GSE/WIDA High School Mathematics Alignment

Introduction

Development of the Alignments

- The high school alignments were drafted in 2007-2008 by a group of volunteer teachers: 13 ESOL (English for speakers of other languages) teachers, one math teacher, and two science teachers; they were revised in 2008-2009 by another volunteer group: 13 ESOL teachers, two math teachers, one science, one social studies, one language arts.
- The project was part of the federally-funded ELL Co-op program, developed by the University of New Hampshire, in partnership with the NH Department of Education Title III Office, the Manchester School District, and the Nashua School District.
- Teachers met four full days each year, working in four groups by content area expertise; special thanks are due to the non-ESOL math, language arts, science, and social studies teachers who provided vital content-area expertise in those subjects.
- The result is an alignment of selected reading, writing, math, science, and social studies Grade Span Expectations (GSEs) with model performance indicators for those subjects, based on the state-adopted ESOL standards.

Goals of the Alignment

1. To give ESOL and content area teachers information that they can use to differentiate instruction by proficiency level.
2. To provide a concrete representation of the WIDA standards so ESOL students have access to all academic content.
3. To define the ESOL curriculum in a way that will focus on the content areas of math, science, social studies, and language arts, and will assist in writing course competencies.
4. To provide a reference of appropriate ESOL teaching strategies for content area teachers.
5. To ensure that ESOL teachers know what content is being taught in content area classes, and that content area teachers know what can be expected of students at a particular level of language proficiency.

New Hampshire's English Language Proficiency Standards

- The standards were developed by the World-Class Instructional Design and Assessment (WIDA) consortium. This consortium consists of 16 states plus the District of Columbia, which have all adopted common English language proficiency standards and assessments.
- There are five standards:
    1. English language learners **communicate** for **Social** and **Instructional** purposes within the school setting.
    2. English language learners **communicate** information, ideas and concepts necessary for academic success in the content area of **Language Arts.**
    3. English language learners **communicate** information, ideas and concepts necessary for academic success in the content area of **Mathematics.**
    4. English language learners **communicate** information, ideas and concepts necessary for academic success in the content area of **Science.**

5. English language learners **communicate** information, ideas and concepts necessary for academic success in the content area of **Social Studies.**

- The format of the WIDA standards is as follows:
  - There are five grade spans: PreK-K, 1-2, 3-5, 6-8, 9-12.
  - Each grade span includes the four language domains of listening, speaking, reading, and writing.
  - There are five language proficiency levels: Entering, Beginning, Developing, Expanding, and Bridging (see Appendix A for descriptions of proficiency levels and a language acquisition chart).
  - Model performance indicators (MPIs) are written for each language domain across the five proficiency levels.
  - WIDA's MPIs provide a framework for creating performance indicators for particular situations, using the topics and strategies needed to implement diverse curricula in diverse classrooms.

<u>Format of the Alignment</u>

- There are three essential elements of a WIDA MPI:
  1. **Language function** is the task ELLs (English language learners) do to demonstrate performance (see Appendix B for appropriate tasks in each subject at each proficiency level).
  2. **Topic** is what the students are expected to communicate (topics come from the selected NH GSEs).
  3. **Support** is the sensory, graphic, or interactive scaffolding used to help ELLs show their knowledge (support is essential through Level 4; optional at Level 5) (see Appendix C for examples of various kinds of supports).
  4. **Examples** are optional; they clarify or extend the meaning of the MPI (e.g., things teachers, students, or texts might say; appropriate supports; subtopics, etc.).
- These WIDA/GSE alignments consist of new MPIs written for the selected GSEs:
  - MPIs were written for all tested GSEs in math, science, reading, writing, and oral communication; they were written for representative topics in social studies.
  - There are five MPIs for each GSE, one for each level of language proficiency.
  - The language domains of listening, speaking, reading, and writing are combined in the alignment MPIs.
  - The MPIs will need to be adapted; they are meant to be **models.** Teachers should change the topic, the language function, and/or the support as needed (lists of tasks and supports in Appendices B and C are helpful for adapting MPIs).

<u>English Language Learners</u>

All students come to school with different backgrounds. Teachers can never assume that all their students have the same background knowledge, or learn in the same way. What is true of all students is doubly true of English language learners, because they come from much more varied backgrounds.

