Achieve, Inc.

# Achieve's Comparison of the American Diploma Project (ADP) Mathematics Benchmarks with the Rhode Island High School Grade-Span Expectations (GSEs) for Mathematics for Grades 9-10, 11-12 and Advanced Mathematics

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ACHIEVE'S
BENCHMARKING INITIATIVE

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## ACHIEVE'S METHODOLOGY

Achieve, Inc. has been asked to review Rhode Island's expectations for high school mathematics to determine the degree to which they align with the American Diploma Project (ADP) Mathematics Benchmarks. For purposes of this analysis, Achieve staff constructed a side-by-side chart comparing Achieve's American Diploma Project (ADP) Mathematics Benchmarks with the Rhode Island Mathematics Grade-Span Expectations (GSEs) for grade spans 9-10, 11-12 and Advanced Mathematics (AM). Since the ADP Benchmarks are cumulative in nature, in select instances content from Grades 5-8 was used to complete the chart.

### THE DEVELOPMENT OF THE AMERICAN DIPLOMA PROJECT (ADP) BENCHMARKS FOR COLLEGE AND WORKPLACE READINESS

The American Diploma Project (ADP) commissioned leading economists to examine labor market projections for the most promising jobs—those that pay enough to support a small family and provide real potential for career advancement—to pinpoint the academic knowledge and skills required for success in those occupations. ADP then surveyed officials from 22 occupations, ranging from manufacturing to financial services, about the skills they believe are most useful for their employees to bring to the job. Following those conversations, ADP worked closely with two- and four-year postsecondary leaders in the partner states to determine the prerequisite English and mathematics knowledge and skills required for success in entry-level, credit-bearing courses in English, mathematics, the sciences and the humanities. The resulting ADP Benchmarks reflect an unprecedented convergence in what these employers and postsecondary faculty say are needed for new employees and freshmen entering credit-bearing coursework to be successful. In mathematics, the Benchmarks reflect a rigorous four-year course sequence that includes content typically taught in Algebra I, Geometry and Algebra II, as well as some data analysis and statistics. In English, the Benchmarks reflect four years of grade-level high school courses that emphasize logic, writing and research. The ADP Benchmarks and sample tasks from employers and postsecondary faculty may be found at www.achieve.org .

Achieve, Education Trust and the Thomas B. Fordham Foundation launched ADP to help states restore the diploma's value by anchoring high school graduation standards to those of jobs and colleges. Toward that end, ADP moves beyond the kinds of standards that reflect experts' consensus view of what is *desirable* for students to learn, to expectations linked directly to the *essential* demands faced by students preparing for college, work and citizenship. These benchmarks are not test blueprints; a fair number of the benchmarks are not able to be assessed through on-demand measures, for example. Rather, they are intended to describe the knowledge and skills that are needed by high school graduates in order to be prepared to achieve in multiple postsecondary venues.

In the area of mathematics, the ADP Benchmarks include expectations that are roughly equivalent to what students should encounter in a 4-year high school mathematics program that includes Algebra I, Geometry, Algebra II and at least a portion of Precalculus. Certain ADP Mathematics Benchmarks are marked with an asterisk (*). These asterisked benchmarks represent content that is recommended for all students but is required for those students who plan to take calculus in college—a requisite for mathematics majors and many mathematics-intensive majors.

## DOCUMENTS USED FOR REVIEW

Reviewed in this report are the draft GSEs for Grades 9-10 and 11-12, plus GSEs for Advanced Mathematics. The ADP Benchmarks in mathematics are used as the basis of comparison in this analysis to help Rhode Island ensure that its high school graduates are ready to succeed in the postsecondary world.

The Rhode Island High School Grade-Span Expectations (GSEs) that are being reviewed in this report have been developed as a means to identify the mathematics content knowledge and skills expected of high school students. The GSEs for Grades 9-10 include expectations that are to be assessed on the state-level assessment administered in the fall of Grade 11, plus other expectations that will be a local curriculum and assessment option. As such, the GSEs defined for Grades 9-10 describe expectations for the end of Grade 10, or the beginning of Grade 11. The high school GSEs are an extension of the expectations defined in the New England Common Assessment Program (NECAP) Mathematics Grade Level Expectations (GLEs) that identify the concepts and skills expected of all students for large-scale assessment of mathematics in Grades 3-8. These expectations are not intended to represent the full mathematics curriculum at each grade level but rather to capture concepts and skills related to the "big ideas" of mathematics that can be assessed in an on-demand setting. They are intended to focus—but not narrow—the curriculum. All expectations are organized into four content strands: Number and Operations, Geometry and Measurement, Functions and Algebra, and Data, Statistics, and Probability. The mathematical process expectations—problem solving, reasoning, connections, and communications—are embedded throughout the GLEs and GSEs instead of being in separate strands.

The College Readiness Standards reviewed in this report consist of guidelines for college readiness in mathematics developed by Rhode Island's PK-16 Mathematics Advisory Council. The committee was chaired by Professor Lewis Pakula of URI and co-chaired by Judith Keeley of RIDE, with representatives from RIC, CCRI, RIDE, and a number of Rhode Island high school. The expectations are comprised of two sets of standards. The first is a set of Basic Skill and Knowledge Expectations that apply to all students entering four-year college programs or intending to transfer to such programs from a community college, and these standards indicate the level of proficiency implicit in Geometry and Algebra II expectations. The second set of standards defines readiness for technical/scientific programs and acknowledges that students intending to enter such programs need more substantial training in precalculus mathematics, including trigonometry and advanced topics in algebra.

## ALIGNMENT OF RHODE ISLAND'S HIGH SCHOOL MATHEMATICS GSEs WITH THE ADP BENCHMARKS

In general, there is very strong alignment between the ADP Mathematics Benchmarks and the Rhode Island GSEs for Grades 9-10, 11-12, and Advanced Mathematics (AM). The state acknowledges that these expectations are not intended to define the complete range of instruction for high school mathematics, and, in the same manner, the ADP Benchmarks are not intended to encompass all instruction, but rather to call out the knowledge and skills necessary for postsecondary success. Although the nuances of the language may be somewhat different, most of the ADP Benchmarks have at least one Rhode Island expectation that aligns with them. There are several Rhode Island expectations that are not explicitly addressed in the ADP Benchmarks, and a number of Rhode Island expectations extend beyond the expectations articulated in the ADP Benchmarks.

What follows is a description of commonalities and differences found between the two sets of standards.

- Both the ADP Benchmarks and the Rhode Island Geometry and Measurement GSEs clearly address the concept of mathematical proof. The ADP Benchmarks, however, are much more explicit in that they provide examples of the types of theorems students should be expected to prove. The Rhode Island GSEs tend to cluster expectations with respect to proof in more generically-worded statements than the ADP Benchmarks, but these expectations can have the same effect if they are presumed to overarch all aspects of the domain. Rhode Island explicitly mentions only two theorems: the Pythagorean Theorem and the Triangle Inequality Theorem. The GSEs related to proof are as follows:

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|---|
| M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle ratios). |
| M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). |
| M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle congruence and similarity). |
| M(G&M)—AM—2 Extends and deepens knowledge and usage of proofs and proof techniques. |

  It is important, however, that teachers have clear and consistent understandings of what types and levels of proof students are expected to perform, and it may be that Rhode Island provides such detail in supporting documents. Simply listing angles, lines, circles, distance, midpoint, triangle ratios, right triangle ratios (sine, cosine, tangent) and polygons including triangle congruence and similarity is not explicit enough to ensure consistent levels of expectation across high school geometry courses in the state. More importantly, it is not clear how the rigor of the expectations changes from grade span 9-10 to grade span 11-12 to AM. This variation in specificity between the ADP

Benchmarks and the Rhode Island GSEs is illustrated in the side-by-side chart included as an appendix to this report. The Rhode Island GSEs involving proof are repeatedly cited as the expectations aligning with the ADP Benchmarks involving proof of basic theorems (K1.2), parallel lines (K2.1), perpendicular lines (K2.2), angles (K2.3), congruence and similarity (K3), circles (K4), Pythagorean Theorem (K5), right triangle trigonometry (K11.1) (K11.2), and again in MR3 (mathematical reasoning involving proof). Regardless, the inclusion of geometric proof in mathematics is a topic of contention in the mathematics world, and it is noteworthy that both Rhode Island and the ADP Benchmarks include it.

- The ADP Benchmarks include geometric construction, which is parsed across a number of Benchmarks, including examples of constructions provided that tie to the content defined in the individual Benchmarks. While Rhode Island includes a reference to construction, it does not include any specificity as to types of geometric construction within its GSEs.

| |
|---|
| M(G&M)—12—10 Perform and justify construction with compass and straightedge or dynamic geometric software. |

  The GSEs would benefit from greater specificity if the intent is to ensure that all students receive the same rich mathematics education with the same level of mathematical rigor.

- Both the ADP Benchmarks and the Rhode Island GSEs clearly expect student to use appropriate technology in the classroom to further their mathematical understandings. The ADP Benchmarks address technology both in stand-alone Benchmarks and embedded within content-focused Benchmarks, calling on students not only to use calculators and computers but also to understand the capabilities and limitations of technology in solving problems. The Rhode Island GSEs tend to be explicit about technology use within the context of content-specific expectations. They also make clear the intent that students meet most expectations with and/or without using technology and that students be able to use dynamic geometry software for spatial reasoning and geometric constructions.

- There are no instances where Rhode Island's expectations for a student completing two years of high school level mathematics, as defined by the GSEs for Grades 9-10, exceed those of the non-asterisked ADP Benchmarks identified as needed by all students.

