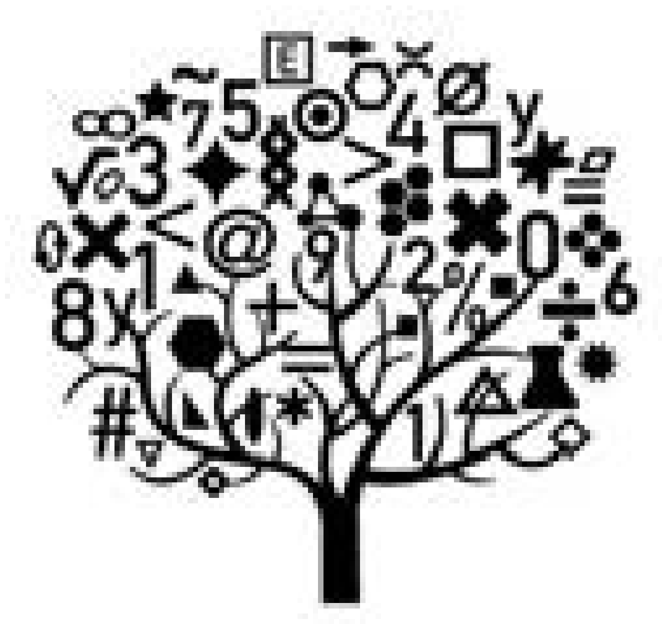
The New Hampshire Adult Education Mathematics Guidebook

A Guide for Teachers Based on College Career Readiness Standards for Adult Education with Examples, Illustrations, and Resources



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NH Adult Education Mathematics Guidebook Contents

Introduction

Helping our adult students become comfortable and competent with mathematics is more important now than in any other time in our country's history. To be competitive in the global economy, our citizens must have a higher level of math competence than they have had. It goes without saying that all our students should have the skills to handle the math they will encounter in their daily lives and on the job.

Unfortunately some of our students come to us with very weak math skills and poor conceptual knowledge of its applications. Still others suffer from math phobia or math anxiety never feeling as though they "got math" while they were in school. So our job to help them to become competent mathematically can be a challenging one. However we can give our students a solid foundation in mathematics while they are in our programs. They may even be the ones who go on to advanced manufacturing classes or earn promotions on the job. Even for our students who are not college bound or on the path to the high school equivalency test, math is ever-essential for the 21st century workforce and the 21st century adult education student. It is no exaggeration to say that acquiring numeracy is just as important as acquiring literacy for all our students.

The NH Math Leadership Team has written this guide to help teachers understand the scope and sequence necessary in the development of our students' basic math skills. The guide is based on Common Core Standards and the Career and College Readiness Standards for Adult Education; however, rather than being organized by grade level or five instructional levels as the aforementioned, this guide has been organized by topic to make it more useful in a typical adult education classroom. It has been compiled by experienced adult education teachers as well as math specialists who have practical knowledge of their students' strengths and weaknesses and ways of learning. It is our hope that this guide will be a valuable resource that you can turn to on a daily or weekly basis. Please take the time to study it carefully so you are familiar with all its contents. If you teach math, it may become one of your best friends!

How to Use this Guide

As you may have noticed from the table of contents, this guide covers eleven different math topics. Within each section you'll find the competencies for that topic. What makes this guide so useful for the adult education classroom where the range of students' math skills can differ widely within one class, is that embedded into every level are algebra and geometry concepts as well as life skills math competencies. This will enable you to teach these more advanced concepts as well as daily living math to every student in your class regardless of his/her incoming math skill set.

To help you understand each of your student's skill set, there is an assessment in each topic which will give you more information about your student's ability solving algorithms and word problems within that topic. In addition to the assessment, supplemental word problems are provided for each topic as well as skill-development worksheets and other resources. Finally, you'll find pages from McGraw Hill's *ETS Official Guide to the HiSET* to help students understand the way a competency may be addressed in the HiSET Exam.

As you dive into the guide, you'll also notice suggestions for using technology in your classroom and useful websites that generate limitless worksheets. At the end of the guide there is an appendix that includes a math glossary and other helpful information.