- Language:
  - All classrooms have students with different dialects and different levels of experience with school language.

- These differences are greater with ELLs because language is the basic medium of communication in the classroom. Second languages must be taught and learned; it is every teacher's responsibility to help students who are still learning English.

- Culture:
  - Any group has multiple cultures and sub-cultures, often determined by ethnicity, but also by family background, interests and abilities, or social class.
  - Cultural differences among students from different language backgrounds are greater than differences among students who all speak the same language because cultures are closely related to languages. ELLs should be encouraged to maintain their native culture as they learn to live comfortably within a new culture, just as they should be encouraged to maintain their native language as they learn English.

- Family and friends:
  - Teachers recognize the influence of students' families and peers on their behavior, and how this influence creates differences in achievement.
  - ELLs may demonstrate greater differences because they are living with two languages and cultures. Some strongly identify with their family's language and culture, and resist assimilation. Others are so anxious to be "American" that they create communication difficulties with their families. Teachers need to be aware of these issues and help students work through them.

- School experiences:
  - All students come to a particular classroom with different school experiences and expectations. For example, some students are used to independent group learning, while others have never done anything except teacher-directed individual work.
  - ELLs may have ways of learning that are very different from any found in U. S. schools, because classroom organization and participation are culturally-determined. Teachers need to respect the school customs that ELLs bring with them, while at the same time helping them learn to participate in different ways.

- Prior knowledge:
  - What one student learns during school is never the same as what another student learns, even if both have gone through the same school.
  - These differences increase greatly for ELLs. They may have gone to schools that put very different emphases on what is important to learn; they may have lost continuity because of attending many different schools; they may have been out of school completely because of wars or other disasters. When teaching ELLs, it is essential to find out what they already know about each new topic introduced, and to provide whatever background knowledge they need to be successful.

- Physical or emotional barriers:
  - Any classroom contains students with special physical, emotional, or learning needs.
  - ELLs may also have physical challenges, emotional problems, or learning difficulties. In addition, language disorders may not be recognized if they are confused with normal second language acquisition processes. Emotional barriers may have been created by trauma or through being uprooted from everything familiar in their lives. Teachers must remember that strategies and accommodations must be adapted for each student's individual needs.

How to Use This Alignment

- Model Performance Indicators are exactly that: *models*; any or all of the three elements can and should be transformed to create PIs that fit specific situations.
- This document will show ESOL teachers exactly what the expectations are for different content areas; the topics of the MPIs in each subject area will help them make decisions about what to emphasize as they help students with material from all content areas.
- This document will help content area teachers teach their subjects to ELLs. It is a valuable resource of appropriate tasks and supports: the tasks provide concrete examples of the kinds of things ELLs can be expected to do at each proficiency level; the supports describe sensory, graphic, and interactive strategies that will help ELLs understand and produce the language needed to accomplish the tasks.
- The strengths and needs of ESOL teachers and content area teachers complement each other; they should use each other as resources in order to get the greatest possible benefit from this document.
- This document will help both ESOL and content teachers avoid the common pitfall of "teaching down" to English language learners. ELLs with the necessary prerequisite knowledge can learn grade-level content while their language is still quite limited. Their language output may be rudimentary, but their understanding of concepts can be quite sophisticated.
- In some cases, the needs of particular ELL students cannot be met with this curriculum alignment. For example, students with limited prior formal education will lack background knowledge and literacy skills, which will need to be acquired before they can accomplish many of the tasks in this high school-level alignment. Alignments for all grade levels are being written; teachers should use the alignments that match their students' working grade levels.

Grade 10 Mathematics/English Language Development Alignment

Number and Operations

Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation(s) to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties.

A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

M(N&O)-10-2	**Demonstrates understanding of the relative magnitude of real numbers** by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., $\sqrt{2}$, π), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.
Level 1 Entering	Matches representations of real numbers and basic mathematical symbols, as spoken by the teacher, with symbols on flashcards (e.g., ≤ ≥ ÷ × + –).
Level 2 Beginning	Demonstrates an understanding of the magnitude of numbers by organizing numbers on a number line, using words and numerical representations.
Level 3 Developing	Compares and orders numbers from sets within the real number system', using words, phrases, and simple sentences.
Level 4 Expanding	Solves simple problems involving ordering or comparing real numbers
Level 5 Bridging	Solves problems involving ordering or comparing real numbers and explains the process used.