- There are several instances where Rhode Island's expectations for a student completing four years of high school level mathematics, as defined by the GSEs for Grades 11-12, exceed those of the non-asterisked Benchmarks. The following Rhode Island GSEs for Grades 11-12 align wholly or in part with ADP Benchmarks identified with asterisks—indicating that they are recommended for all students but required for students intending to take calculus. The expectations identified below do not include those GSEs identified as AM, which exceed those articulated for Grades 9-10 and 11-12. Bolded text indicates those portions of GSEs that extend beyond the ADP Benchmarks for all students. GSEs containing no bolded text align in their entirety with asterisked ADP Benchmarks.

| RHODE ISLAND EXPECTATIONS (GRADES 11-12) ALIGNING WITH ASTERISKED ADP BENCHMARKS |
|---|
| **Number and Operations** |
| M(N&O)—12—4 Solves problems involving scientific notation or uses significant digits to assess the precision of an answer. **Interprets rational exponents and their relation to radicals**. Computes by hand in simple cases (e.g.4 $^{3/2}$), and using a calculator when appropriate. Interprets numbers given in scientific notation and carries out computations of them with and without a calculator. |
| **Functions and Algebra** |
| M(F&A)—12—1 Identifies arithmetic and geometric sequences and finds the nth term; then uses the generalization to find a specific term. |
| M(F&A)—12—3 Understands properties of logarithms and can convert between logarithmic and exponential forms. |
| M(F&A)—12—3 Manipulates, evaluates, and simplifies expressions involving rational exponents and radicals and can convert between expressions with rational exponents and expressions with radicals. |
| M(F&A)—12—4 Understands and applies the various processes of solving equations and systems of equations or inequalities. Interprets the solutions algebraically and graphically. Solves 2x2 and **3x3 systems of linear equations** and graphically interprets the solutions. |
| **Geometry and Measurement** |
| M(G&M)—12—3 Knows the characterization of circles as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for circles |
| M(G&M)—12—6 Applies trigonometric formulas (law of sines/cosines, A= 1/2 ab sinC ) to find angles, lengths and areas of polygons. |

- There are also several asterisked ADP Benchmarks that align with expectations defined in Rhode Island's AM topics. All Rhode Island students are currently required to pass two years of mathematics, with requirements increasing in 2008 to three years of high school mathematics and a fourth year of an applied course. Students who are intending to pursue mathematics-related careers will likely take courses that include these expectations but only if they complete the expectations listed in the GSEs for Grades 9-10 and 11-12 in three years or less. With the state's assessment occurring in the fall of Grade 11, it is likely that a number of students will not exceed the content expectations defined in the GSEs for Grades 9-10, with fewer encountering the Grade 11-12 GSEs, and only the most mathematically advanced students enrolling in courses that address the AM expectations.

| RHODE ISLAND EXPECTATIONS FROM ADVANCED MATHEMATICS THAT ALIGN WITH ASTERISKED ADP BENCHMARKS |
|---|
| **Functions and Algebra** |
| M(F&A)—AM—1 Computes partial sums of infinite arithmetic and geometric sequences, determines when an infinite geometric series converges, and finds its sum. Connects arithmetic and geometric sequences to linear and exponential functions, respectively.<br>(Study of this concept begins in 11-12) |
| M(F&A)—AM—2 Understands domain restriction and the effects of it on the function and its properties. |
| M(F&A)—AM—2 Understands functions and relations from a set-theoretic perspective, and **operations on functions including composition** and **inverse including computing inverses algebraically.** |
| M(F&A)—AM—2 Analyzing characteristic of classes of functions and **inverse functions (exponential, logarithmic**, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes.<br>(Study of this concept begins in 11-12) |
| M(F&A)—AM—2 Recognizes properties of families of functions including logarithmic and **trigonometric, and graphs them.** |
| M(F&A)—AM—4 Solves equations and verifies identities involving trigonometric expressions. |
| **Geometry and Measurement** |
| M(G&M)—AM—3 Explores and interprets the characteristics of conic sections graphically and algebraically. Understands how different planar slices of a double cone yield different conic sections. Knows the characterization of conic sections as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for the conic sections. |
| M(G&M)—AM—7 Understands why radian measure is useful and converts between radian measure of angles and degrees. |
| M(G&M)—AM— 9 Solves specific problems using analytic geometry (including in three dimensions) and **circular trigonometry** (e.g. find the equation of a circle inscribed in a triangle given the coordinates of the vertices; the distance between opposite vertices in a rectangular solid). |

- There are a few ADP Benchmarks defined for all students that are not included in the Rhode Island expectations prior to the AM topics.

| |
|---|
| M(N&O)—AM—1 Understands the structure of the real number system as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number eventually repeats or terminates (without using a calculator). |
| M(N&O)—AM—8 Knows and uses the principle of mathematical induction |
| M(G&M)—AM—6 Derives and uses formulas for lengths of arcs and areas of sectors and segments. |
| M(DSP)—AM—2 Analyzes and interprets measures of dispersion (standard deviation, variance, and percentiles) and central tendency for normal distributions. |

- There are several instances where the Rhode Island GSEs are more detailed and explicit than the ADP Benchmarks. For example, ADP Benchmark J2 states that students need to be able to understand functions, their representations, and their properties. More detailed expectations categorized under this broad ADP Benchmark reference students recognizing whether a relationship is a function, determining domain, using notation, performing composition and operations on functions, understanding inverses, and knowing/applying the inverse relationship between exponential and logarithmic functions. The Rhode Island GSEs include all of the ADP Benchmarks but also include:

| |
|---|
| M(F&A)—10—2 Demonstrates conceptual understanding of linear and nonlinear functions and relations (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation). |
| M(F&A)—12—2 Represents and analyzes functions in several ways. Recognizes properties of functions and characteristics properties of families of functions. Applies knowledge of functions to interpret, model, and solve problems.<br><br>Analyzes characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change.<br><br>Represents functions numerically, algebraically, graphically, and verbally (i.e. in written words). Recognizes properties of a function from these representations, and transfers information from one representation to another.<br><br>Graphs polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes.<br><br>Applies knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences |
| M(F&A)—12—4 Finds approximate solutions to equations by graphing each side as a function using technology. Understand that any equation in $x$ can be interpreted as the equation $f(x) = g(x)$ and interpret the solutions of the equation as the $x$-value(s) of the intersection point(s) of the graphs of<br>$y = f(x)$ and $y = g(x)$. |
| M(F&A)—AM—2 Analyzes properties of functions including injectivity (1-1), surjectivity (onto), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither. Analyzes informally the idea of continuity and limits. |
| M(F&A)—AM—2 Analyzes characteristics of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes |
| M(F&A)—AM—2 Recognizes properties of families of functions including logarithmic and trigonometric, and graphs them. |

Rhode Island exceeds the ADP Benchmarks' expectations within this strand particularly with respect to the families of functions students are to identify and work with. Included in the GSEs are function families that the ADP Benchmarks exclude, such as logarithmic, trigonometric, polynomial, rational, and exponential functions. Students graph polynomial, rational, and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes. These expectations, although consistent with the ADP Benchmarks, extend beyond the comparable ADP Benchmarks.

Rhode Island includes relatively sophisticated expectations with respect to properties of functions and their inverses, including the domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, asymptotes, injectivity (1-1), surjectivity (onto), critical points, and inflection points. Rhode Island also expects students to determine graphically and analytically whether a function is even, odd, or neither and to analyze informally the idea of continuity and limits. This comprehensive set of expectations involving function analysis will build strong mathematics reasoning skills in students and equip them with powerful tools for recognizing and modeling functions arising in real world applications across disciplines.

- In addition to the function extensions mentioned above, the Rhode Island standards include additional content in each strand that is not part of the ADP Benchmarks. The majority of this content is contained in the AM topics. This additional content includes:

| **Rhode Island Expectations That are Either not Included in the ADP Benchmarks or are Extensions of ADP Benchmarks** |
|---|
| **Number and Operation** |
| M(N&O)—**10**—6 Mentally calculates benchmark perfect squares and related square roots (e.g., 12, 22 , …, 122, 152, 202, 252, 1002, 10002). Determines any whole number percentage of a number or any multiples of 100% up to 500%. **Determines benchmark fractions of a number.** (IMPORTANT: The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.) |
| M(N&O)—**12**—4 Demonstrates understanding of complex numbers by interpreting them geometrically and by computing with them (e.g., adding, multiplying, dividing, finding the nth root, or by finding conjugates). Understand complex numbers as an extension of the real numbers (e.g. arising in solutions of polynomial equations). Manipulates complex numbers using rectangular and polar coordinates. Knows the fundamental theorem of algebra and knows that non-constant polynomials always factor into linear factors over the complex numbers. |
| M(N&O)—**12**—4 Solves problems involving scientific notation or **uses significant digits to assess the precision of an answer.** Interprets rational exponents and their relation to radicals. Computes by hand in simple cases (e.g. $4^{3/2}$), and using a calculator when appropriate. Interprets numbers given in scientific notation and carries out computations of them with and without a calculator. |
| M(N&O)—**12**—8 Determine whether a given subset of numbers is closed under a given arithmetic operation. |
| M(N&O)—**AM**—8 Add and multiply numerical matrices with attention to the arithmetic properties of these operations. Algebraically and geometrically interpret vectors, vector addition, and scalar multiplication in the plane, with attention to arithmetic properties. |

| Functions and Algebra |
|---|
| M(F&A)—**10**—1 **Identifies, extends, and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs to solve problems. |
| M(F&A)—**12**—4 **Solves equations involving polynomial, rational, and radical expressions**. Graphs and interprets the solutions. |
| M(F&A)—**12**—4 Solves **systems of equations involving nonlinear expressions** and graphically interprets the solutions. |
| M(F&A)—**12**—4 Solves **systems of linear** and quadratic **inequalities.** |
| M(F&A)—**AM**—3 Uses the **remainder theorem, the factor theorem and rational root theorem** for polynomials. |
| M(F&A)—**AM**—3 Understands the difference between factoring polynomials over integer, rational, real and complex numbers |
| M(F&A)—AM—4 Solves equations involving exponential and logarithmic expressions. Graphs and interprets the solutions. |
| M(F&A)—**AM**—4 Knows and applies the **intermediate value theorem** to find exact or approximate solutions of equations or zeros of continuous functions. |
| M(F&A)—**AM**—4 Interprets **systems as matrix equations** and solves them by computing the appropriate matrix inverse and multiplication, with or without technology. |
| **Geometry and Measurement** |
| M(G&M)—**10**—7 Applies informal concepts of **successive approximation, upper and lower bounds, and limits in measurement situations** (e.g., use successive approximation to find the area of a pond); uses measurement conversion strategies (e.g., unit/dimensional analysis). |
| M(G&M)—**AM**—3 Explores and interprets the characteristics of **conic sections** graphically and algebraically. Understands how different planar slices of a double cone yield different conic sections. Knows the characterization of conic sections as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for the conic sections. |
| M(G&M)—**AM**—4 Uses matrices to represent reflections, translations, rotations. |
| M(G&M)—**AM**—6 Knows Cavalieri's principle and uses it to find volumes. |
| M(G&M)——**AM**—9 Solves specific problems using analytic geometry (including in **three dimensions**) and circular trigonometry (e.g. find the equation of a circle inscribed in a triangle given the coordinates of the vertices; **the distance between opposite vertices in a rectangular solid).** |
| **Data, Statistics and Probability** |
| M(DSP)—**10**—4 Uses counting techniques to solve contextualized problems involving combinations or permutations (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or others). |
| M(DSP)—**12**—1 Given a regression function (linear, **quadratic, and exponential**), analyze the data to make inferences and to formulate, justify, and critique conclusions. |
| M(DSP)—**12**—3 Find or estimate linear, **quadratic, and exponential regression** functions by organizing and displaying data with or without using technology. |
| M(DSP)—**12**—4 Solves problems involving combinations and permutations using a variety of strategies including nCr, nPr, or n!. Finds unions, intersections, and complements of sets. |
| M(DSP)—**AM**—3 Uses technology to explore the method of **least squares and median-median** for linear regression. |