Learning math doesn't have to be scary for our students. If presented in a non-threatening and logical fashion, many students can improve their math competence in the time that they are with us. It is our hope that this guide will help you pave the way for a smooth journey for all of your students who are discovering that math is not only doable, but fun too!

I. Whole Numbers and Number Concepts

1. Understand Place Value

a. Understand place value to the billions
b. Read and write whole numbers using knowledge of place value
c. Write numbers in expanded form *(1,209 = 1,000 + 200 + 9)*
d. Compare and order whole numbers using =, ≠, <, >, ≤ and ≥
e. Use place value understanding to round whole numbers to the nearest ten, hundred, thousand…..trillions

2. Develop an Understanding of Integers

a. Use positive and negative numbers to represent real-world quantities such as temperature above and below 0, elevation above and below sea level, and credit/debit
b. Locate and place whole integers on a number line
c. Use a number line to identify opposites such as 5 and -5
d. Understand absolute value of an integer (distance from 0 on a number line)

3. Add and Subtract Whole Numbers

a. Use knowledge of place value to mentally add and subtract by 10, 100, 1000
b. Understand the relationship between addition and subtraction; they are inverse operations
c. Fluently add and subtract multi-digit numbers written both vertically and horizontally
d. Check accuracy of answers using inverse operation
e. Understand properties of addition and subtraction and create examples:
   - Commutative Property *2 + 5 = 5 + 2*
   - Associative Property *2 + 6 + 4 = 2 + (6 + 4) = 2 + 10 = 12 or*
     2 + 6 + 4 = (2 + 6) + 4 = 8 + 4 = 12
f. Use a calculator to add and subtract and to check the accuracy of computation
g. Develop algebraic thinking skills by working with addition and subtraction equations
   - Determine if equations involving addition and subtraction are true or false:
     Does 5 + 2 = 4 + 3, Does 10 +1 0 = 5 + 5 + 5, Is 10 + 5 > 20 – 6 ?
   - Determine the unknown number that makes an equation true
     If 8 + n = 12, what does n equal, If 20 – n = 5, what does n equal?

4. Solve Problems Using Addition and Subtraction

a. Understand and use the language of addition and subtraction *(sum, difference, total, combine, etc.)*
b. Solve "real-world" problems using addition and subtraction, one step and multi-step
c. Assess the reasonableness of answers using mental computation and estimation strategies including rounding
d. Use different strategies to solve problems: draw a picture, make a list, work backwards, look for a pattern, create a table, use a variable, write an equation, guess and check
e. Develop algebraic thinking skills by representing word problems using an equation with letters standing for unknown quantities
    - *Example: Tom needs $185.00 to buy the phone that he likes. He has already saved $125.00. How much more money does Tom need to buy the phone?*
      $125.00 + n = $185.00

5. Multiply and Divide Whole Numbers

a. Know the various ways that multiplication is represented: *5(2), 5•2, ab*
b. Fluently know basic multiplication facts or compensation techniques
c. Mentally multiply and divide by 10, 100, 1000
d. Multiply multi-digit whole numbers
e. Use a calculator to multiply multi-digit numbers and to check the accuracy of answers when not using a calculator
f. Understand the Commutative Property of Multiplication
    - *The product of two numbers is unaffected by the order in which they are multiplied, 5x2 = 2x5*
g. Understand the Distributive Property of Multiplication
    - *The sum of two numbers times a third number is equal to the sum of each addend times the third number: 2(3 + 4) = 2(3) + 2(4)*
h. Understand that division is the inverse of multiplication
i. Know basic division facts or compensation techniques, such as using a multiplication table
j. Know that division is most often represented as $\frac{5}{2}$ = (5÷2)
k. Divide numbers using short and/or long division

l. Divide numbers using a calculator and check the accuracy of answers using a calculator *(and understand the actual number remainder is different from the decimal conversion using a calculator, 5÷2=2R1 or 2.5, not 2R5 or 2.1)*
m. Develop algebraic thinking skills by working with multiplication and division equations
    - *Determine if equations involving multiplication and division are true or false: Does 5 x 4 = 20 ÷ 4? , Does 10 x 10 = 5 x 5 x 4?, Is 10 x 5 > 100 ÷ 4?, If n = 5 does 5n = 25?*
    - Determine the unknown number that makes an equation true: If *8 x n = 32, what does n equal?, If 20 ÷ n = 5, what does n equal?*