M (N&O) 10-4	**Accurately solves problems involving** rational numbers within mathematics, across content strands, disciplines or contexts (with emphasis on, but not limited to proportions, percents, ratios, and rates).
Level 1 Entering	Identifies key words within the context of a simple problem and matches key words with their visual representations.
Level 2 Beginning	Rephrases or recites sentences involved in problem solving, using models and visual support.
Level 3 Developing	Organizes a set of written sentences to show how to solve problems, using visual supports.
Level 4 Expanding	Describes, in written form, two or more approaches to solving problems, using visual support.
Level 5 Bridging	Explains strategies for solving problems to peers by demonstrating the step-by-step process, with details and visual support.

Geometry and Measurement

Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.

We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.

Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.

In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.

M (G&M) 10-2	**Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems** involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem).
Level 1 Entering	Identifies and names basic components of multi-dimensional shapes from visually supported words or phrases (e.g., segment, angle, side, diagonal).
Level 2 Beginning	Draws or constructs geometric figures (e.g., bisected angle, isosceles/right/equilateral triangle, sphere, cylinder), using a word bank.
Level 3 Developing	Compares and contrasts properties of multi-dimensional shapes by defending conjectures based on angles, parallel/perpendicular sides, or diagonals (e.g., *At least one pair of…),* with the aid of visually supported text and word banks.
Level 4 Expanding	Makes conjectures and constructs geometric arguments with, the aid of visually supported text and word banks.
Level 5 Bridging	Makes and defends geometric conjectures and proofs.

M (G&M) 10-4	**Applies the concepts of congruency to solving problems** on or off a coordinate plane involving reflections, translations, or rotations; or solves problems using congruency involving problems within mathematics or across disciplines or contexts.
Level 1 Entering	Matches geometric figures with their changes in symmetry by labeling visual representations, using a word bank.
Level 2 Beginning	Draws or constructs congruent figures on or off the coordinate plane, based on oral descriptions; then matches the drawing or construction to a correct visual representation (e.g., with geoboards, dot paper, or dynamic geometry software).
Level 3 Developing	Uses concepts of reflection, rotation, and translation to demonstrate congruency in geometric figures, with visual and graphic support.
Level 4 Expanding	Uses concepts of congruency to solve problems, and explains the process, using models, with occasional visual and graphic support.
Level 5 Bridging	Explains to peers detailed strategies for using the concepts of congruency for solving problems.

M (F&A) 10-1	**Identifies, extends and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations.
Level 1 Entering	Identifies a pattern or a sequence and continues the sequence, using visual and graphic support and number models.
Level 2 Beginning	Describes a rule for a pattern, represented by a model, a sequence, a table, or a graph, using word or phrase banks and visual support.
Level 3 Developing	Describes rules for arithmetic and geometric sequences, using some technical language (e.g., *variable* and n*th term),* with visual and graphic support.
Level 4 Expanding	Organizes sentences in logical order, with sequential language, to show how to solve problems involving patterns, using tables, models, and graphs.
Level 5 Bridging	Summarizes, implements and explains procedures for solving problems involving linear and nonlinear patterns.

M (G&M) 10-6	**Solves problems involving perimeter, circumference, or area** of two-dimensional figures (including composite figures) or **surface area or volume** of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.
Level 1 Entering	Identifies basic components of multidimensional shapes, and produces elements of equations or formulas from visually supported word/phrase banks and models, (e.g., matching or labeling diagrams).
Level 2 Beginning	Produces the equations or formulas of perimeter, area, volume, for multi-dimensional shapes, from visually supported word/phrase banks and models.
Level 3 Developing	Determines the correct application of the equations and/or formulas by matching key information given in a problem with words from a word bank.
Level 4	Explains applications of equations and/or formulas and distinguishes among

Expanding	perimeter, area and volume for a given problem, using figures and notations.
Level 5 Bridging	Summarizes procedures for solving problems and distinguishes when to use formulas involving perimeters, circumference, area, surface area, and volume, with examples..

