

## New Hampshire High School Mathematics Grade-Span Expectations

### About the Mathematics GSEs

This set of New Hampshire Grade-Span Expectations (GSEs) for high school includes expectations that will be assessed on the state-level assessment and others that will be a local curriculum and assessment option (i.e., guidance for local curriculum and assessment but not assessed on the state assessment). Those GSEs that will be assessed on the state assessment are boxed with bold outlines. Furthermore, these assessment GSEs are common across NH, RI, and VT and will be assessed through the New England Common Assessment Program (NECAP). **The assessment GSEs in this document can be interpreted as describing expectations for the end of grade 10, or the beginning of grade 11.** The local content GSEs were developed in collaboration with the Rhode Island Department of Education. As you review the DRAFT New Hampshire GSEs in Mathematics the following are important to understand.

- 1) The grade 8 Grade-Level Expectations (GLEs) are included in this document in order to see how the concepts and skills develop from grade 8 to high school.
- 2) The GSEs and GLEs are organized into four content strands (Number and Operations; Functions and Algebra; Data, Statistics, and Probability; Geometry and Measurement) and two process strands (Problem-Solving, Reasoning, and Proof; Communications, Representations, and Connections).
- 3) Since it is crucial that process standards (problem-solving, reasoning, proof, communication, connections, and representations) are not seen as completely separate from content standards, the process standards have been imbedded throughout the content strands (e.g., M(F&A)–10–1 **Identifies, extends, and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations). This mirrors classroom instruction in most classes. As students are learning content knowledge, instruction is focusing on improving their abilities in problem solving, reasoning, and communication. Students should be looking for and making appropriate connections, and should be able to understand and use multiple representations of mathematical ideas. Since it is crucial that students are strong in both content and process knowledge, we have included two local process strands following the content strands. These process strands are in addition to the process standards that are embedded in the content standards and are included to help guide local curriculum, assessment, and instruction. The process standards have been separated by grade-spans (K–2, 3–5, 6–8, and 9–12). The 6–8 and 9–12 spans have been included in this document. Each grade span should be thought of as building upon the skills and concepts in the previous grade span.
- 4) Each GSE and GLE includes a **bolded** statement called the “stem” which is the same or similar across the grades for a given GSE or GLE. Each stem is meant to communicate the main curricular and instructional focus of the GSE or GLE across the grades.
- 5) The unbolded text within a GSE or GLE indicates the proficiencies for that given grade-level or grade-span.
- 6) The underlined text at grade 8 indicates the concepts and skills that are new to grade 8 from grade 7. (Note: Sometimes nothing is underlined within a GLE. In these situations examine other GLEs across the strand to identify the differences or check the Depth of Knowledge table in the test specifications.)
- 7) Each GSE is coded for the content strand, grade level, and the GSE “stem” number (e.g., M(F&A)–X–3 where X = 10, HS, or AM: The “M” stands for mathematics, the “F&A” stands for the Functions and Algebra strand, the “10” indicates that the standard is a state assessment standard for the end of grade 10 or beginning of grade 11, “HS” indicates a local high school expectation for all students, an “AM” indicates a local expectation for students preparing for advanced mathematics courses (e.g., calculus), and the “3” stands for stem 3 or the 3<sup>rd</sup> big idea in the Functions and Algebra strand). See the diagram on page 2.
- 8) Unless otherwise specified the number parameters defined in the Number and Operations strand for a particular grade-level or grade-span apply to all GLEs at that grade-level or all GSEs at that grade-span.
- 9) All the concepts and skills identified in the assessment GSEs are fair game for assessment purposes. However, while all parts of a GSE may be assessed each year, it is more likely that all parts of a GSE will be assessed over several years.
- 10) In some places you will see “or<sup>sc</sup>” (“sc” stands for “student choice”). While, in general, students have choices as to the strategies that they use to solve problems, “or<sup>sc</sup>” was used to explicitly state to the testing contractor that a particular strategy could not be required. For example, as in GSE M(DSP)–10–4: “...using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or<sup>sc</sup> others).”
- 11) While the use of dynamic software will not be assessed on the state-level assessment at this time, it is expected that students at the high school level would receive ample opportunity to solve problems using dynamic software at the local level.

New Hampshire High School Mathematics Grade-Span Expectations  
**Reading the Mathematics GSEs**

**STEM** – Bolded statement communicating main curricular focus and is the same or similar across grades (K-12)

**CONTENT AREA: “M”** stands for mathematics

**STRAND:** In this case, “DSP” stands for Data, Statistics, and Probability

**GRADE LEVEL:** “10” indicates a state assessment standard for the end of grade 10 or beginning of grade 11; “HS” indicates a local high school expectation for all students; “AM” indicates a local expectation for students preparing for an advanced mathematics course (e.g., calculus)

**STEM Number:** The big ideas in each strand are numbered. In this case, “2” indicates the 2<sup>nd</sup> big idea in the Data, Statistics, and Probability strand.