- There are also instances in which content that is clearly addressed in the ADP Benchmarks is not **explicitly** cited in the Rhode Island expectations. For example, it is apparent that Rhode Island values mathematical modeling, but the ADP Benchmarks are much more specific about the types of functions to be modeled and cite examples that clarify the intent.

| **ADP Benchmarks That are not Explicitly Cited in the Rhode Island Expectations** |
|---|
| **Algebra** |
| **J5.3.** Recognize and solve problems that can be modeled using a quadratic equation, such as the motion of an object under the force of gravity. |
| **J5.5. *** Recognize and solve problems that can be modeled using an exponential function but whose solution requires facility with logarithms, such as exponential growth and decay problems. |
| **J5.6.** Recognize and solve problems that can be modeled using a finite geometric series, such as home mortgage problems and other compound interest problems. |
| **J6. * Understand the binomial theorem and its connections to combinatorics, Pascal's triangle and probability.** |
| **Geometry** |
| **K1.1.** Identify, explain the necessity of and give examples of definitions, axioms and theorems. |
| **Data Interpretation, Statistics and Probability** |
| **L1.4.** Compare data sets using graphs and summary statistics. |
| **L2.1.** Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed. |
| **L2.2.** Identify and explain misleading uses of data. |
| **L2.3.** Recognize when arguments based on data confuse correlation with causation. |
| **L3.3.** Explain the differences between randomized experiments and observational studies. |
| **L4.2.** Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome. |
| **L4.3.** Explain how the law of large numbers can be applied in simple examples. |
| **Mathematical Reasoning** |
| **MR4.** Using the special symbols of mathematics correctly and precisely. |
| **MR6.** Distinguishing relevant from irrelevant information, identifying missing information and either finding what is needed or making appropriate estimates. |
| **MR8.** When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results. |

- The ADP Benchmarks and the Rhode Island expectations tend to be "packaged" differently. Many of the Rhode Island expectations are compound in nature, encompassing multiple ADP Benchmarks in one expectation statement. They are, therefore, repeated multiple times in the side-by-side chart. For example, the ADP Benchmarks have separate expectations in algebra for the laws of integer exponents and roots (J1.1), operations with and the simplification of rational expressions (J1.5), and evaluation of polynomial, rational, radical, and absolute value expressions (J1.6). The Rhode Island GSE M(F&A)—10—3 appears to address all three topics in one expectation.

| |
|---|
| M(F&A)—10—3 Demonstrates conceptual understanding of algebraic expressions by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions. |

Similarly, the ADP Benchmarks tend to have separate expectations for solving, graphing, and modeling various types of equations and functions while the Rhode Island expectations tend to combine these performances in one objective. For example, aspects of Rhode Island's GSE M(F&A)—10—4 align with nine separate ADP Benchmarks.

| |
|---|
| M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. |

Other examples of compound expectations from the Rhode Island standards document that tend to be repeated several times in the side-by-side chart include:

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|---|
| M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem).<br><br>M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle ratios).<br><br>*(Used as a match for 10 ADP Benchmarks)* |
| M(DSP)—10—1 Interprets a given representation (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g. media, workplace, social and environmental situations).<br><br>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)—10—2.) |
| M(DSP)—10—2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed. |

| |
|---|
| M(DSP)—10—3 Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in M(DSP)—10—1.<br><br>M(DSP)—10—3 Organizes and displays one- and two-variable data using a variety of representations (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts). Analyzes the data to formulate or justify conclusions, make predictions, or to solve problems. Identifies representations that best display a given set of data. |

The organization and grain size of Rhode Island's GSEs vary—most noticeably between the GSEs for Grades 9-10 and 11-12. As seen in the preceding table, a number of GSEs for Grades 9-10, which guide development of the state assessment, are compound in nature. It would seem that this grade span might benefit from expectations with more specificity and clarity about exactly what is expected with respect to each skill and concept defined in the GSEs. In contrast, the GSEs for Grades 11-12 tend to be composed of more finely grained statements that are often listed separately but identified using a single GSE label, such as M(F&A)—12—3. Examples of more finely grained expectation statements—from across the various GSE levels—include:

| |
|---|
| M(F&A)—12—3 Factors quadratic and higher degree polynomials, including difference of squares. |
| M(F&A)—12—3 Adds, subtracts, multiplies and divides rational expressions. |
| M(F&A)-12—3 Simplifies complex fractions. |
| M(F&A)—AM—2 Understands domain restriction and the effects of it on the function and its properties. |
| M(F&A)—AM—3 Uses the remainder theorem, the factor theorem and rational root theorem for polynomials. |
| M(G&M)—10—5 Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts. |

- The Rhode Island GSEs are to be commended for clearly placing considerable emphasis on conceptual understanding of content. For example, the following expectations specifically highlight conceptual understanding:

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| M(F&A)—10—2 Demonstrates conceptual understanding of linear and nonlinear functions and relations (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation). |
| M(F&A)—10—3 Demonstrates conceptual understanding of algebraic expressions by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic |

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| expressions. |
| M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. |
| M(G&M)—10—10 Demonstrates conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three-dimensional objects, or by solving related problems. |

Other Rhode Island expectations are explicit about how conceptual understanding will be demonstrated. The expectations often include the phrases such as "demonstrate understanding of... by..." and "understand... by...".

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| M(N&O)— AM—1 Understands the structure of the real number system as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number eventually repeats or terminates (without using a calculator). |
| M(F&A)—12—3 Understands properties of logarithms and can convert between logarithmic and exponential forms. |

When the verb "understand" is used alone, it would be wise to include modifiers explaining how the understanding should be demonstrated. Examples of Rhode Island expectations that would benefit from such modification include:

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| M(F&A)—12—4 Understands the effect of simplifying radical or rational expressions on the solution set of equations involving such expressions. (e.g. $x$^2/$x$ =$x$ for $x \neq 0$). |
| M(F&A)—AM—2 Understands domain restriction and the effects of it on the function and its properties. |
| M(F&A)—AM—2 Understands functions and relations from a set-theoretic perspective, and operations on functions including composition and inverse including computing inverses algebraically. |
| M(F&A)—AM—3 Understands the difference between factoring polynomials over integer, rational, real and complex numbers |

- The performance expectations in the Rhode Island GSEs, through the use of certain verbs, clearly convey a high level of cognitive demand. Students are asked to represent, analyze, solve, apply, design and critique, graph, describe, construct, interpret, model, create, make and defend, perform and justify, and derive and use. Even the verb "know," which usually is ambiguous and can be interpreted as low level recall, tends to be coupled with "use," implying a higher level of demand. Similarly, the verb "understand" is often clarified in the GSEs by explanation of how the student will be expected to show understanding. There are very few expectations that are unclear with regard to the performances expected.  The only ones noted are:

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| M(N&O)—12—1 Knows that rational numbers are precisely those with eventually repeating or terminating decimal expansions |
| M(F&A)—12—4 Understands the effect of simplifying radical or rational expressions on the solution set of equations involving such expressions. (e.g. $x^2/x = x$ for $x \neq 0$). |
| M(F&A)—AM—3 Understands the difference between factoring polynomials over integer, rational, real and complex numbers |

- Rhode Island is quite specific about its intent that mental mathematics and estimation are to be embedded throughout the curriculum.  This is a commendable goal, and the GSEs are much more specific about this aspect of mathematics than the ADP Benchmarks. Specifically, these two GSEs are as follows:

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| M(N&O)—10—6  Mentally calculates benchmark perfect squares and related square roots (e.g., 12, 22 , …, 122, 152, 202, 252, 1002, 10002). Determines any whole number percentage of a number or any multiples of 100% up to 500%. Determines benchmark fractions of a number.<br>(IMPORTANT: The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.) |
| M(N&O)—10—7 Makes appropriate estimates in a given situation by determining the level of accuracy needed and analyzing the accuracy of results. Estimates tips, discounts, and tax and estimates the value of a non-perfect square root or cube root.<br>(IMPORTANT: The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.) |

- The Rhode Island high school expectations tend to be less comprehensive than the ADP Benchmarks with respect to Data Interpretation, Statistics, and Probability.  For example, ADP Benchmark L2 and its subcomponents address the need for students to be able to explain and critique alternative ways of presenting and using information.  Specifically, the ADP Benchmarks call for students to be able to evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed, identify and explain misleading uses of data, and recognize when arguments based on data confuse correlation with causation.  The Rhode Island GSE that most closely aligns with this Benchmark is neither explicit about precisely what is expected nor about the depth of understanding expected, as seen in the language below:

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| M(DSP)—10—1 Interprets a given representation (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g. media, workplace, social and environmental situations).(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)—10—2.) |

Likewise, the ADP Benchmarks call for students to be explicit in explaining the differences between randomized experiments and observational studies, to explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome, and to explain how the law of large numbers can be applied in simple examples. The Rhode Island expectations are not explicit about these concepts but rather place more emphasis on solving probability problems using concepts not explicitly addressed in the ADP Benchmarks.

- The ADP Benchmarks articulated under Benchmark J5 are very explicit about the types of real world problems students should be able to model and solve (i.e., time/rate/distance problems, percentage increase or decrease problems, ratio and proportion problems, mixture problems, motion of an object under the force of gravity, compound interest problems, exponential growth and decay problems, home mortgage problems, and other compound interest problems). Rhode Island does not specifically list types of problems but instead uses the generic phrase "model and solve problems." Greater specificity—either in this document or in a supplementary resource—would prove helpful to teachers.

**Alignment of Rhode Island's High School Mathematics GSEs with the Rhode Island College Readiness Standards**

Rhode Island's PK-16 Mathematics Advisory Committee has developed guidelines for college readiness in mathematics that define two tiers of expectations. First, a set of basic skill and knowledge expectations defines the level of mathematics needed for all students to enter college and be successful. These standards indicate the scope and level of proficiency implicit in Geometry and Algebra II expectations. By design, they do not include all of the mathematics—such as statistics—that constitutes a strong high school mathematics program. Rather, they focus on areas that college mathematics professors define as critical underpinnings for success in entry-level college mathematics courses. The second set of standards defines readiness for technical/scientific programs and acknowledges that students intending to enter mathematics-intensive programs need substantially more exposure in high school to precalculus topics, including trigonometry and more advanced topics in algebra. The College Readiness Standards are intentionally light on data, probability, and statistics—unlike the Rhode Island GSEs and the ADP Benchmarks. In fact, they contain only one expectation addressing this area—an expectation involving permutations, combinations, and probability calculations that is in the basic skill and knowledge expectations. With respect to the more advanced levels of knowledge and skill, there is only one asterisked ADP Benchmark—dealing with the binomial theorem—that is not included in the Rhode Island College Readiness Standards.