M (G&M) 10-7	**Uses units of measure appropriately and consistently when solving problems across content strands; makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations** involving measurement in other GSEs.
Level 1 Entering	Matches a variety of objects with measuring tools from a word bank, and chooses the appropriate unit of measure.
Level 2 Beginning	Converts units of measure from one system to another or within the same system, using models or conversion factor tables and word banks.
Level 3 Developing	Organizes a set of written steps for solving problems involving measurement and conversion within or across systems of measure, using models or conversion factor tables and word banks.
Level 4 Expanding	Interprets a given conversion problem situation and chooses an appropriate method for solving it, using models or conversion factor tables and word banks.
Level 5 Bridging	Analyzes a given conversion problem situation and explains decisions about the degree of accuracy, using models or conversion factor tables and word banks.

M (G&M) 10-9	**Solves problems on and off the coordinate plane** involving distance, midpoint, perpendicular and parallel lines, or slope.
Level 1 Entering	Identifies the basic vocabulary of coordinate planes, graphs, or equations in oral statements (e.g.*, x-axis, y-axis, coordinates of a point, variables),* by matching pictures or examples correctly.
Level 2 Beginning	Recognizes the key elements of appropriate equations needed to solve a given set of problems (e.g., distance and midpoint).
Level 3 Developing	Matches specific language of complex graphs, equations, or coordinate planes (e.g., *zero,* y*-intercept, slope, rise over run)* with figures and detailed oral descriptions.
Level 4 Expanding	Compares and contrasts graphs, equations, or coordinate planes from figures and oral scenarios, using appropriate language from word banks (e.g., *trends, exponential growth* and *wave motion)*.
Level 5 Bridging	Describes graphical models using technical language (e.g., *best fit lines, connections between multiple representations).*

Functions and Algebra

Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.

M (F&A) 10-1	**Identifies, extends and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations.
Level 1 Entering	Identifies a pattern or a sequence and continues the sequence, using visual and graphical support and number models.
Level 2 Beginning	Describes a rule for a pattern, represented by a model, a sequence, a table, or a graph, using word or phrase banks and visual support.
Level 3 Developing	Describes rules for arithmetic and geometric sequences, using some technical language (e.g., *variable* and n*th term),* with visual and graphic support.

Level 4 Expanding	Organizes sentences in a logical order to show how to solve problems involving patterns, using tables, models, graphs, and sequential language, with visual and graphic support.
Level 5 Bridging	Summarizes, implements and explains procedures for solving problems involving linear and nonlinear patterns.

M (F&A) 10-2	**Demonstrates conceptual understanding of linear and nonlinear functions and relations** (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).
Level 1 Entering	Names and identifies variables and constants from illustrations and notation; recognizes and matches increasing and decreasing rates, with teacher support.
Level 2 Beginning	Relates functions of two variables by describing how change in the value of one variable relates to change in the value in the second variable, using illustrations of graphs, table of values, equations, and/or function notations.
Level 3 Developing	Gives examples of representations of functions and/or relations of two variables (e.g., graphing on a coordinate plane, or using function notation), from illustrations of representations and notation.
Level 4 Expanding	Interprets representations of functions and/or relations with tables, equations, or function notations of two variables, using a word bank.
Level 5 Bridging	Analyzes the relationship of one variable to another within a function and/or relation, using technical language (e.g., *rates of change, intercepts, zeros, asymptotes),* and representations of graphs, table of values, equations or function notation with or without a word bank.

M (F&A) 10-3	**Demonstrates conceptual understanding of algebraic expressions** by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions.
Level 1 Entering	Identifies and names the elements of algebraic expressions (e.g., *bases, square root, absolute values, exponents, coefficients, variables),* using word or phrase banks and manipulatives.
Level 2 Beginning	Manipulates, evaluates, and simplifies algebraic expressions, from oral or written directions.
Level 3 Developing	Organizes the steps used to simplify and evaluate algebraic expressions.
Level 4 Expanding	Identifies key words in a problem situation in order to match an expression to that situation, solve the problem, and explain the representations in one's own words
Level 5 Bridging	Simplifies or solves problems by translating situations into algebraic expressions and summarizes procedures, using technical language.