Grade 8	High School	Advanced Mathematics
<p>M(DSP)–8–2 <b>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using</b> measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, <u>quartile values</u>, or <u>estimated line of best fit</u> to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, <u>random</u>, or <u>non-random</u>).</p>	<p>M(DSP)–HS–2 <b>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using</b> measures of dispersion (standard deviation, variance, and percentiles).</p> <div data-bbox="964 751 1704 1089" style="border: 2px solid black; padding: 5px;"> <p>M(DSP)–10–2 <b>Analyzes patterns, trends, or distributions in data in a variety of contexts by</b> determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed.</p> </div>	<p>M(DSP)–AM–2 <b>Analyzes and interprets measures of dispersion</b> (standard deviation, variance, and percentiles) and <b>central tendency</b> for the normal distribution; and <b>interprets</b> the correlation coefficient and the coefficient of determination in the context of data.</p>

**UNDERLINING** – Indicates concepts and skills that are new to the grade level in the K-8 GLEs

**BOLD BOX** – Indicates the standards to be assessed on the NECAP assessment; all other standards are guidance for local curriculum and assessment

New Hampshire High School Mathematics Grade-Span Expectations

**Number and Operations**

**Purpose:** Numbers and operations remain a cornerstone for the study of mathematics in grades K – 12. Students use numbers to quantify sets, identify location, measure, quantify the probability of an event, analyze data, and describe and interpret real-world phenomena. Having students know basic facts and having students compute fluently (i.e., accurately and efficiently) continues to be an important goal in mathematics education. However, knowing basic facts should be incorporated into a rich mathematics curriculum that builds conceptual understanding of these facts.

Through the school years, the amount of time spent on numbers and their operations will decrease and the types of numbers studied will change. As students progress through the elementary grades and into middle school, they will need to develop an in-depth conceptual understanding of fractions, decimals, and percents prior to doing algorithmic computations with these numbers. Conceptual development of integers and meaningful computation with them are also goals for middle grade students. The study of irrational numbers and the real number system will begin in eighth grade and continue through high school. Imaginary and complex numbers are introduced in advanced mathematics. It is important for students to model and represent the different types of numbers they study.

Students cannot appreciate the power of numbers unless they also understand the operations upon those numbers. Students need to recognize which operation(s) to apply to a given problem situation they encounter. They need to know what effect the various operations will have on different types of numbers. They need to know the relationships among the operations and among the operations and their properties. A deep understanding of the operations and their properties will help students make sense of computation algorithms and lead to fluency in computation. A firm understanding of numbers as well as operations and their properties will provide a good foundation for the study of algebra.

Grade 8	High School	Advanced Mathematics
<p>M(N&amp;O)–8–1 <b>Demonstrates conceptual understanding of rational numbers with respect to</b> absolute values, perfect square and cube roots, and percents as a way of describing change (percent increase and decrease) <b>using explanations, models, or other representations.</b></p>	<p>M(N&amp;O)–HS–1 <b>Demonstrates conceptual understanding of rational numbers</b> by knowing why a real number is rational if and only if the number’s decimal expansion eventually repeats or terminates.</p>	<p>M(N&amp;O)–AM–1 <b>Demonstrates conceptual understanding of the real number system</b> as an extension of the rational numbers by representing real numbers as infinite decimal expansions (that provide successive rational approximations to the number) and as points on a number line. Determines whether the decimal expansion of a rational number given in fractional form eventually repeats or terminates (without using a calculator).</p>
<p>M(N&amp;O)–8–2 <b>Demonstrates understanding of the relative magnitude of numbers</b> by ordering or comparing rational numbers, <u>common irrational numbers</u> (e.g., <math>\sqrt{2}</math>, <math>\pi</math>), numbers with whole number or <u>fractional bases</u> and whole number exponents, <u>square roots</u>, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.</p>	<p>M(N&amp;O)–HS–2 <b>Demonstrates understanding of the relative magnitude of real numbers</b> by solving problems that involve ordering or comparing any subset of the real numbers.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(N&amp;O)–10–2 <b>Demonstrates understanding of the relative magnitude of real numbers</b> by solving problems involving ordering or comparing rational numbers, common irrational numbers (e.g., <math>\sqrt{2}</math>, <math>\pi</math>), rational bases with integer exponents, square roots, absolute values, integers, or numbers represented in scientific notation using number lines or equality and inequality symbols.</p> </div>	<p>M(N&amp;O)–AM–2 No standard at this level</p>
<p>M(N&amp;O)–8–3 No standard at this grade</p>	<p>M(N&amp;O)–HS–3 No standard at this level</p>	<p>M(N&amp;O)–AM–3 No standard at this level</p>