6. Solve Problems Using Multiplication and Division

a. Understand and use the language of multiplication and division *(product, quotient, dividend, divisor, factor, per, triple, etc.)*
b. Solve "real world problems" using multiplication and division, one step and multistep
c. Assess the reasonableness of answers using mental computation and estimation strategies including rounding
d. Use different strategies to solve problems: draw a picture, make a list, work backwards, look for a pattern, create a table, use a variable, write an equation, guess and check
e. Develop algebraic thinking skills by representing word problems using an equation with letters standing for unknown quantities
    - *Example: John owes $500.00 more dollars on his loan. He pays the same amount every month, and it will take him 4 more months to pay off the loan. How much does John pay each month? 4 x n = 500*

7. Develop Beginning Algebra Skills using Whole Numbers

a. Write and interpret simple expressions using whole numbers
b. Solve simple equations using whole numbers
c. Follow the Order of Operations
d. Write and evaluate numerical expressions using whole number exponents
    - *5^2 = 5 X 5 and is read 5 to the second power or 5 squared*
    - *5^3 = 5 X 5 X 5 and is read 5 to the third power or 5 cubed*
e. Calculate the square root of a number using perfect square roots
f. Be aware of patterns and extend repeating and growing patterns

***** *Refer to Algebra Section 11 for more information*

8. Develop Beginning Geometry Skills using Whole Numbers

a. Name and identify simple shapes: triangle, square, circle, rectangle
b. Find perimeter using whole numbers
c. Find area using whole numbers
d. When given the area or perimeter and the length of one or more sides of a square or rectangle, find the length of a missing side
e. Find the volume of a rectangular solid and cube using whole numbers

*****Refer to Geometry Section 10 for more information*

9. Develop Measurement and Data Skills Using Whole Numbers

a. Use a ruler and tape measure to measure *inches, feet, yards, centimeters, and meters*
b. Compare, contrast, and convert customary units of length: *inch, foot, yard, mile* and use to solve real life problems
c. Compare, contrast, and convert customary units of capacity: *ounces, cups, pints, quarts, gallons* and use to solve real life problems
d. Compare, contrast, and convert customary units of weight: *ounces, pounds, tons* and use to solve real life problems
e. Compare, contrast, and convert units of time: *seconds, minutes, hours, days, weeks, months, years* and use to solve real life problems
f. Solve elapsed time problems
   Example: If you start work at 8am and finish at 2pm, how many hours did you work? If it is10am now, what time will it be in 6 hours?
g. Become familiar with metric units of length: *millimeter, centimeter, meter, kilometer*
h. Become familiar with metric units of weight: *milligram, gram, kilogram*
i. Become familiar with metric units of liquid measure: *milliliter, liter*
j. Find mean, median and mode using whole numbers and apply to real life
k. Identify line graphs, bar graphs, scatter plots, pie charts, pictographs, and tables
l. Accurately "read" charts and graphs using whole numbers

*****Refer to Measurement Section 8 and Data Analysis Section 6 for more information*

Whole Numbers Assessment Part I

Name________________________________

Date_______________

Write each as a numeral.

1) seven billion, three hundred

2) ninety-two billion, one hundred

3) nine billion, three million

4) nine hundred billion, nine thousand, twenty

5) two billion, five hundred million, ninety-five

6) nine hundred sixty billion, fifty million, four thousand, six hundred

Round each to the place indicated.

7) 25,521,948; ten millions

8) 90; tens

9) 9,866,298; tens

10) 6,365,178,159; ten millions

11) 5,002,232; millions

12) 89,529,238; millions

Evaluate each expression.

13) $993 + 653 + 908$

14) $412 + 101 + 231$

15) $364 + 464 + 308$

16) $996 + 510 - 793$

Find each product.

17) $(7)(10)(3)$

18) $(43)(17)(24)$

19) $(3)(34)(15)$

20) $(49)(32)(13)$

Find each quotient.