M (F&A) 10-4	**Demonstrates conceptual understanding of equality** by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.
Level 1 Entering	Identifies and names the elements of algebraically solving equations and functions, getting at the concept of 'equivalence' using word or phrase banks, manipulatives, and graphs.
Level 2 Beginning	Solves basic linear equations symbolically and graphically, given an example or a mathematical model developing the concept of *equivalence.*

Level 3 Developing	Organizes steps by identifying the properties of equivalence used to solve basic linear equations and systems of equations, and describes the solution set symbolically or graphically, using word banks, pictures, models, etc.
Level 4 Expanding	Identifies key words and operations in order to translate problem situations into equations or systems of equations and uses the properties of equivalence to solve them.
Level 5 Bridging	Solves problems by translating situations into algebraic equations, using the properties of equivalence and summarizes procedures, using technical language.

Data, Statistics, and Probability

Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.

M (DSP) 10-1	**Interprets a given representation(s)** (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g., media, workplace, social and environmental situations). (IMPORTANT: *Analyzes data consistent with concepts and skills in M(DSP)–10–2*.)
Level 1 Entering	Identifies or matches a variety of data representations (e.g., scatter plots, histograms, bar graphs, circle graphs, frequency charts, etc.), using visual representations and a word bank.
Level 2 Beginning	Names and makes observations about data representations, given word/phrase banks and visual graphical representations (e.g., matches appropriate titles/characteristics for the representations).
Level 3 Developing	Answers questions about a data representation, orally or in writing, in pairs.
Level 4 Expanding	Analyzes and formulates conclusions about a data representation, orally or in writing, in small groups.
Level 5 Bridging	Justifies conclusions and/or make predictions based on interpretations, with minimal support

M (DSP) 10-2	**Analyzes patterns, trends, or distributions in data in a variety of contexts** by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed.
Level 1 Entering	Matches graphic representations with technical vocabulary to identify key concepts.
Level 2 Beginning	Identifies and names patterns and trends observed in a data set (e.g., mean, median, mode, outliers, etc.), with visual and graphic support.
Level 3 Developing	Answers questions about patterns, trends, or distributions found in the data, using visual and graphic support, both orally and in writing.
Level 4 Expanding	Describes and interprets more complex meanings of data, in pairs, with visual and graphic support.
Level 5 Bridging	Determines and defends a prediction about the impact of changes in data displayed in charts, tables, or graphs, orally or in writing.

M (DSP) 10-3	**Identifies or describes representations or elements of representations that best display a given set of data or situation,** consistent with the representations required in M(DSP)-10-1.
Level 1 Entering	Matches data sets with corresponding data representations, using visual models and oral descriptions.
Level 2 Beginning	Selects appropriate vocabulary to identify elements of data representations, and chooses the appropriate graph to represent the data, using a word bank, examples, and visual representations.
Level 3 Developing	Chooses and displays data sets in charts, tables, or graphs, given a context for the data and using a set of possible choices.
Level 4 Expanding	Describes the data display, using some technical language (e.g., *continuity, discrete);* identifies questions that may be raised from review of the data. .
Level 5 Bridging	Explains the data display and choice of representation using technical language, and uses the data display to formulate conclusions and make predictions.

M (DSP) 10-4	**Uses counting techniques to solve problems in context** involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or others).
Level 1 Entering	Organizes pictures to show all possible combinations of a given problem and records results.
Level 2 Beginning	Organizes pictures and labels those pictures to show and describe all possible combinations in a given problem, and describes a rule that represents the solution to the problem.
Level 3 Developing	Creates a mathematical representation from a written or oral problem; identifies the technique appropriate for the context of a problem, using some specific and technical language (e.g., *combinations, permutations, factorials).*
Level 4 Expanding	Explains steps for solving problems using a variety of counting techniques, and describes the solutions, in pairs,.
Level 5 Bridging	Summarizes and explains procedures for solving problems involving counting techniques, orally or in writing.

M (DSP) 10-5	**Solves problems involving experimental or theoretical probability.**
Level 1 Entering	Collects and records data using physical models (e.g., spinners, dice), and answers simple questions about outcomes to the experiment.
Level 2 Beginning	Matches numerical probability with a visual model (e.g., 1/4 matched to a four-color spinner); names or describes the matches.
Level 3 Developing	Describes steps for determining theoretical probability based on collecting experimental probability results, working in small groups and using a graphic representation.
Level 4 Expanding	Identifies, within the context of a probability problem, the important information needed to solve the problem; solves the problem and describes the solution orally and/or in writing.
Level 5 Bridging	Solves problems involving experimental or theoretical probability, makes predictions based on their solutions, and explains the solutions.