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<b>Number and Operations Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(N&amp;O)–8–4 <b>Accurately solves problems involving</b> proportional reasoning (<u>percent increase or decrease</u>, <u>interest rates</u>, <u>markups</u>, or rates); multiplication or division of integers; and <u>squares</u>, <u>cubes</u>, and <u>taking square or cube roots</u>.</p> <p>(IMPORTANT: <i>Applies the conventions of order of operations.</i>)</p>	<p>M(N&amp;O)–HS–4 <b>Accurately solves problems.</b></p> <ul style="list-style-type: none"> <li>• Interprets and computes with rational exponents and their relation to radicals, by hand in simple cases (e.g., <math>4^{3/2}</math>), and using a calculator when appropriate.</li> <li>• Interprets and computes in scientific notation with and without a calculator.</li> <li>• Solves compound interest problems using <math>A = P\left(1 + \frac{r}{n}\right)^m</math>, where <math>n</math> is finite.</li> </ul> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(N&amp;O)–10–4 <b>Accurately solves problems involving</b> rational numbers within mathematics, across content strands, disciplines or contexts (with emphasis on, but not limited to, proportions, percents, ratios, and rates).</p> </div>	<p>M(N&amp;O)–AM–4 <b>Accurately solves problems.</b></p> <ul style="list-style-type: none"> <li>• Solves problems involving scientific notation and assesses the precision of the answer in terms of significant digits.</li> <li>• Solves problems involving complex numbers by adding, multiplying, dividing, and computing conjugates; interprets complex numbers geometrically; and understands complex numbers as an extension of the real numbers (e.g., arising in solutions of polynomial equations).</li> <li>• Manipulates complex numbers using rectangular and polar coordinates.</li> <li>• Finds the <math>n</math>th roots of a complex number using De Moivre’s Theorem.</li> <li>• Solves compound interest problems including continuously compounded interest.</li> </ul>
M(N&O)–8–5 No standard at this grade	M(N&O)–HS–5 No standard at this level	M(N&O)–AM–5 No standard at this level
<p>M(N&amp;O)–8–6 <b>Uses a variety of mental computation strategies to solve problems</b> (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to <b>determine the reasonableness of answers</b>; <b>mentally calculates</b> benchmark perfect squares and related square roots (e.g., <math>1^2</math>, <math>2^2</math>, ... , <math>12^2</math>, <math>15^2</math>, <math>20^2</math>, <math>25^2</math>, <math>100^2</math>, <math>1000^2</math>); determines the part of a number using benchmark percents and related fractions (1%, 10%, 25%, <math>33\frac{1}{3}\%</math>, 50%, <math>66\frac{2}{3}\%</math>, 75%, and <u>100%</u>) (e.g., 25% of 16; <math>33\frac{1}{3}\%</math> of 330).</p> <p>(IMPORTANT: <i>The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>	<p>M(N&amp;O)–HS–6 <b>Uses a variety of mental computation strategies to solve problems</b> (e.g., using compatible numbers, applying properties of operations, using mental imagery, using patterns) and to <b>determine the reasonableness of answers.</b></p> <p>(IMPORTANT: <i>The intent of this GSE is to embed mental arithmetic throughout the instructional program, not to teach it as a separate unit.</i>)</p>	<p>M(N&amp;O)–AM–6 No standard at this level</p>

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<b>Number and Operations Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(N&amp;O)–8–7 <b>Makes estimates</b> in a given situation (including tips, discounts, tax, and the value of a non-perfect square root as between two whole numbers) by identifying when estimation is appropriate, selecting the appropriate method of estimation; determining the level of accuracy needed given the situation; analyzing the effect of the estimation method on the accuracy of results; and evaluating the reasonableness of solutions appropriate to <u>grade level GLEs</u> across content strands.</p> <p>(IMPORTANT: <i>The intent of this GLE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>	<p>M(N&amp;O)–HS–7 <b>Makes estimates</b> in a given situation (e.g., tips, discounts, tax, the value of a non-perfect square root or cube root) by identifying when estimation is appropriate, selecting the appropriate method of estimation; determining the level of accuracy needed given the situation; analyzing the effect of the estimation method on the accuracy of results; evaluating the reasonableness of solutions appropriate to GSEs across content strands.</p> <p>(IMPORTANT: <i>The intent of this GSE is to embed estimation throughout the instructional program, not to teach it as a separate unit.</i>)</p>	<p>M(N&amp;O)–AM–7 No standard at this level</p>
<p>M(N&amp;O)–8–8 <b>Applies properties of numbers</b> (odd, even, remainders, divisibility, and <u>prime factorization</u>) and <b>field properties</b> (commutative, associative, identity [<u>including the multiplicative property of one, e.g., <math>2^0 \times 2^3 = 2^{0+3} = 2^3</math>, so <math>2^0 = 1</math></u>], distributive, inverses) <b>to solve problems and to simplify computations</b>, and <b>demonstrates conceptual understanding of field properties</b> as they apply to subsets of real numbers <u>when addition and multiplication are not defined in the traditional ways (e.g., If <math>a \Delta b = a + b - 1</math>, is <math>\Delta</math> a commutative operation?)</u></p>	<p>M(N&amp;O)–HS–8 <b>Applies properties of numbers and field properties</b> (including determining whether a given subset of numbers is closed under a given arithmetic operation) <b>to solve problems or to simplify computations</b>; and <b>compares and contrasts the properties of numbers and number systems</b>; adds and multiplies numerical matrices with attention to the arithmetic properties of these operations.</p>	<p>M(N&amp;O)–AM–8 <b>Explores field properties</b> with vectors and matrices; <b>algebraically/geometrically interpret</b> vectors, vector addition, and scalar multiplication in the plane; and <b>solves problems</b> using matrices (e.g., inverses, identity, determinants, and augmented matrices).</p>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Geometry and Measurement</b>		
<p><b>Purpose:</b> Geometry and the related area of measurement help students represent, describe, and make sense of the world in which they live. Geometry is also a natural place for students to develop their reasoning and justification skills.</p> <p>We live in a three-dimensional world. To interpret, understand, and appreciate that world, students need to develop an understanding of space. In addition, success in mathematics depends, in part, on the development of spatial abilities. Spatial skills include making and interpreting drawings, forming mental images, and visualizing changes.</p> <p>Measurement is the process of assigning a numerical value to an attribute of an object. The study of measurement provides students with techniques and tools they will need to describe and analyze their world. It also provides an opportunity to make connections within mathematics and between mathematics and other curricular areas. High school students must develop more mature insights into the essential role of measurement as a link between the abstractness of mathematics and the concreteness of the real-world.</p> <p>In both areas, geometry and measurement, students need to investigate, experiment, and explore geometric properties using both technology and hands-on materials.</p>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
M(G&M)–8–1 <b>Uses properties of angle</b> relationships resulting from two or three intersecting lines (adjacent angles, vertical angles, straight angles, or angle relationships formed by two non-parallel lines cut by a transversal), or two parallel lines cut by a transversal to solve problems.	M(G&M)–HS–1 No standard at this level	M(G&M)–AM–1 No standard at this level
M(G&M)–8–2 <u>Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations.</u>	<p>M(G&amp;M)–HS–2 <b>Creates formal proofs</b> of propositions (e.g., angles, lines, circles, distance, midpoint and polygons including triangle congruence and similarity).</p> <p><i>(IMPORTANT: It is the intent that students are creating formal proofs as articulated in the process standards, independent of the topic being studied. Furthermore, students should not be limited to any particular method of proof, but rather use a variety of strategies and those that work best for them. Some topics may be treated more formally than others.)</i></p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(G&amp;M)–10–2 <b>Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems</b> involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem).</p> </div>	M(G&M)–AM–2 <b>Extends and deepens knowledge and usage of proofs and proof techniques;</b> and uses geometric models to represent and distinguish between Euclidean and Non-Euclidean Systems.
M(G&M)–8–3 No standard at this grade	M(G&M)–HS–3 No standard at this level	M(G&M)–AM–3 No standard at this level