A student who completes a course of study in high school that encompasses the expectations defined in the GSEs for Grades 9-10 and 11-12 should be prepared for college—as defined by the basic skill and knowledge expectations component of Rhode Island's College Readiness Standards. Such a student will have even been taught content—most particularly in data, probability, and statistics—that is not deemed by the PK-16 Mathematics Advisory Committee that drafted the College Readiness Standards—to be an important precursor to success in college. An issue seems to be, however, whether all students completing high school—even those with four credits of mathematics—and intending to enter college with a mathematics-intensive major will be able to meet the expectations defined in the College Readiness Standards. Many of the expectations defined in the College Readiness Standards applicable to these students will only be studied by students who have taken courses that address the AM expectations defined to build on the GSEs for Grades 9-10 and 11-12. Rhode Island's plan to move from a graduation requirement of two years of mathematics to three years of mathematics plus a fourth year of an applied course will be instrumental in ensuring that students have adequate preparation to pursue college majors and careers of their choosing.

## Conclusion

In general, the Rhode Island GSEs and AM expectations are comprehensive in scope and rigorous in their level of cognitive demand. They tend to align well with the ADP Benchmarks, and students who learn the content and skills defined in the GSEs should be prepared for success in entry-level mathematics in college. It is only those students completing a more rigorous mathematics program in high school—one that addresses the expectations defined for AM—who will be prepared for success in mathematics-intensive majors. As revisions are made to the high school GSEs, consideration should be given to adding detail in the areas of mathematical modeling and geometry. Another recommendation is to separate the compound expectations that comprise many of the GSEs for Grades 9-10 into more manageable subsets that would clearly delineate the specific expectations with regard to how students will demonstrate their knowledge and more clearly outline what they will be expected to know. This detail will help teachers focus on the important mathematics and ensure that all students across the state are not only being held to the same high expectations but also engaged with challenging tasks and rich applications of mathematics.