21) $\frac{351}{13}$

22) $\frac{160}{10}$

23) $\frac{286}{26}$

24) $\frac{638}{29}$

PART II Using Whole Numbers:
Evaluate each expression.

25) $(6+1)(2+2)-\frac{10}{5}$

26) $6^2-\frac{8+12}{4}+6$

27) $(6+1)\left(1+\frac{16}{3+1}\right)$

28) $\frac{11-2}{(3)(1+5-5)}$

29) $\frac{(4+4)(2)}{4}-(6-5)$

30) $4+\frac{5-4+2}{2+1}$

Evaluate each using the values given.

31) $(3)(p+p)-q^2$; use $p=6$, and $q=3$

32) $j+j-\frac{3}{3}+k$; use $j=4$, and $k=1$

33) $\left(\frac{yz}{6}\right)(x-z)$; use $x=6$, $y=3$, and $z=4$

34) $2j-\left(3+\frac{h}{2}\right)$; use $h=2$, and $j=5$

35) $n+(n)(3-n+p)$; use $n=3$, and $p=2$

36) $mp-(m+3+m)$; use $m=4$, and $p=4$

Answers to

1) 7,000,000,300	2) 92,000,000,100	3) 9,003,000,000	4) 900,000,009,020
5) 2,500,000,095	6) 960,050,004,600	7) 30,000,000	8) 90
9) 9,866,300	10) 6,370,000,000	11) 5,000,000	12) 90,000,000
13) 2554	14) 744	15) 1136	16) 713
17) 210	18) 17544	19) 1530	20) 20384
21) 27	22) 16	23) 11	24) 22
25) 26	26) 37	27) 35	28) 3
29) 3	30) 5	31) 27	32) 8
33) 4	34) 6	35) 9	36) 5

Name_________________________ Date_________________

WHOLE NUMBER WORD PROBLEMS

1. Cait has driven 908 miles. She needs to drive 1,200 more miles. How many miles will Cait drive in all?
    1. 3,086
    2. 2,108
    3. 292
    4. 2,018

2. Cait needs to drive 870 miles to get home. She has already driven 599 miles. How many more miles does she need to drive to get home?
    1. 1,469
    2. 1,500
    3. 329
    4. 271

3. Jamie bought a used car for $5,500. She paid a $1,000.00 down payment. She will pay the balance in 12 equal payments over the year. How much will she pay each month?
    1. $4,400
    2. $375
    3. $458
    4. $360.

4. Peri needs to work 45 hours this week. She has already worked 8 hours on Monday, 6 hours on Tuesday, and 8 hours on Wednesday. How many more hours does she need to work this week?
    1. 23
    2. 18
    3. 25
    4. 27

5. A whole peach pie has 1200 calories. If 5 friends share the pie, how many calories will each person consume?
    1. 600
    2. 800
    3. 240
    4. 2400

6. There were 8,499 people at the baseball game. Round this number to the nearest thousand. ____________________

7. Cory bought $85.00 worth of groceries and $28.00 of gas. He started out with $150.00 in his wallet. How much does he have left? ______________________

8. Aggie ordered 10 boxes of pencils. Each box holds 4 dozen pencils. How many pencils will she receive in all?
   1. 40
   2. 500
   3. 480
   4. 48

9. Becky's gross pay is $320.00. $30.00 was taken out for state taxes and $15.50 was deducted for social security. What is her net pay?
   1. $35.50
   2. $365.50
   3. $45.50
   4. $274.50

10. Three friends went out to eat. The bill was $16.00 and the tax was $1.28. They split the cost equally 3 ways. How much did each friend pay?
    1. $17.28
    2. $5.76
    3. $5.30
    4. $8.65

11. A dozen donuts cost $4.50. Lisa bought 3 dozen donuts and 1 cup of coffee for $1.89. How much did she spend?
    1. $15.39
    2. $18.00
    3. $13.50
    4. $11.61

Whole Number Word Problem Answers:

1. 2
2. 4
3. 2
4. 23
5. 3
6. 8,000
7. 37
8. 3
9. 4
10. 2
11. 1

ETS

The Official Guide to the HiSET Exam

McGraw Hill Education

Whole Number and Operations Drills & Number Sense

1. **This week Laura worked 40 hours regular time and 3 hours overtime. If she earns $12.00 per hour regularly and gets 1½ times that for overtime, how much did Laura earn last week?**