New Hampshire High School Mathematics Grade-Span Expectations

<b>Geometry and Measurement Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
M(G&M)–8–4 No standard at this grade	<p>M(G&amp;M)–HS–4 <b>Applies the concepts of congruency</b> by using matrices to represent reflections, translations, and rotations.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(G&amp;M)–10–4 <b>Applies the concepts of congruency by solving problems</b> on or off a coordinate plane involving reflections, translations, or rotations; or solves problems using congruency involving problems within mathematics or across disciplines or contexts.</p> </div>	M(G&M)–AM–4 No standard at this level
M(G&M)–8–5 <b>Applies concepts of similarity</b> to determine the <u>impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor</u> ; to <u>determine the length of sides of similar triangles</u> , or to <u>solve problems involving growth and rate</u> .	<p>M(G&amp;M)–HS–5 <b>Applies concepts of similarity</b> to define the trigonometric functions as ratios of sides of right triangles; <b>uses the ratios of the sides of special right triangles</b> (30° - 60° - 90° and 45° - 45° - 90°) to determine the sine, cosine and tangent of 30°, 45°, and 60°; and solves related problems.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(G&amp;M)–10–5 <b>Applies concepts of similarity by solving problems</b> within mathematics or across disciplines or contexts.</p> </div>	M(G&M)–AM–5 No standard at this level
M(G&M)–8–6 <b>Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume</b> of rectangular prisms, triangular prisms, <u>cylinders</u> , <u>pyramids</u> , or <u>cones</u> . Expresses all measures using appropriate units.	<p>M(G&amp;M)–HS–6 <b>Applies trigonometric formulas</b> (e.g., Law of Sines, Law of Cosines, <math>A = \frac{1}{2} ab \sin C</math>) to find angles, lengths and areas of polygons.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(G&amp;M)–10–6 <b>Solves problems involving perimeter, circumference, or area</b> of two-dimensional figures (including composite figures) or <b>surface area or volume</b> of three-dimensional figures (including composite figures) within mathematics or across disciplines or contexts.</p> </div>	M(G&M)–AM–6 <b>Derives and uses formulas for lengths of arcs and areas of sectors and areas of segments of circles.</b>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Geometry and Measurement Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
M(G&M)–8–7 No standard at this grade	<p>M(G&amp;M)–HS–7 <b>Applies informal concepts of successive approximation, upper and lower bounds, and limits in measurement situations</b> (e.g., use successive approximation to find the area of a pond); and <b>uses measurement conversion strategies</b> (e.g., unit/dimensional analysis).</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(G&amp;M)–10–7 <b>Uses units of measure appropriately and consistently when solving problems across content strands; makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations</b> involving measurement in other GSEs.</p> </div>	M(G&M)–AM–7 <b>Uses radian measure appropriately</b> when solving problems; <b>converts</b> between radian measure and degree measure; and understands why radian measure is useful.
M(G&M)–8–8 No standard at this grade	M(G&M)–HS–8 No standard at this level	M(G&M)–AM–8 No standard at this level
M(G&M)–8–9 No standard at this grade	<div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(G&amp;M)–10–9 <b>Solves problems on and off the coordinate plane</b> involving distance, midpoint, perpendicular and parallel lines, or slope.</p> </div>	M(G&M)–AM–9 <b>Solves problems using analytic geometry</b> (including three-dimensions) and <b>circular trigonometry</b> (e.g., find the equation of a circle inscribed in a triangle; find the distance between opposite vertices in a rectangular solid); <b>explores and interprets the characteristics of conic sections graphically and algebraically</b> including understanding how different planar slices of a double cone yield different conic sections; knows the characterization of conic sections as loci of points in the plane satisfying certain distance requirements, and uses the distance formula to obtain equations for the conic sections.
M(G&M)–8–10 No standard at this grade	M(G&M)–HS–10 <b>Demonstrates conceptual understanding of spatial reasoning and visualization</b> by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three-dimensional objects, and by solving related problems; perform and justify constructions with a compass and straightedge or dynamic geometric software.	M(G&M)–AM–10 No standard at this level