## APPENDIX A: SIDE-BY-SIDE COMPARISON OF THE ADP MATHEMATICS BENCHMARKS TO THE RHODE ISLAND GRADE SPAN EXPECTATIONS

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| Certain mathematics benchmarks are marked with an **asterisk (*).** These asterisked benchmarks represent **content that is recommended for all students, but is required for those students who plan to take calculus** in college, a requisite for mathematics and many mathematics intensive majors. | In the case of RI expectations with compound performance and/or content expectations the subset of content that aligns with the ADP benchmark is **bolded**.<br><br>Shaded rows denote extensions of ADP benchmarks. | **COMMENTS ARE DIRECTED TOWARDS ISSUES WITH ADP ALIGNMENT.** |
| **I. Number Sense and Numerical Operations - ** The high school graduate can: | **Numbers and Operations** | |
| **I1. Compute with rational numbers fluently and accurately without a calculator:** | | |
| **I1.1.** Add, subtract, multiply and divide integers, fractions and decimals. | M(N&O)—10—6 Mentally calculates benchmark perfect squares and related square roots (e.g., 12, 22 , …, 122, 152, 202, 252, 1002, 10002). Determines any whole number percentage of a number or any multiples of 100% up to 500%. **Determines benchmark fractions of a number.** (IMPORTANT: The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.) | **Mental math is not included in the ADP Benchmarks.** |
| | M(N&O)—10—8 Applies properties of numbers to solve problems, to **simplify computations**, or to compare and contrast the properties of numbers and number systems. | |
| | M(N&O)—6—4 Accurately solves problems involving single or multiple operations on fractions (proper, improper, and mixed), or decimals; and addition or subtraction of integers; percent of a whole; or problems involving greatest common factor or least common multiple. | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| | M(N&O)—8—4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. | |
| **I1.2.** Calculate and apply ratios, proportions, rates and percentages to solve problems. | M(N&O)—8—4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots.(IMPORTANT: Applies the conventions of order of operations.) | |
| | M(N&O)—10—4 Accurately solves problems that involve but are not limited to proportional relationships, percents, ratios, and rates. (The problems might be drawn from contexts outside of and within mathematics including those that cut across content strands or disciplines.)<br>Solves problems involving compound interest | |
| | M(N&O)—10—6 Mentally calculates benchmark perfect squares and related square roots (e.g., 12, 22 , …, 122, 152, 202, 252, 1002, 10002). **Determines any whole number percentage of a number or any multiples of 100% up to 500%**. Determines benchmark fractions of a number. (IMPORTANT: The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.) | |
| | M(N&O)—10—7 Makes appropriate estimates in a given situation by determining the level of accuracy needed and analyzing the accuracy of results. **Estimates tips, discounts, and tax** and estimates the value of a non-perfect square root or cube root.<br>(IMPORTANT: The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.) | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| **I1.3.** Use the correct order of operations to evaluate arithmetic expressions, including those containing parentheses. | M(N&O)—8—4 Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots.**(IMPORTANT: Applies the conventions of order of operations.)** | |
| **I1.4.** Explain and apply basic number theory concepts such as prime number, factor, divisibility, least common multiple and greatest common divisor. | M(N&O)—8—8 Applies **properties of numbers (odd, even, remainders, divisibility, and prime factorization)** and field properties (commutative, associative, identity [including the multiplicative property of one, e.g. 20 x 23 = 20+3 = 23, so 20 = 1], distributive, inverses) to solve problems and to simplify computations, and demonstrates conceptual understanding of field properties as they apply to subsets of real numbers when addition and multiplication are not defined in the traditional ways. | |
| | M(N&O)—5—4 Accurately solves problems involving multiple operations on whole numbers or the use of the properties of **factors, multiples, prime, or composite numbers**; and addition or subtraction of fractions (proper) and decimals to the hundredths place. (Division of whole numbers by up to a two-digit divisor.) | |
| | M(N&O)—6—4 Accurately solves problems involving single or multiple operations on fractions (proper, improper, and mixed), or decimals; and addition or subtraction of integers; percent of a whole; or problems involving **greatest common factor or least common multiple.** | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| **I1.5.** Multiply and divide numbers expressed in scientific notation. | M(N&O)—12—4 **Solves problems involving scientific notation** or uses significant digits to assess the precision of an answer. Interprets rational exponents and their relation to radicals. Computes by hand in simple cases (e.g.$4^{3/2}$), and using a calculator when appropriate. **Interprets numbers given in scientific notation and carries out computations of them with and without a calculator.** | **ADP designates this Benchmark for all.**<br>**RI includes this Benchmark at the 11-12th grade level.** |
| **I2. Recognize and apply magnitude (absolute value) and ordering of real numbers:** | | |
| **I2.1.** Locate the position of a number on the number line, know that its distance from the origin is its absolute value and know that the distance between two numbers on the number line is the absolute value of their difference. | M(N&O)—10—2 Demonstrates understanding of the relative magnitude of real numbers by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., square root of 2 , π ), rational bases with integer exponents, square roots, **absolute values**, integers, or numbers represented in scientific notation **using number lines** or equality and inequality symbols. | |
| **I2.2.** Determine the relative position on the number line of numbers and the relative magnitude of numbers expressed in fractional form, in decimal form, as roots or in scientific notation. | M(N&O)—10—2 Demonstrates understanding of the relative magnitude of real numbers by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., square root of 2 , π ), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols. | |
| | M(N&O)—12—2 Demonstrates understanding of the relative magnitude of real numbers by solving problems that involve ordering or comparing. | **How does this expectation differ from the expectation at Grades 9-10?** |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| | M(N&O)— AM—1 Understands the structure of the real number system as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and **as points on a number line**. Determines whether the decimal expansion of a rational number eventually repeats or terminates (without using a calculator). | **RI designates this standard as advanced. ADP holds this expectation for all.** |
| **I3. Understand that to solve certain problems and equations, number systems need to be extended from whole numbers to the set of all integers (positive, negative and zero), from integers to rational numbers, from rational numbers to real numbers (rational and irrational numbers) and from real numbers to complex numbers; define and give examples of each of these types of numbers.** | M(N&O)—10—8 Applies properties of numbers to solve problems, to simplify computations, or to **compare and contrast the properties of numbers and number systems**. | |
| | M(N&O)—12—1 Knows that rational numbers are precisely those with eventually repeating or terminating decimal expansions | **All students will not study the structure of the number system since these expectations are included at grade span 11-12 and advanced mathematics.** |
| | M(N&O)— AM—1 Understands the structure of the real number system as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number eventually repeats or terminates (without using a calculator). | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| | M(N&O)—12—4 Demonstrates understanding of complex numbers by interpreting them geometrically and by computing with them (e.g., adding, multiplying, dividing, finding the nth root, or by finding conjugates). Understand complex numbers as an extension of the real numbers (e.g. arising in solutions of polynomial equations). Manipulates complex numbers using rectangular and polar coordinates. Knows the fundamental theorem of algebra and knows that non-constant polynomials always factor into linear factors over the complex numbers. | **This Benchmark is an extension of ADP. The study of complex numbers is a much more in-depth in the RI standards.** |
| | M(N&O)—12—8 Determine whether a given subset of numbers is closed under a given arithmetic operation | **This concept is an extension of ADP.** |
| | M(N&O)—AM—8 Add and multiply numerical matrices with attention to the arithmetic properties of these operations. Algebraically and geometrically interpret vectors, vector addition, and scalar multiplication in the plane, with attention to arithmetic properties.<br><br>Knows and uses the principle of mathematical induction. | **Matrices and Vectors are not included in ADP.** |
| **I4. Understand the capabilities and the limitations of calculators and computers in solving problems:** | | |
| **I4.1.** Use calculators appropriately and make estimations without a calculator regularly to detect potential errors. | M(N&O)—10—7 Makes appropriate estimates in a given situation by determining the level of accuracy needed and analyzing the accuracy of results. Estimates tips, discounts, and tax and estimates the value of a non-perfect square root or cube root.<br>(IMPORTANT: The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.) | **ADP does not explicitly include the intent of "embedding estimation throughout the instructional program."** |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| | M(N&O)—12—7 Makes appropriate estimates in a given situation by determining the level of accuracy needed and analyzing the accuracy of results. (IMPORTANT: The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.) | |
| **I4.2.** Use graphing calculators and computer spreadsheets. | M(DSP)—12—3 Find or estimate linear, quadratic, and exponential regression functions by organizing and displaying data **with or without using technology** | |
| | M(N&O)—AM—1 Understands the structure of the real number system as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number eventually repeats or terminates (**without using a calculator)**. | |
| | M(N&O)—12—4 Solves problems involving scientific notation or uses significant digits to assess the precision of an answer. Interprets rational exponents and their relation to radicals. Computes by hand in simple cases (e.g. ), and **using a calculator when appropriate**. Interprets numbers given in scientific notation and carries out computations of them **with and without a calculator** | |
| | M(G&M)—10—10 Demonstrates conceptual understanding of spatial reasoning and visualization by sketching or using **dynamic geometric software** to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three- dimensional objects, or by solving related problems. | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| | M(G&M)—12—10 Perform and justify construction with compass and straightedge or **dynamic geometric software.** | |
| | M(F&A)—12—4 Finds approximate solutions to equations by graphing each side as a function **using technology**. Understand that any equation in x can be interpreted as the equation f(x) = g(x) and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of y = f(x) and y = g(x). | |
| | M(F&A)—AM—4 Interprets systems as matrix equations and solves them by computing the appropriate matrix inverse and multiplication, **with or without technology**. | |
| | M(DSP)—AM—3 **Uses technology** to explore the method of least squares and median-median for linear regression | |
| | M(DSP)—12—5 Designs and critiques experimental models **(with or without technology)** to approximate desired probabilities. | |
| **J. Algebra -** The high school graduate can: | **Functions and Algebra** | |
| **J1. Perform basic operations on algebraic expressions fluently and accurately:** | | |
| | M(F&A)—10—1 Identifies, extends, and generalizes a variety of patterns (linear and nonlinear) represented by models, tables, sequences, or graphs to solve problems. | **Generalizing patterns is not included in ADP.** |
| **J1.1.** Understand the properties of integer exponents and roots and apply these properties to simplify algebraic expressions. | M(F&A)—10—3 Demonstrates conceptual understanding of algebraic expressions by solving problems involving algebraic expressions, by simplifying expressions (e.g., **simplifying** polynomial or rational expressions, or **expressions involving integer exponents, square roots,** or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions. | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| **J1.2. *** Understand the properties of rational exponents and apply these properties to simplify algebraic expressions. | M(N&O)—12—4 Solves problems involving scientific notation or uses significant digits to assess the precision of an answer. **Interprets rational exponents and their relation to radicals**. Computes by hand in simple cases (e.g.4 $^{3/2}$), and using a calculator when appropriate. Interprets numbers given in scientific notation and carries out computations of them with and without a calculator. | |
| | M(F&A)—12—3 Manipulates, evaluates, and simplifies expressions involving rational exponents and radicals and can convert between expressions with rational exponents and expressions with radicals. | |
| **J1.3.** Add, subtract and multiply polynomials; divide a polynomial by a low degree polynomial. | M(F&A)—12—3 Manipulates, evaluates, and simplifies algebraic and numerical expressions.<br><br>Adds, subtracts, multiplies and divides polynomials. | **ADP designates this Benchmark for all. RI includes this Benchmark at the 11-12th grade level.** |
| **Polynomial Theorems** | M(F&A)—AM—3 Uses the remainder theorem, the factor theorem and rational root theorem for polynomials. | **The polynomial theorems are an extension of ADP.** |
| **J1.4.** Factor polynomials by removing the greatest common factor; factor quadratic polynomials. | M(F&A)—12—3 Factors quadratic and higher degree polynomials, including difference of squares. | **ADP designates this Benchmark for all. RI includes this Benchmark at the 11-12th grade level.** |
| | M(F&A)—AM—3 Understands the difference between factoring polynomials over integer, rational, real and complex numbers | **This concept is an extension of ADP.** |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| **J1.5.** Add, subtract, multiply, divide and simplify rational expressions. | M(F&A)—10—3 Demonstrates conceptual understanding of algebraic expressions by solving problems involving algebraic expressions, by simplifying expressions (e.g., **simplifying** polynomial or **rational expressions**, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions. | |
| | M(F&A)—12—3 Adds, subtracts, multiplies and divides rational expressions.<br><br>Simplifies complex fractions. | **ADP designates this Benchmark for all.. RI includes this Benchmark at the 11-12th grade level.** |
| **J1.6.** Evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified values of their variables. | M(F&A)—10—3 Demonstrates conceptual understanding of algebraic expressions by solving problems involving algebraic expressions, by simplifying expressions (e.g., **simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values)**, by **evaluating expressions,** or by translating problem situations into algebraic expressions. | |
| | M(F&A)—12—4 Understands the effect of simplifying radical or rational expressions on the solution set of equations involving such expressions. (e.g. x^2/x =x for x ≠0). | **Consistent with the intent of ADP but not explicitly referenced in ADP.** |
| **J1.7. *** Derive and use the formulas for the general term and summation of finite arithmetic and geometric series; find the sum of an infinite geometric series whose common ratio, r, is in the interval (-1, 1). | M(F&A)—12—1 Identifies arithmetic and geometric sequences and finds the nth term; then uses the generalization to find a specific term. | |
| | M(F&A)—AM—1 Computes partial sums of infinite arithmetic and geometric sequences, determines when an infinite geometric series converges, and finds its sum. Connects arithmetic and geometric sequences to linear and exponential functions, | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| | respectively. | |
| **J2 Understand functions, their representations and their properties:** | | |
| **Conceptual understanding of linear and nonlinear functions** | M(F&A)—10—2 Demonstrates conceptual understanding of linear and nonlinear functions and relations (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation). | **The general study of classes or families of functions is not referenced in the ADP Benchmarks.**<br><br>**These expectations are laudable. The goal of building conceptual understanding is explicitly highlighted and thus valued. Looking across functions for analysis of characteristics and properties such as constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change and working between and among different representations** |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| **Families of functions** | M(F&A)—12—2 Represents and analyzes functions in several ways. Recognizes properties of functions and characteristics properties of families of functions. Applies knowledge of functions to interpret, model, and solve problems.<br><br>Analyzes characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change.<br><br>Represents functions numerically, algebraically, graphically, and verbally (i.e. in written words). Recognizes properties of a function from these representations, and transfers information from one representation to another.<br><br>Graphs polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes.<br><br>Applies knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences | **of functions and relations (e.g., graphs, tables, equations, function notation) will give students the foundations of a more generalized function study that can be more readily applied to real world problems across the disciplines.** |
| **Classes of functions/inverses** | M(F&A)—AM—2 Analyzing characteristic of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes | **Consistent with the intent of ADP, but not all of these characteristics are explicitly referenced in ADP.** |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
|---|---|---|
| **Properties of Functions** | M(F&A —AM—2 Analyzes properties of functions including injectivity (1-1), surjectivity (onto), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither.<br>Analyzes informally the idea of continuity and limits. | **These properties of functions are beyond the scope of ADP.** |
| **Understanding solutions to equations as two functions** | M(F&A)—12—4 Finds approximate solutions to equations by graphing each side as a function using technology. Understand that any equation in x can be interpreted as the equation f($x$) = g($x$) and interpret the solutions of the equation as the x-value(s) of the intersection point(s) of the graphs of<br>$y$ = f($x$) and $y$ = g($x$). | **This concept is not explicitly referenced in ADP but was a MAP 6-8 expectation.** |
| **J2.1** Recognize whether a relationship given in symbolic or graphical form is a function | M(F&A)—12—2 Represents and analyzes functions in several ways. **Recognizes properties of functions** and characteristics properties of families of functions. Applies knowledge of functions to interpret, model, and solve problems. | **ADP designates this Benchmark for all.**<br>**RI includes this Benchmark at the 11-12th grade level.** |
| **J2.2. *** Determine the domain of a function represented in either symbolic or graphical form. | M(F&A)—AM—2 Understands domain restriction and the effects of it on the function and its properties. | **Only students in Advanced Mathematics will encounter this concept, it is not included in the four year expectations.** |
| **J2.3.** Understand functional notation and evaluate a function at a specified point in its domain. | M(F&A)—10—2 describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, **function notation**). | |
| **J2.4. *** Combine functions by composition, as well as by addition, subtraction, multiplication and division. | M(F&A)—AM—2 Understands functions and relations from a set-theoretic perspective, and **operations on functions including composition** and inverse including computing inverses algebraically. | **Only students in Advanced Mathematics will encounter this concept, it is not included in the four year expectations.** |

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| **J2.5. * I**dentify whether a function has an inverse and when functions are inverses of each other; explain why the graph of a function and its inverse are reflections of one another over the line y = x. | M(F&A)—AM—2 Understands functions and relations from a set-theoretic perspective, and operations on functions including composition and **inverse including computing inverses algebraically.** | **Only students in Advanced Mathematics will encounter this concept; it is not included in the four year expectations.** |
| **J2.6. *** Know the inverse of an exponential function is a logarithm, prove basic properties of a logarithm using properties of its inverse and apply those properties to solve problems. | M(F&A)—12—3 Understands properties of logarithms and can convert between logarithmic and exponential forms. | |
| | M(F&A)—AM—2 Analyzing characteristic of classes of functions and **inverse functions** (**exponential, logarithmic**, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes | |
| **J3. Apply basic algebraic operations to solve equations and inequalities:** | M(F&A)—10—4 Works with a wide variety of equations. | |
| **J3.1.** Solve linear equations and inequalities in one variable including those involving the absolute value of a linear function. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by **solving linear equations** (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. | |
| **J3.2.** Solve an equation involving several variables for one variable in terms of the others. | M(F&A)—10—2 d**escribes how change in the value of one variable relates to change in the value of a second variable**; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation). | |

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| | M(F&A)—8—4 Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, **solving formulas for a variable requiring one transformation (e.g., *d* = *rt*; *d*/*r* = *t*);** by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context. | |
| **J3.3.** Solve systems of two linear equations in two variables. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or **by solving problems involving systems of linear equations** in a context (using equations or graphs) or using models or representations. | |
| **J3.4. *** Solve systems of three linear equations in three variables. | M(F&A)—12—4 Understands and applies the various processes of solving equations and systems of equations or inequalities. Interprets the solutions algebraically and graphically.<br><br>Solves 2x2 and 3x3 systems of linear equations and graphically interprets the solutions. | |

