graphs.

M (F&A) 7-2	**Demonstrates conceptual understanding of linear relationships** *(y = kx; y = mx + b)* **as a constant rate of change** by solving problems involving the relationship between slope and rate of change, by describing the meaning of slope in concrete situations, or informally determining the slope of a line from a table or graph; **and distinguishes between constant and varying rates** of change in concrete situations represented in tables or graphs; **or describes how change in the value of one variable relates to change in the value of a second variable** in problem situations with constant rates of change.
Level 1 Entering	Uses general vocabulary related to slope and coordinate graphs to describe a variety of graphs and pictures (e.g., *steep, uphill, downhill, rise, run, flat, same, changing).*
Level 2 Beginning	Identifies which graphs show a constant rate of change given a selection of linear and nonlinear graphs and classifies their slopes as positive, negative, zero, or undefined.
Level 3 Developing	Identifies corresponding examples of *x-y* tables, graphs, equations, and word sentences, given a vocabulary list (e.g., *independent variable, dependent variable, function, slope, rate of change, y-intercept),* with a partner.
Level 4 Expanding	Completes an *x-y* table, graph, and/or equation, given an oral description of changes in *x* and *y* (e.g., y *increases by three every time* x *increases by one),* with support such as a word wall, student-made glossary, and examples.
Level 5 Bridging	Interprets a table of values from a real-world problem situation to determine if a constant rate of change exists, writes a linear equation, creates a graph if one exists, and summarizes the situation.

M (F&A) 7-3	**Demonstrates conceptual understanding of algebraic expressions** by using letters to represent unknown quantities to write algebraic expressions (including those with whole number exponents or more than one variable; (or by evaluating algebraic expressions (including those with whole number exponents or more than one variable); or by evaluating an expression within an equation (e.g., determine the value of *y* when *x* = 4 given y = $5x^3 - 2$).
Level 1 Entering	Matches words with symbols (e.g., *sum, more, less, add, minus, times, product,* etc.), and sorts by operation.
Level 2 Beginning	Recognizes the variable in an expression or equation, and identifies the operation that is performed on it, using word banks and examples.
Level 3 Developing	Evaluates algebraic expressions with one variable, with and without using a calculator, with a partner.
Level 4 Expanding	Evaluates algebraic expressions with more than one variable, with and without a calculator, in small groups.
Level 5 Bridging	Explains the order of the steps involved in evaluating an algebraic expression, both orally and in writing.

M (F&A) 7-4	**Demonstrates conceptual understanding of equality** by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right- hand sides of the equations being solved at this grade level). Using models or different representations of the expressions, solving multi-step linear equations of the form $ax \pm b = c$ with a not equal to 0, $ax \pm b = cx \pm d$ with a and c not equal to 0, and $(x/a) \pm b = c$ with a not equal to 0, where a, b, c and d are whole numbers; or by translating a problem-solving situation into an equation consistent with the parameters of the type of equations being solved for this grade level.
Level 1 Entering	Identifies equivalent and non-equivalent expressions from a set of numerical and algebraic expressions, and reads expressions using general math vocabulary (e.g., *plus, minus, times, equals,* letters and numbers, etc.).
Level 2 Beginning	Sequences steps to solve an equation (e.g., by organizing cards picturing the steps to show how to get to the solution).
Level 3 Developing	Selects appropriate math terms or numerical values to complete sentence prompts that describe steps in solving equations (e.g., *Subtract ____ from both sides, then _____ both sides by 5).*
Level 4 Expanding	Describes orally or in writing the steps to solve an equation, using mathematical vocabulary (e.g., *Undo the addition by subtracting,* or *Divide both sides by three,* or *The opposite of multiplication is division).*
Level 5 Bridging	Translates a word problem into an equation and solves the equation, with the support of a dictionary, pictures, and examples.

Data, Statistics, and Probability

Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

M (DSP) 7-1	**Interprets a given representation** (circle graphs, scatter plots that represent discrete linear relationships or histograms) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems. (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP) 7–2.*)
Level 1 Entering	Labels pictures of graphs to show their names and parts *(bars, points, axes, scale).*
Level 2 Beginning	Matches basic descriptive statements with a graph (e.g., *The most popular pet is a dog. Which graph shows that?).*
Level 3 Developing	Responds to basic questions that involve interpreting a graph, with the support of word banks and examples.
Level 4 Expanding	Makes predictions based on a given graph, with a partner.
Level 5 Bridging	Justifies the relationship between the data shown on a graph and the corresponding conclusions, orally or in writing.

M (DSP) 7-2	**Analyzes patterns, trends, or distributions in data in a variety of contexts by solving problems using** measures of central tendency (mean, median, or mode), dispersion (range or variation), or outliers to analyze situations to

	determine their effect on mean, median, or mode; and evaluates the sample from which the statistics were developed (bias).
Level 1 Entering	Matches general oral language associated with measures of central tendency with visual or graphic displays (e.g., *highest number, middle number, the number you see the most).*
Level 2 Beginning	Illustrates or identifies examples of measures of central tendency or dispersion, based on oral directions and visual or graphic displays (e.g., *mean, median, mode, outlier)*.
Level 3 Developing	Selects measures of central tendency and/or dispersion, based on visual or graphic displays of real-life situations from magazines and newspapers.
Level 4 Expanding	Makes predictions or estimates based on measures of central tendency, dispersion, and/or sample (bias) from visual or graphic displays, in a small group.
Level 5 Bridging	Explains in writing how a change in an outlier can affect a measure of central tendency using data from a graph.

M (DSP) 7-3	**Identifies or describes representations or elements of representations that best display a given set of data or situation,** consistent with the representations required in M(DSP) 7-1. (IMPORTANT: *Analyzes* data *consistent with concepts and skills in M(DSP) 6–2*.)
Level 1 Entering	Identifies different types of graphical representations, with a word bank.
Level 2 Beginning	Matches data sets with appropriate ways to display them graphically (e.g., either a circle graph or a bar graph to show a discrete set of data; a line graph to show change over time).
Level 3 Developing	Compares and contrasts two graphs that represent the same data, with the support of word banks and examples.
Level 4 Expanding	Discusses reasons for choosing one graph over another for a persuasive presentation, with a partner.
Level 5 Bridging	Explains, orally or in writing, with details, why one display is better than another to represent a particular set of data.

M (DSP) 7-5	**For a probability event in which the sample space may or may not contain equally likely outcomes, determines** the experimental or theoretical probability of an event in a problem-solving situation.
Level 1 Entering	Records and labels outcomes of events involving chance, using real objects (e.g., coin flips) or virtual manipulatives.
Level 2 Beginning	Reports outcomes of events involving probability, using real objects, with a partner.
Level 3 Developing	Discusses probability based on observed outcomes, and describes results in a series of written sentences, in a small group.
Level 4 Expanding	Discusses possible combinations based on probability, and compares against observed outcomes, with a partner.
Level 5 Bridging	Interprets a real-life problem situation involving probability by identifying key words within the text, and solves the problem.