New Hampshire High School Mathematics Grade-Span Expectations

<b>Functions and Algebra</b>		
<p><b>Purpose:</b> Algebra is the language through which much of mathematics is communicated. Students in Kindergarten begin to explore algebraic concepts using informal representations (e.g., words, physical models, tables, graphs). In later years students progress to more abstract representations. The study of patterns is one of the central themes of algebraic thinking and leads to an understanding of relations and functions. Students at all grade-levels should recognize, describe, and generalize patterns and build mathematical models to describe, interpret, and predict the behavior of real-world phenomenon. Algebraic processes are important tools that students can use throughout their lives.</p>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(F&amp;A)–8–1 <b>Identifies and extends to specific cases a variety of patterns</b> (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; <b>and generalizes</b> a linear relationship (non-recursive explicit equation); generalizes a linear relationship to find a specific case; <b>generalizes a nonlinear relationship using words or<sup>sc</sup> symbols</b>; or <b>generalizes a common nonlinear relationship to find a specific case</b>.</p>	<p>M(F&amp;A)–HS–1 <b>Identifies arithmetic and geometric sequences</b> and finds the <math>n</math>th term; then <b>uses the generalization</b> to find a specific term.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(F&amp;A)–10–1 <b>Identifies, extends, and generalizes a variety of patterns</b> (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations.</p> </div>	<p>M(F&amp;A)–AM–1 Computes partial sums of infinite arithmetic and geometric sequences, determines when an infinite geometric series converges, and finds its sum. Connects arithmetic and geometric sequences to linear and exponential functions, respectively. Works between recursive and explicit representations.</p>