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| **J3.5.** Solve quadratic equations in one variable. | M(F&A)—12—4 Factors, completes the square, uses the quadratic formula, and graphs quadratic functions to solve quadratic equations. | **ADP designates this Benchmark for all. RI includes this Benchmark at the 11-12th grade level.** |
| **Polynomial, Rational, and Radical Equations** | M(F&A)—12—4 Solves equations involving polynomial, rational, and radical expressions. Graphs and interprets the solutions. | **These expectations are extensions of ADP J3 Benchmarks involving solving equations.** |
| **Exponential and Logarithmic Equations** | M(F&A)—AM—4 Solves equations involving exponential and logarithmic expressions. Graphs and interprets the solutions. | |
| **Intermediate Value Theorem** | M(F&A)—AM–4 Knows and applies the intermediate value theorem to find exact or approximate solutions of equations or zeros of continuous functions. | |
| **J4. Graph a variety of equations and inequalities in two variables, demonstrate understanding of the relationships between the algebraic properties of an equation and the geometric properties of its graph, and interpret a graph:** | | |
| **J4.1.** Graph a linear equation and demonstrate that it has a constant rate of change. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving **linear equations (symbolically and graphically)** and expressing the solution set symbolically or **graphically,** or p**rovides the meaning of the graphical interpretations of solution(s)** in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
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| | M(F&A)—8—2 Demonstrates conceptual understanding of linear relationships ($y$ = k$x$; $y$ = m$x$ + b) **as a constant rate of change** by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or describing the meaning of slope and intercept in context; and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change. | |
| **J4.2.** Understand the relationship between the coefficients of a linear equation and the slope and x- and y-intercepts of its graph. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and **graphically)** and expressing the solution set symbolically or **graphically,** or p**rovides the meaning of the graphical interpretations of solution(s)** in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
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| | M(F&A)—8—2 Demonstrates conceptual understanding of linear relationships (*y* = k*x*; *y* = m*x* + b) as a constant rate of change by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or **describing the meaning of slope and intercept in context;** and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change. | |
| **J4.3.** Understand the relationship between a solution of a system of two linear equations in two variables and the graphs of the corresponding lines. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or **provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.** | |
| | M(F&A)—12—4 Solves 2x2 and 3x3 systems of linear equations and graphically interprets the solutions. | |
| **Solving using matrices** | M(F&A)—AM—4 Interprets systems as matrix equations and solves them by computing the appropriate matrix inverse and multiplication, with or without technology. | **Matrices are not included in ADP.** |
| **Systems of nonlinear equations** | M(F&A)—12—4 Solves systems of equations involving nonlinear expressions and graphically | **Only linear systems are included in ADP.** |

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| | interprets the solutions. | |
| **J4.4.** Graph the solution set of a linear inequality and identify whether the solution set is an open or a closed half-plane; graph the solution set of a system of two or three linear inequalities. | M(F&A)—12—4 Solves **systems of linear** and quadratic **inequalities.** | **Systems of quadratic inequalities are not included in ADP**<br>**ADP expects this Benchmark from all students. RI includes linear inequalities at 11-12.** |
| **J4.5.** Graph a quadratic function and understand the relationship between its real zeros and the x-intercepts of its graph. | M(F&A)—12—4 Factors, completes the square, uses the quadratic formula, and g**raphs quadratic functions** to solve quadratic equations. | **ADP designates this Benchmark for all.**<br>**RI includes this Benchmark at the 11-12th grade level.** |
| **J4.6. *** Graph ellipses and hyperbolas whose axes are parallel to the x and y axes and demonstrate understanding of the relationship between their standard algebraic form and their graphical characteristics. | M(G&M)—AM—3 Explores and interprets the characteristics of conic sections graphically and algebraically. Understands how different planar slices of a double cone yield different conic sections. Knows the characterization of conic sections as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for the conic sections. | **Only students in Advanced Mathematics will encounter this concept, it is not included in the four year expectations.** |
| **J4.7.** Graph exponential functions and identify their key characteristics. | M(F&A)—12—2 Graphs polynomial, rational and **exponential functions,** including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes. | **ADP designates this Benchmark for all.**<br>**RI includes this Benchmark at the 11-12th and advanced level.**<br><br>**Graphing polynomial, rational and logarithmic equations are not included in the ADP Benchmarks under J4.** |
| | M(F&A)—AM—4 Solves equations involving **exponential** and logarithmic expressions. **Graphs and interprets the solutions**. | |
| **J4.8.** Read information and draw conclusions from graphs; identify properties of a graph that provide useful information about the original problem. | This ADP Benchmark is embedded throughout the expectations in the RI Function and Algebra strand. | |
| | M(F&A)—12—4 Understands and applies the various processes of solving equations and systems of equations or inequalities. **Interprets the solutions algebraically and graphically.** | |
| **Logrithmic Function Graphs** | M(F&A)—AM—2 Recognizes properties of families of functions including logarithmic and trigonometric, and graphs them. | **ADP does not explicitly include logarithmic graphs.** |

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| **J5. Solve problems by converting the verbal information given into an appropriate mathematical model involving equations or systems of equations; apply appropriate mathematical techniques to analyze these mathematical models; and interpret the solution obtained in written form using appropriate units of measurement:** | M(F&A)—12—2 Applies knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences | **This expectation is not specific enough to know which classes of functions students will be expected to apply.** |
| **J5.1.** Recognize and solve problems that can be modeled using a linear equation in one variable, such as time/rate/distance problems, percentage increase or decrease problems, and ratio and proportion problems. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) **in problem-solving situations**; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. | |
| **J5.2.** Recognize and solve problems that can be modeled using a system of two equations in two variables, such as mixture problems. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by **solving problems involving systems of linear equations in a context (using equations or graphs) or using models or** representations. | |
| **J5.3.** Recognize and solve problems that can be modeled using a quadratic equation, such as the motion of an object under the force of gravity. | | **There is not explicit mention of using quadratic equations to model problems.** |

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| **J5.4.** Recognize and solve problems that can be modeled using an exponential function, such as compound interest problems. | M(N&O)—10—4 Solves problems involving compound interest | |
| **J5.5. *** Recognize and solve problems that can be modeled using an exponential function but whose solution requires facility with logarithms, such as exponential growth and decay problems. | | **There is not explicit mention of using exponential functions to model problems.** |
| **J5.6.** Recognize and solve problems that can be modeled using a finite geometric series, such as home mortgage problems and other compound interest problems. | | **There is not explicit mention of using finite geometric series to model situations.** |
| **J6. * Understand the binomial theorem and its connections to combinatorics, Pascal's triangle and probability.** | | **These concepts are not explicitly addressed in RI.** |
| **K. Geometry -** The high school graduate can: | **Geometry and Measurement** | |
| **K1. Understand the different roles played by axioms, definitions and theorems in the logical structure of mathematics, especially in geometry:** | | |
| **K1.1.** Identify, explain the necessity of and give examples of definitions, axioms and theorems. | M(G&M)—AM—2 Extends and deepens knowledge and usage of proofs and proof techniques. | **Definitions, axioms and theorems are not referenced in the RI expectations with the intent of ADP.** |
| **K1.2.** State and prove key basic theorems in geometry such as the Pythagorean theorem, the sum of the angles of a triangle is 180 degrees, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length. | M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle ratios). | |
| | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
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| | Theorem). | |
| | M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle congruence and similarity). | |
| **K1.3.** Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. | M(G&M)—10—2 Uses geometric models to represent and distinguish between Euclidian and non-Euclidian systems. | |
| **K2. Identify and apply the definitions related to lines and angles and use them to prove theorems in (Euclidean) geometry, solve problems, and perform basic geometric constructions using a straight edge and compass:** | M(G&M)—12—10 Perform and justify construction with compass and straightedge or dynamic geometric software. | **Constructions are included in ADP for all students. RI includes constructions at grade span 11-12.** |
| **K2.1.** Identify and apply properties of and theorems about parallel lines and use them to prove theorems such as two lines parallel to a third are parallel to each other and to perform constructions such as a line parallel to a given line through a point not on the line. | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, **lines**, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |
| | M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, **lines**, circles, distance, midpoint and polygons including triangle ratios). | |
| | M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, **lines,** circles, distance, midpoint and polygons including triangle congruence and similarity). | |

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| **K2.2.** Identify and apply properties of and theorems about perpendicular lines and use them to prove theorems such as the perpendicular bisectors of line segments are the set of all points equidistant from the two end points and to perform constructions such as the perpendicular bisector of a line segment. | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, **lines,** polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |
| | M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, **lines,** circles, distance, midpoint and polygons including triangle ratios). | |
| | M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, **lines**, circles, distance, midpoint and polygons including triangle congruence and similarity). | |
| **K2.3.** Identify and apply properties of and theorems about angles and use them to prove theorems such as two lines are parallel exactly when the alternate interior angles they make with a transversal are equal and to perform constructions such as the bisector of an angle | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving **angles**, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |
| | M(G&M)—10—2 Creates formal proofs of propositions (e.g. **angles**, lines, circles, distance, midpoint and polygons including triangle ratios). | |
| | M(G&M)—12—2 Creates formal proofs of propositions (e.g. **angles**, lines, circles, distance, midpoint and polygons including triangle congruence and similarity). | |
| **K3. Know the basic theorems about congruent and similar triangles and use them to prove additional theorems and solve problems.** | M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including **triangle congruence and similarity**). | **ADP expects proof involving congruence and similarity for all. RI places these proofs at grade span 11-12.** |

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| **K4. Know the definitions and basic properties of a circle and use them to prove basic theorems and solve problems.** | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, **circles**, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |
| | M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, lines, **circles**, distance, midpoint and polygons including triangle ratios). | |
| | M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, lines, **circles**, distance, midpoint and polygons including triangle congruence and similarity). | |
| | M(G&M)—AM—6 Derives and uses formulas for lengths of arcs and areas of sectors and segments. | **ADP expects the properties of circles for all. RI places these properties in advanced mathematics.** |
| **K5. Apply the Pythagorean theorem, its converse and properties of special right triangles to solve problems.** | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., **Pythagorean Theorem**, Triangle Inequality Theorem). | |
| | M(G&M)—8—2 Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. | |
| **K6. Use rigid motions (compositions of reflections, translations and rotations) to determine whether two geometric figures are congruent and to create and analyze geometric designs.** | M(G&M)—10—4 Applies the concepts of congruency by solving problems on or off a coordinate plane involving reflections, translations, or rotations; or solves problems using congruency involving problems within mathematics or across disciplines or contexts | |
| **Matrices** | M(G&M)—AM—4 Uses matrices to represent | **Matrices are not included in ADP** |