<b>Functions and Algebra Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(F&amp;A)–8–2 <b>Demonstrates conceptual understanding of linear relationships</b> (<math>y = kx</math>; <math>y = mx + b</math>) <b>as a constant rate of change</b> by solving problems involving the relationship between slope and rate of change; <u>informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations</u>; or <u>describing the meaning of slope and intercept in context</u>; <b>and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change)</b> represented in tables, graphs, equations, or problem situations; or <b>describes how change in the value of one variable relates to change in the value of a second variable</b> in problem situations with constant and <u>varying rates</u> of change.</p>	<p>M(F&amp;A)–HS–2 <b>Demonstrates conceptual understanding of linear and nonlinear functions and relations.</b></p> <ul style="list-style-type: none"> <li>• Analyzes characteristics of classes of functions (polynomial, rational, and exponential) to include domain, range, intercepts, increasing and decreasing intervals and rates of change.</li> <li>• Understands one-to-one (injective) functions and that a function that is one-to-one has a converse that is also a function; and finds inverses algebraically and graphically.</li> <li>• Graphs polynomial, rational and exponential functions, including vertical and horizontal shifts, stretches, and compressions as well as reflections across vertical and horizontal axes.</li> <li>• Applies knowledge of functions to interpret and understand situations, design mathematical models, and solve problems in mathematics as well as in the natural and social sciences.</li> </ul> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(F&amp;A)–10–2 <b>Demonstrates conceptual understanding of linear and nonlinear functions and relations</b> (including characteristics of classes of functions) through an analysis of constant, variable, or average rates of change, intercepts, domain, range, maximum and minimum values, increasing and decreasing intervals and rates of change (e.g., the height is increasing at a decreasing rate); describes how change in the value of one variable relates to change in the value of a second variable; or works between and among different representations of functions and relations (e.g., graphs, tables, equations, function notation).</p> </div>	<p>M(F&amp;A)–AM–2 <b>Demonstrates conceptual understanding of linear and nonlinear functions and relations.</b></p> <ul style="list-style-type: none"> <li>• Understands functions and relations from a set-theoretic perspective, and operations on functions including composition.</li> <li>• Analyzes characteristics of classes of functions and inverse functions (exponential, logarithmic, trigonometric) to include domain, range, intercepts, increasing and decreasing intervals and rates of change, periodicity, end behavior, maximum and minimum values, continuity, and asymptotes; graphs classes of functions; and understands domain restrictions and their effects on functions.</li> <li>• Analyzes properties of functions including onto (surjectivity), critical points and inflection points. Determine graphically and analytically whether a function is even, odd or neither.</li> <li>• Analyzes informally the idea of continuity and limits.</li> </ul>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Functions and Algebra Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(F&amp;A)–8–3 <b>Demonstrates conceptual understanding of algebraic expressions</b> by evaluating and <u>simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers)</u>; or by evaluating an expression within an equation (e.g., determine the value of <math>y</math> when <math>x = 4</math> given <math>y = 7\sqrt{x} + 2x</math>).</p>	<p>M(F&amp;A)–HS–3 <b>Demonstrates conceptual understanding of algebraic expressions.</b></p> <ul style="list-style-type: none"> <li>• Manipulates, evaluates, and simplifies algebraic and numerical expressions.</li> <li>• Adds, subtracts, multiplies and divides polynomials and rational expressions.</li> <li>• Factors quadratic and higher degree polynomials.</li> <li>• Understands properties of logarithms and converts between logarithmic and exponential forms.</li> <li>• Manipulates, evaluates, and simplifies expressions involving rational exponents and radicals and converts between expressions with rational exponents and expressions with radicals.</li> <li>• Understands the effect of simplifying rational expressions on the domain of the related functions (e.g., <math>x^2/x = x</math> for <math>x \neq 0</math>).</li> </ul> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(F&amp;A)–10–3 <b>Demonstrates conceptual understanding of algebraic expressions</b> by solving problems involving algebraic expressions, by simplifying expressions (e.g., simplifying polynomial or rational expressions, or expressions involving integer exponents, square roots, or absolute values), by evaluating expressions, or by translating problem situations into algebraic expressions.</p> </div>	<p>M(F&amp;A)–AM–3 <b>Demonstrates conceptual understanding of algebraic expressions.</b></p> <ul style="list-style-type: none"> <li>• Simplifies complex fractions.</li> <li>• Uses the Remainder Theorem, the Factor Theorem and Rational Root Theorem for polynomials.</li> <li>• Knows the Fundamental Theorem of Algebra and that non-constant polynomials always factor into linear factors over the complex numbers.</li> <li>• Understands the difference between factoring polynomials over integer, rational, real, and complex numbers.</li> </ul>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Functions and Algebra Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(F&amp;A)–8–4 <b>Demonstrates conceptual understanding of equality</b> by showing equivalence between two expressions (<u>expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level</u>) using models or different representations of the expressions, <u>solving formulas for a variable requiring one transformation (e.g., <math>d = rt</math>; <math>d/r = t</math>)</u>; by <u>solving multi-step linear equations with integer coefficients</u>; by <u>showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution</u>; and by <u>informally solving problems involving systems of linear equations in a context</u>.</p>	<p>M(F&amp;A)–HS–4 <b>Demonstrates conceptual understanding of equality.</b></p> <ul style="list-style-type: none"> <li>• Factors, completes the square, uses the quadratic formula, and graphs quadratic functions to solve quadratic equations.</li> <li>• Solves equations involving polynomial, rational, and radical expressions. Graphs and interprets the solutions.</li> <li>• Understands extraneous solutions.</li> <li>• Finds approximate solutions to equations by graphing each side as a function using technology. Understands that any equation in <math>x</math> can be interpreted as the equation <math>f(x) = g(x)</math> and interpret the solutions of the equation as the <math>x</math>-value(s) of the intersection point(s) of the graphs of <math>y = f(x)</math> and <math>y = g(x)</math>.</li> <li>• Solves 2x2 and 3x3 systems of linear equations and graphically interprets the solutions.</li> <li>• Solves systems of linear and quadratic inequalities.</li> <li>• Solves systems of equations involving nonlinear expressions and graphically interprets the solutions.</li> <li>• Translates problem situations into inequalities; and solves linear and non-linear inequalities (symbolically and graphically).</li> </ul> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(F&amp;A)–10–4 <b>Demonstrates conceptual understanding of equality</b> by solving problems involving algebraic reasoning about equality; by translating problem situations into equations; by solving linear equations (symbolically and graphically) and expressing the solution set symbolically or graphically, or provides the meaning of the graphical interpretations of solution(s) in problem-solving situations; or by solving problems involving systems of linear equations in a context (using equations or graphs) or using models or representations.</p> </div>	<p>M(F&amp;A)–AM–4 <b>Demonstrates conceptual understanding of equality.</b></p> <ul style="list-style-type: none"> <li>• Solves equations and verifies/proves identities involving trigonometric expressions.</li> <li>• Solves equations involving exponential and logarithmic expressions; graphs and interprets the solutions.</li> <li>• Uses matrices or determinants to solve systems of equations with or without technology.</li> <li>• Knows and applies the Intermediate Value Theorem to find exact or approximate solutions of equations or zeros of continuous functions.</li> </ul>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Data, Statistics, and Probability</b>		