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| | reflections, translations, rotations. | |
| **K7. Know about the similarity of figures and use the scale factor to solve problems.** | M(G&M)—10—5 Applies concepts of similarity by solving problems within mathematics or across disciplines or contexts. | |
| **K8. Know that geometric measurements (length, area, perimeter, volume) depend on the choice of a unit and that measurements made on physical objects are approximations; calculate the measurements of common plane and solid geometric figures:** | ; | |
| **K8.1.** Understand that numerical values associated with measurements of physical quantities must be assigned units of measurement or dimensions; apply such units correctly in expressions, equations and problem solutions that involve measurements; and convert a measurement using one unit of measurement to another unit of measurement. | M(G&M)—10—7 Uses units of measure appropriately and consistently when solving problems across content strands; makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations involving measurement in other GSEs. | |
| | M(G&M)—10—6 Applies the appropriate unit of measure | |
| **K8.2.** Determine the perimeter of a polygon and the circumference of a circle; the area of a rectangle, a circle, a triangle and a polygon with more than four sides by decomposing it into triangles; the surface area of a prism, a pyramid, a cone and a sphere; and the volume of a rectangular box, a prism, a pyramid, a cone and a sphere. | M(G&M)—10—6 Solves problems involving perimeter, circumference, or area of two-dimensional figures (including composite figures) or surface area or volume of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts. | |
| **Cavalieri's principle** | M(G&M)—AM—6 Knows Cavalieri's principle and uses it to find volumes. | **Cavalieri's principle is not included in ADP.** |

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| **K8.3.** Know that the effect of a scale factor *k* on length, area and volume is to multiply each by $k$, $k^2$ and $k^3$, respectively. | M(G&M)—6—5 Demonstrates conceptual understanding of similarity by **describing the proportional effect on the linear dimensions of polygons** or circles when scaling up or down while preserving the angles of polygons, or by solving related problems (including applying scales on maps). Describes effects using models or explanations | |
| | M(G&M)—7—5 Applies concepts of similarity by solving problems involving scaling up or down and their **impact on angle measures, linear dimensions and areas** of polygons, and circles when the linear dimensions are multiplied by a constant factor. Describes effects using models orsc explanations. | |
| | M(G&M)—8—5 Applies concepts of similarity to determine the **impact of scaling on the volume or surface area** of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. | |
| **K9. Visualize solids and surfaces in three-dimensional space when given two-dimensional representations (e.g., nets, multiple views) and create two-dimensional representations for the surfaces of three-dimensional objects.** | M(G&M)—10—10 Demonstrates conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three- dimensional objects, or by solving related problems. | |
| | M(G&M)—7—10 Demonstrates conceptual understanding of spatial reasoning and visualization by sketching three-dimensional solids; and draws nets of rectangular and triangular prisms, cylinders, and pyramids and uses the nets as a technique for finding surface area. | |

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| **K10. Represent geometric objects and figures algebraically using coordinates; use algebra to solve geometric problems:** | M(G&M)—–AM—9 Solves specific problems using analytic geometry (including in three dimensions) and circular trigonometry (e.g. find the equation of a circle inscribed in a triangle given the coordinates of the vertices; the distance between opposite vertices in a rectangular solid). | **These concepts though not inconsistent with ADP are not explicitly addressed in ADP.** |
| **K10.1.** Express the intuitive concept of the "slant" of a line in terms of the precise concept of slope, use the coordinates of two points on a line to define its slope, and use slope to express the parallelism and perpendicularity of lines. | M(G&M)—10—9 Solves problems on and off the coordinate plane involving distance, midpoint, **perpendicular and parallel lines, or slope.** | |
| **K10.2.** Describe a line by a linear equation. | M(F&A)—10—4 Demonstrates conceptual understanding of equality by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations. | |
| **K10.3.** Find the distance between two points using their coordinates and the Pythagorean theorem. | M(G&M)—10—9 Solves problems on and off the coordinate plane involving **distance**, midpoint, perpendicular and parallel lines, or slope. | |
| **K10.4. *** Find an equation of a circle given its center and radius and, given an equation of a circle, find its center and radius. | M(G&M)—12—3 Knows the characterization of circles as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for circles | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
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| **Conic Sections** | M(G&M)—AM—3 Explores and interprets the characteristics of conic sections graphically and algebraically. Understands how different planar slices of a double cone yield different conic sections. Knows the characterization of conic sections as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for the conic sections. | **Conic Sections in general are not a benchmark in ADP. ADP does include circles K10.4, ellipses and hyperbolas J4.6 and parabolas J4.5.** |
| **K11. Understand basic right-triangle trigonometry and apply it to solve problems:** | | |
| **K11.1.** Understand how similarity of right triangles allows the trigonometric functions sine, cosine and tangent to be defined as ratios of sides and be able to use these functions to solve problems. | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or **right triangle ratios (sine, cosine, tangent)** within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |
| | M(G&M)—12—5 Knows that similarity of right triangles allows the trigonometric functions to be defined as ratios of sides of triangles, and uses the ratios of the sides of special right triangles (30-60-90 and 45-45-90) to determine the sine, cosine and tangent ( 30,45, 60) and solve related problems. | **ADP includes this Benchmark for all. RI places this expectation at 11-12.** |
| **K11.2.** Apply the trigonometric functions sine, cosine and tangent to solve for an unknown length of a side of a right triangle, given one of the acute angles and the length of another side. | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or **right triangle ratios (sine, cosine, tangent)** within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
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| **K11.3.** Use the standard formula for the area of a triangle, $A = ½bh$, to explain the area formula, $A = ½ab\sin C$ where a and b are the lengths of two sides of a triangle and C is the measure of the included angle formed by these two sides, and use it to find the area of a triangle when given the lengths of two of its sides and the included angle. | M(G&M)—12—6 Applies trigonometric formulas (law of sines/cosines, A= 1/2 *ab* sinC ) to find angles, lengths and areas of polygons. | **ADP includes this Benchmark for all. RI places this expectation at 11-12.** |
| **K12. * Know how the trigonometric functions can be extended to periodic functions on the real line, derive basic formulas involving these functions, and use these functions and formulas to solve problems:** | | |
| **K12.1. *** Know that the trigonometric functions sine and cosine, and thus all trigonometric functions, can be extended to periodic functions on the real line by defining them as functions on the unit circle, that radian measure of an angle between 0 and 360 degrees is the arc length of the unit circle subtended by that central angle, and that by similarity, the arc length $s$ of a circle of radius $r$ subtended by a central angle of measure $t$ radians is $s = rt$. | M(G&M)—AM—7 Understands why radian measure is useful and converts between radian measure of angles and degrees. | **ADP 12.1-ADP 12.3 will only be studied by students in the advanced level.** |
| | M(G&M)—–AM—9 Solves specific problems using analytic geometry (including in three dimensions) and **circular trigonometry** (e.g. find the equation of a circle inscribed in a triangle given the coordinates of the vertices; the distance between opposite vertices in a rectangular solid). | |
| **K12.2. *** Know and use the basic identities, such as $\sin^2(x) + \cos^2(x) = 1$ and $\cos(\Pi/2-x) = \sin(x)$ and formulas for sine and cosine, such as addition and double angle formulas. | M(F&A)—AM—4 Solves equations and verifies identities involving trigonometric expressions. | |
| **K12.3. *** Graph sine, cosine and tangent as well as their reciprocals, secant, cosecant and | M(F&A)—AM—2 Recognizes properties of families of functions including logarithmic and | |

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| cotangent; identify key characteristics. | trigonometric, and graphs them. | |
| **K12.4. * Know** and use the law of cosines and the law of sines to find missing sides and angles of a triangle. | M(G&M)—12—6 Applies trigonometric formulas (law of sines/cosines, A= 1/2 *ab* sinC ) to find angles, lengths and areas of polygons. | |
| **L. Data Interpretation, Statistics and Probability -** The high school graduate can: | **Data, Statistics, and Probability** | |
| **L1. Explain and apply quantitative information:** | | |
| **L1.1.** Organize and display data using appropriate methods (including spreadsheets) to detect patterns and departures from patterns. | M(DSP)—10—1 **Interprets** a given representation (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to **analyze** the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g. media, workplace, social and environmental situations).(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)—10—2.) | |
| **L1.2.** Read and interpret tables, charts and graphs. | | |
| | M(DSP)—10—3 Identifies or describes representations or elements of representations that best display a given set of data or situation, consistent with the representations required in M(DSP)—10—1. | |
| | M(DSP)—10—3 Organizes and displays one- and two-variable data using a variety of representations (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts). Analyzes the data to formulate or justify conclusions, make predictions, or to solve problems. Identifies representations that best display a given set of data. | |

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| **L1.3.** Compute and explain summary statistics for distributions of data including measures of center (mean, median) and spread (range, percentiles, variance, standard deviation). | M(DSP)—10—2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed. | |
| | M(DSP)—12—2 Calculates and analyzes measures of dispersion (standard deviation, variance, and percentiles). | **ADP includes this Benchmark for all. RI places this expectation at 11-12.** |
| **L1.4.** Compare data sets using graphs and summary statistics. | | **Comparing data sets is not explicitly included in RI.** |
| **L1.5.** Create scatter plots, analyze patterns and describe relationships in paired data. | M(DSP)—10—3 Organizes and displays one- and two-variable data using a variety of representations (e.g., box-and-whisker plots, **scatter plots,** bar graphs, line graphs, circle graphs, histograms, frequency charts). Analyzes the data to formulate or justify conclusions, make predictions, or to solve problems. Identifies representations that best display a given set of data. | |
| | M(DSP)—10—2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, **estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation)** to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed. | |

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| **L1.6.** Know the characteristics of the Gaussian normal distribution (bell-shaped curve). | M(DSP)—AM—2 Analyzes and interprets measures of dispersion (standard deviation, variance, and percentiles) and central tendency for normal distributions. | **ADP expects the characteristics of the Gaussian normal distribution for all. RI places this distribution in advanced mathematics.** |
| **L2. Explain and critique alternative ways of presenting and using information:** | M(DSP)—10—1 Interprets a given representation (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or **justify conclusions, critique conclusions,** make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g. **media, workplace, social and environmental situations)**.(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)—10—2.) | **ADP L2 does not have parallel expectations with strong alignments in RI.** |
| **L2.1.** Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed. | | |
| **L2.2.** Identify and explain misleading uses of data. | | |
| **L2.3.** Recognize when arguments based on data confuse correlation with causation. | | |
| **L3. Explain the use of data and statistical thinking to draw inferences, make predictions and justify conclusions:** | | |