<p><b>Purpose:</b> Collecting, organizing, and displaying data, as well as interpreting and analyzing the information to make decisions and predictions, have become very important in our society. Statistical instruction should be carried out in a spirit of investigation and exploration so students can answer and formulate questions about data. Probability should be studied in familiar contexts. Students need to investigate fairness, chances of winning, and uncertainty. Technology should be used as a tool throughout the investigation process.</p>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(DSP)–8–1 <b>Interprets a given representation</b> (line graphs, scatter plots, histograms, or box-and-whisker plots) to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–8–2.)</p>	<p>M(DSP)–HS–1 <b>Interprets a given representation(s)</b> (e.g., regression function including linear, quadratic, and exponential) to analyze the data to make inferences and to formulate, justify, and critique conclusions.</p> <p>(IMPORTANT: Analyze data consistent with concepts and skills in M(DSP)–HS–2).</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(DSP)–10–1 <b>Interprets a given representation(s)</b> (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g., media, workplace, social and environmental situations).</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–10–2.)</p> </div>	<p>M(DSP)–AM–1 No standard at this level</p>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Data, Statistics, and Probability Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(DSP)–8–2 <b>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using</b> measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, <u>quartile values</u>, or <u>estimated line of best fit</u> to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, <u>random</u>, or <u>non-random</u>).</p>	<p>M(DSP)–HS–2 <b>Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using</b> measures of dispersion (standard deviation, variance, and percentiles).</p> <div style="border: 2px solid black; padding: 5px;"> <p>M(DSP)–10–2 <b>Analyzes patterns, trends, or distributions in data in a variety of contexts</b> by determining, using, or analyzing measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, estimated line of best fit, regression line, or correlation (strong positive, strong negative, or no correlation) to solve problems; and solve problems involving conceptual understanding of the sample from which the statistics were developed.</p> </div>	<p>M(DSP)–AM–2 <b>Analyzes and interprets measures of dispersion</b> (standard deviation, variance, and percentiles) and <b>central tendency</b> for the normal distribution; and <b>interprets</b> the correlation coefficient and the coefficient of determination in the context of data.</p>
<p>M(DSP)–8–3 <b>Organizes and displays data using</b> scatter plots to answer questions related to the data, to analyze the data to formulate or justify conclusions, to make predictions, or to solve problems; or identifies representations or elements of representations that best display a given set of data or situation, consistent with the representations required in <u>M(DSP)–8–1</u>.</p> <p><i>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–8–2.)</i></p>	<p>M(DSP)–HS–3 <b>Organizes and displays one- and two-variable data using a variety of representations</b> (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts, linear, quadratic, and exponential regression functions) to analyze the data to formulate or justify conclusions, make predictions, or to solve problems with or without using technology.</p> <div style="border: 2px solid black; padding: 5px;"> <p>M(DSP)–10–3 <b>Identifies or describes representations or elements of representations that best display a given set of data or situation</b>, consistent with the representations required in M(DSP)–10–1.</p> </div>	<p>M(DSP)–AM–3 <b>Uses technology to explore</b> the method of least squares and median-median for linear regression.</p>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Data, Statistics, and Probability Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(DSP)–8–4 <b>Uses counting techniques to solve problems</b> in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or<sup>sc</sup> others).</p>	<p>M(DSP)–HS–4 <b>Uses counting techniques to solve problems</b> in context involving combination or permutations using a variety of strategies (e.g., <math>nCr</math>, <math>nPr</math>, or <math>n!</math>); and finds unions, intersections, and complements of sets.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(DSP)–10–4 <b>Uses counting techniques to solve problems</b> in context involving combinations or permutations using a variety of strategies (e.g., organized lists, tables, tree diagrams, models, Fundamental Counting Principle, or<sup>sc</sup> others).</p> </div>	<p>M(DSP)–AM–4 No standard at this level</p>
<p>M(DSP)–8–5 <b>For a probability event in which the sample space may or may not contain equally likely outcomes, determines</b> the experimental or theoretical probability of an event in a problem-solving situation; and <b>predicts</b> the theoretical probability of an event and tests the prediction through experiments and simulations; compares and contrasts theoretical and experimental probabilities; finds the odds of an event and understands the relationship between probability and odds.</p>	<p>M(DSP)–HS–5 <b>For a probability event in which the sample space may or may not contain equally likely outcomes, predicts</b> the theoretical probability of an event and tests the prediction through experiments and simulations; compares and contrasts theoretical and experimental probabilities; finds the odds of an event and understands the relationship between probability and odds.</p> <div style="border: 2px solid black; padding: 5px; margin-top: 10px;"> <p>M(DSP)–10–5 <b>Solves problems involving experimental or theoretical probability.</b></p> </div>	<p>M(DSP)–AM–5 <b>Solves probability problems</b> (e.g., by applying concepts of counting, random variables, independence/dependence of events, and conditional probability).</p>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Data, Statistics, and Probability Continued...</b>		
<b>Grade 8</b>	<b>High School</b>	<b>Advanced Mathematics</b>
<p>M(DSP)–8–6 <b>In response to a teacher or student generated question or hypothesis</b> decides the most effective method (e.g., survey, observation, experimentation) to collect the data (numerical or categorical) necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the question or hypothesis being tested while <u>considering the limitations that could affect interpretations</u>; and when appropriate makes predictions; and asks new questions and makes connections to real-world situations.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–8–2.)</p>	<p>M(DSP)–HS–6 <b>In response to a teacher or student generated question or hypothesis</b> decides the most effective method (e.g., survey, observation, research, experimentation) and sampling techniques (e.g., random sample, stratified random sample) to collect the data necessary to answer the question; collects, organizes, and appropriately displays the data; analyzes the data to draw conclusions about the questions or hypotheses being tested while considering the limitations of the data that could effect interpretations; and when appropriate makes predications, asks new questions, or makes connections to real-world situations.</p> <p>(IMPORTANT: Analyzes data consistent with concepts and skills in M(DSP)–10–2.)</p>	<p>M(DSP)–AM–6 No standard at this level</p>