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| **L3.1.** Explain the impact of sampling methods, bias and the phrasing of questions asked during data collection and the conclusions that can rightfully be made. | M(DSP)—10—2 Analyzes patterns, trends, or distributions in data in a variety of contexts by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual **understanding of the sample from which the statistics were developed.** | |
| **L3.2.** Design simple experiments or investigations to collect data to answer questions of interest. | M(DSP)—10—6 Designs an experiment in response to a teacher or student generated question or hypothesis. Designs an effective methodology to answer the questions (e.g., survey, observation, research, experimentation). **Uses an appropriate sampling techniques to collect the data necessary to answer the question (e.g., random sample, stratified random sample)**. Collects, organizes, and appropriately displays the data. Analyzes the data to draw conclusions about the questions or hypothesis being tested while considering the limitations of the data that could affect interpretations. Finally when appropriate makes predictions, asks new questions, or makes connections to real-world situations. (IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)—10—2.) | |
| **L3.3.** Explain the differences between randomized experiments and observational studies. | | **This expectation is not included in RI.** |

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| **L3.4.** Construct a scatter plot of a set of paired data, and if it demonstrates a linear trend, use a graphing calculator to find the regression line that best fits this data; recognize that the correlation coefficient measures goodness of fit and explain when it is appropriate to use the regression line to make predictions | M(DSP)—AM—3 Uses technology to explore the method of least squares and median-median for linear regression | **Linear regression is expected for all students in ADP.**<br><br>**RI designates this expectation as advanced.**<br><br>**RI includes quadratic and exponential regression.** |
| | M(DSP)—12—3 Find or estimate linear, quadratic, and exponential regression functions by organizing and displaying data with or without using technology. | |
| | M(DSP)—12—1 Given a regression function (linear, quadratic, and exponential), analyze the data to make inferences and to formulate, justify, and critique conclusions. | |
| **L4. Explain and apply probability concepts and calculate simple probabilities:** | | |
| **L4.1.** Explain how probability quantifies the likelihood that an event occurs in terms of numbers. | M(DSP)—5—5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the experimental or theoretical probability of an event and expresses the result as a fraction; and predicts the likelihood of an event as a fraction and tests the prediction through experiments; and determines if a game is fair. | |
| | M(DSP)—3—5 For a probability event in which the sample space may or may not contain equally likely outcomes, determines the likelihood of the occurrence of an event (using "more likely", "less likely", or "equally likely"); and predicts the likelihood of an event using "more likely," "less likely," "equally likely," certain, or impossible and tests the prediction through experiments; and determines if a game is fair. | |
| **L4.2.** Explain how the relative frequency of a specified outcome of an event can be used to estimate the probability of the outcome. | | **Not specifically mentioned in RI** |

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| **L4.3.** Explain how the law of large numbers can be applied in simple examples. | | **Not specifically mentioned in RI** |
| **L4.4.** Apply probability concepts such as conditional probability and independent events to calculate simple probabilities. | M(DSP)—12—5 Designs and critiques experimental models (with or without technology) to approximate desired probabilities. Solves probability problems by applying concepts of counting, random variables, independence/dependence of events, and conditional probability. | **Conditional probability is included for all students in ADP. RI includes it in 11-12.** |
| | M(DSP)—10—4 Uses counting techniques to solve contextualized problems involving combinations or permutations (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or others). | |
| | M(DSP)—10—5 Solves problems involving experimental or theoretical probability. | |
| | M(DSP)—12—4 Solves problems involving combinations and permutations using a variety of strategies including nCr, nPr, or n!.Finds unions, intersections, and complements of sets. | **All extensions of ADP** |
| **L4.5.** Apply probability concepts to practical situations to make informed decisions. | M(DSP)—10—4 Uses counting techniques to solve contextualized problems involving combinations or permutations (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or others). | |
| | M(DSP)—12—5 Designs and critiques experimental models (with or without technology) to approximate desired probabilities. Solves probability problems by applying concepts of counting, random variables, independence/dependence of events, and conditional probability. | |

| ADP Benchmarks: Mathematics | Rhode Island GSEs 9-10, 11-12 and Advanced Mathematics | COMMENTARY |
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| **Mathematical Reasoning:** Woven throughout the four domains of mathematics — Number Sense and Numerical Operations; Algebra; Geometry; and Data Interpretation, Statistics and Probability — are the following mathematical reasoning skills: | | |
| **MR1.** Using inductive and deductive reasoning to arrive at valid conclusions. | M(N&O)—AM—8 Knows and uses the principle of mathematical induction | **ADP includes this Benchmark for all. RI places this expectation in advanced mathematics.** |
| **MR2.** Using multiple representations (literal, symbolic, graphic) to represent problems and solutions. | M(F&A)—10—2 Demonstrates conceptual understanding of linear and nonlinear functions and relations (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or w**orks between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).** | |
| **MR3.** Understanding the role of definitions, proofs and counterexamples in mathematical reasoning; constructing simple proofs. | M(G&M)—AM—2 Extends and deepens knowledge and usage of proofs and proof techniques. | |
| | M(G&M)—10—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle ratios). | |
| | M(G&M)—10—2 Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem). | |

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| | M(G&M)—12—2 Creates formal proofs of propositions (e.g. angles, lines, circles, distance, midpoint and polygons including triangle congruence and similarity). | |
| **MR4.** Using the special symbols of mathematics correctly and precisely. | | **This process standard is not specifically mentioned in RI.** |
| **MR5.** Recognizing when an estimate or approximation is more appropriate than an exact answer and understanding the limits on precision of approximations. | M(N&O)—12—4 Solves problems involving scientific notation or **uses significant digits to assess the precision of an answer.** Interprets rational exponents and their relation to radicals. Computes by hand in simple cases (e.g. $4^{3/2}$ ), and using a calculator when appropriate. Interprets numbers given in scientific notation and carries out computations of them with and without a calculator. | |
| | M(N&O)—10—7 **Makes appropriate estimates in a given situation by determining the level of accuracy needed and analyzing the accuracy of results**. Estimates tips, discounts, and tax and estimates the value of a non-perfect square root or cube root. | |
| | M(N&O)—12—7 Makes appropriate estimates in a given situation by determining the level of accuracy needed and analyzing the accuracy of results. (IMPORTANT: The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.) | |
| | M(G&M)—10—7 Applies informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations (e.g., use successive approximation to find the area of a pond); uses measurement conversion strategies (e.g., unit/dimensional analysis). | |
| **MR6.** Distinguishing relevant from irrelevant information, identifying missing information and either finding what is needed or making appropriate estimates. | | **This process standard is not specifically mentioned in RI.** |

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| **MR7.** Recognizing and using the process of mathematical modeling: recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem. | M(F&A)—12—2 Applies knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in natural and social sciences | **ADP includes this Benchmark for all. RI places this expectation in 11-12.** |
| **MR8.** When solving problems, thinking ahead about strategy, testing ideas with special cases, trying different approaches, checking for errors and reasonableness of solutions as a regular part of routine work, and devising independent ways to verify results. | | **This process standard is not specifically mentioned in RI.** |
| **MR9.** Shifting regularly between the specific and the general, using examples to understand general ideas, and extending specific results to more general cases to gain insight. | M(N&O)—AM—8 Knows and uses the principle of mathematical induction | **ADP includes this Benchmark for all. RI places this expectation in advanced mathematics.** |

## APPENDIX B: BIOGRAPHIES

### ACHIEVE STAFF

#### KAYE FORGIONE, SENIOR ASSOCIATE, MATHEMATICS, ACHIEVE

**Kaye Forgione** joined Achieve as senior associate for mathematics in March 2001 where she leads Achieve's Standards and Benchmarking Initiatives involving mathematics. Prior to joining Achieve, Kaye served as assistant director of the Systemic Research Collaborative for Mathematics, Science and Technology Education (SYRCE), a project at the University of Texas at Austin funded by the National Science Foundation. Her responsibilities at the University of Texas also included management and design responsibilities for UTeach, a collaborative project of the College of Education and the College of Natural Sciences to train and support the next generation of mathematics and science teachers in Texas. Before her work at the University of Texas, Kaye was director of academic standards programs at the Council for Basic Education, a nonprofit education organization located in Washington, DC. Prior to joining the Council for Basic Education in 1997, Kaye worked in the K–12 arena in a variety of roles, including several leadership positions with the Delaware Department of Education. Kaye began her education career as a high school mathematics teacher. She taught mathematics at the secondary and college levels as part of adult continuing education programs. Kaye received a bachelor's degree in mathematics and education from the University of Delaware, a master's degree in systems management from the University of Southern California, and a doctorate in educational leadership from the University of Delaware.

#### LAURA MCGIFFERT SLOVER, DIRECTOR, CONTENT & POLICY RESEARCH, ACHIEVE

**Laura McGiffert Slover** is director of Content & Policy Research at Achieve, where she has senior responsibility for overseeing a number of Achieve's major initiatives. She supervises Achieve's Benchmarking Initiative, leads its work with states on building mathematics capacity, and oversees the organization's research agenda. Laura has extensive experience reviewing academic standards and education policies in the United States and abroad, and she has written a number of reports and articles on the topic. Before joining Achieve in 1998, Laura was a high school English teacher in Eagle County, Colorado, where she was involved in the district's early efforts to develop standards and benchmark assessments. She also taught writing and composition at Colorado Mountain College. A native Washingtonian, Laura earned a bachelor's degree in English and American Literature from Harvard University; a master's in Education Curriculum and Instruction from the University of Colorado at Boulder; and a master's in Education Policy from Georgetown University. She is a mentor and a member of the Board of Directors of Project Northstar, an organization that provides mentoring and tutoring to homeless and at-risk students in the District of Columbia.

## CONTENT EXPERTS IN MATHEMATICS

**MARY LYNN RAITH**

**Mary Lynn Raith** received her B.S in mathematics from Indiana University at Pittsburgh and her M.Ed. in mathematics education from the University of Pittsburgh. She is recently retired from the position of Mathematics Specialist in the Division of Instructional Support of the Pittsburgh Public Schools. As such, her responsibilities included leadership roles in curriculum development, textbook selection, design of alternative assessments, in-service program design and implementation, and coordination of mathematics programs across levels and schools. Ms. Raith was also the Co-Director of the Pittsburgh Reform in Mathematics Education project (PRIME), a K-12 professional development system. She has also been involved with a number of national projects, including the development of both the New Standards Reference Examination and the Portfolio project for the middle grades, the Assessment Communities of Teachers project (ACT), and the Alternative Assessment in Mathematics project ($A^2IM$). She has also worked extensively with both NCTM and NCEE on its America's Choice school design and has presented at numerous national conferences.