## New Hampshire High School Mathematics Grade-Span Expectations

Since it is crucial that process standards (problem-solving, reasoning, proof, communication, connections, and representations) are not seen as completely separate from content standards, the process standards have been imbedded throughout the content strands (e.g., M(F&A)–10–1 **Identifies, extends, and generalizes a variety of patterns** (linear and nonlinear) represented by models, tables, sequences, or graphs in problem solving situations). This mirrors classroom instruction in most classes. As students are learning content knowledge, instruction is focusing on improving their abilities in problem solving, reasoning, and communication. Students should be looking for and making appropriate connections, and should be able to understand and use multiple representations of mathematical ideas. Since it is crucial that students are strong in both content and process knowledge, we have included two local process strands following the content strands. These process strands are in addition to the process standards that are embedded in the content standards and are included to help guide local curriculum, assessment, and instruction. The process standards have been separated by grade-spans (K–2, 3–5, 6–8, and 9–12). The 6–8 and 9–12 spans have been included in this document. Each grade span should be thought of as building upon the skills and concepts in the previous grade span.

<b>Problem Solving, Reasoning, and Proof</b>	
<p><b>Purpose:</b> Problem solving should serve as the organizing feature of the mathematics curriculum (as well as other areas of study) and be applied to everyday activities. Thus, problem solving approaches should be used to investigate and understand new mathematical content, with students working sometimes independently and sometimes in groups. Students should have many experiences in posing and solving problems from their world using data that are meaningful to them and performing mathematical investigations. Students should build a positive disposition toward problem solving, including the confidence needed to explore unique problems and increasingly complex tasks.</p>	
6 – 8	High School
<p>M(PRP)–8–1 <b>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Use problem-solving strategies appropriately and effectively for a given situation.</li> <li>• Determine, collect and organize the relevant information needed to solve real-world problems.</li> <li>• Apply integrated problem-solving strategies to solve problems in the physical, natural and social sciences, and in pure mathematics.</li> <li>• Use technology when appropriate to solve problems.</li> <li>• Reflect on solutions and the problem-solving process for a given situation and refine strategies as needed.</li> </ul>	<p>M(PRP)–HS–1 <b>Students will use problem-solving strategies to investigate and understand increasingly complex mathematical content</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Expand the repertoire of problem-solving strategies and use those strategies in more sophisticated ways.</li> <li>• Use technology whenever appropriate to solve real-world problems (e.g., personal finance, wages, banking and credit, home improvement problems, measurement, taxes, business situations, purchasing, and transportation).</li> <li>• Formulate and redefine problem situations as needed to arrive at appropriate conclusions.</li> </ul>
<p>M(PRP)–8–2 <b>Students will use mathematical reasoning and proof</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Draw logical conclusions and make generalizations using deductive and inductive reasoning.</li> <li>• Formulate, test, and justify mathematical conjectures and arguments.</li> <li>• Construct and determine the validity of a mathematical argument or a solution.</li> <li>• Apply mathematical reasoning skills in other disciplines.</li> </ul>	<p>M(PRP)–HS–2 <b>Students will use mathematical reasoning and proof</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Expand the repertoire of proof techniques and use those techniques in more sophisticated ways.</li> <li>• Use informal and formal reasoning and proof to explain and justify conclusions.</li> <li>• Formalize mathematical arguments through the use of deductive reasoning.</li> <li>• Use the principle of mathematical induction.</li> <li>• Use reasoning and proof throughout classroom discussions independent of the mathematical topic being studied.</li> <li>• Recognize how reasoning and proof influence the structure of mathematics.</li> </ul>

**Communication, Connections, and Representations**

**Purpose:** Reading, writing, talking, listening, and modeling provide students with the opportunity to develop deeper mathematical understanding and to integrate the language of mathematics into their world. Actively exploring, investigating, describing, and explaining mathematical ideas promote communication which leads to a greater comprehension of mathematical concepts.

Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics. Representations make mathematical ideas more concrete and available for reflection, and they help students recognize the common mathematical nature of different situations. Students can develop and deepen their understanding of mathematical concepts and relationships as they create, compare, and use various representations.

Mathematical topics, ideas, and procedures must be connected to each other and to the students' everyday experiences, both in and out of school. In particular, mathematics must be connected to all other curriculum areas. Mathematical connections will help students become aware of the usefulness of mathematics, serve to bridge the concrete and the abstract, and enable deeper understanding of important ideas.

6 – 8	High School
<p>M(CCR)–8–1 <b>Students will communicate their understanding of mathematics</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Articulate ideas clearly and logically in both written and oral form.</li> <li>• Present, share, explain, and justify thinking with others and build upon the ideas of others to solve problems.</li> <li>• Use mathematical symbols and notation.</li> <li>• Formulate questions, conjectures, definitions, and generalizations about data, information, and problem situations.</li> </ul>	<p>M(CCR)–HS–1 <b>Students will communicate their understanding of mathematics</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Explain and justify their thinking and develop increasingly sophisticated questions for given problem-situations.</li> <li>• Critique and follow the logic of arguments presented within mathematics and across disciplines.</li> </ul>
<p>M(CCR)–8–2 <b>Students will create and use representations to communicate mathematical ideas and to solve problems</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Use models and technology to develop equivalent representations of the same mathematical concept.</li> <li>• Use and create representations to solve problems and organize their thoughts and ideas.</li> <li>• Convert between representations (e.g., a table of values, an equation, and a graph may all be representations of the same function).</li> </ul>	<p>M(CCR)–HS–2 <b>Students will create and use representations to communicate mathematical ideas and to solve problems</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Choose appropriate representations and mathematical language (e.g., spreadsheets, geometric models, algebraic symbols, tables, graphs, matrices) to present ideas clearly and logically for a given situation.</li> <li>• See a common structure in mathematical phenomena that come from very different contexts (e.g., the sum of the first <math>n</math> odd natural numbers, the areas of square gardens, and the distance traveled by a vehicle that starts at rest and accelerates at a constant rate can be represented by functions of the form <math>f(x) = ax^2</math>).</li> <li>• Find representations that model essential features of a mathematical situation (e.g., cost of postage can be modeled by a step-function).</li> <li>• Use representations as a primary means for expressing and understanding more abstract mathematical concepts.</li> </ul>

New Hampshire High School Mathematics Grade-Span Expectations

<b>Communication, Connections, and Representations Continued...</b>	
<b>6 – 8</b>	<b>High School</b>
<p>M(CCR)–8–3 <b>Students will recognize, explore, and develop mathematical connections</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Connect new mathematical ideas to those already studied and build upon them.</li> <li>• Understand that many real-world applications require an understanding of mathematical concepts (e.g., personal finance, running a business, building a house, following a recipe, or sending a rocket to the moon).</li> <li>• Explain in oral and written form the relationships between a real-world problem and an appropriate mathematical model.</li> <li>• Explain in oral and written form the relationships among various mathematical concepts (e.g., the relationship between exponentiation and multiplication).</li> </ul>	<p>M(CCR)–HS–3 <b>Students will recognize, explore, and develop mathematical connections</b> and be able to:</p> <ul style="list-style-type: none"> <li>• Explain in oral or written form how mathematics connects to other disciplines, to daily life, careers, and society (e.g., geometry in art and literature, data analysis in social studies, and exponential growth in finance).</li> <li>• Explain multiple approaches that lead to equivalent results when solving problems.</li> </ul>

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