   A $480
   B $516
   C $534
   D $552
   E $774

2. **Which of the following problems has an answer called a *product*?**

   A 12(8)
   B $25 - 17$
   C $\sqrt{64}$
   D 81/9
   E $15 + 13$

3. **Which of the following is equal to $144 \div (9 - 3)$?**
   - A $6 \times 2 + 1$
   - B $4 + 5 \times 4$
   - C $36 \div 2 - 2$
   - D $2 \times (12 + 4)$
   - E $36 \div (5 - 2)$

4. **Fay is buying a new washing machine, which is on sale for $1,479. She can either pay the entire amount at the time of purchase or make a down payment of $250 and make monthly payments of $125 for one year. How much more will it cost her to choose the payment plan than to pay the full sale price?**
   - A $21
   - B $102
   - C $229
   - D $271
   - E $3,021

5. **Evaluate the expression: $7 + 8(15 - 9)^2$**
   - A 8,100
   - B 2,311
   - C 540
   - D 295
   - E 70

6. **Charlie bought a new computer desk that was on clearance for $485 and a chair for $199. The desk was originally $350 more than the price he paid for it. What was the original price of the desk?**
   - A $835
   - B $684
   - C $450
   - D $334
   - E $135

Questions 7 and 8 are based on the following information.

The owners of an apartment complex took out a loan for $35,000 to remodel some of the units. They put new carpeting in 12 of the units for $975 each and bought new appliances for 8 of the units for $1,350 each. They also repainted the interior of 15 of the units.

7. **How much of the loan money do they have left after purchasing the carpeting and appliances?**
   - A $22,500
   - B $16,400
   - C $12,500
   - D $11,700
   - E $10,800

8. **The owners paid a total of $2,460 for paint. If the same amount of paint was used for each of the apartments, what was the cost of paint per unit?**
   - **A** $70.29
   - **B** $123
   - **C** $164
   - **D** $205
   - **E** $307.50

9. **On Friday, Saturday, and Sunday, a theater sold 665 tickets each day for a newly released movie. The theater sold 220 fewer tickets per day from Monday through Thursday. Which shows how to find the total number of tickets sold during the seven-day period?**
   - **A** $3 \times 665 + 4 \times 665 - 220$
   - **B** $3 \times 665 + 4(665 - 220)$
   - **C** $3 \times 665 + 4 \times 220$
   - **D** $7 \times (665 - 220)$
   - **E** $7 \times 665 - 220$

10. **A chef purchased 26 pounds of beef, 38 pounds of chicken, and 16 pounds of pork. He prepared an equal amount of meat for each of four dinner parties. Which explains how to find the number of pounds of meat he served at each party?**
    - **A** add 26, 38, and 16; then divide the total by 4
    - **B** add 26 and 38; then add 16 divided by 4
    - **C** add 26, 38, and 16; then subtract 4
    - **D** divide 26 and 38 by 4; then add 16
    - **E** multiply 26, 38, and 16 by 4

Answers are on page 723.

1. C 2. A 3. B 4. D 5. D 6. A 7. C 8. C 9. B 10. A

NUMBER SENSE DRILLS

For each question, choose the best answer.

1. **Which is the best estimate of the sum of 2,402 + 101,873 + 75,601?**
   - **A** 170,000
   - **B** 182,000
   - **C** 193,000
   - **D** 200,000
   - **E** 202,000

The following information will be used to answer questions 2 and 3.

The Earth's orbit around the sun is a distance of 92,956,050 miles.

2. **What is the value of the underlined digit?**

   $\mathbf{9\underline{2},956,050}$
   - **A** two
   - **B** two hundred
   - **C** two thousand
   - **D** two million
   - **E** two hundred million

3. **What is the distance of the Earth's orbit around the sun, rounded to the nearest hundred thousand?**

   A 90,000,000
   B 92,000,000
   C 93,000,000
   D 95,000,000
   E 1,000,000,000

The following information will be used to answer questions 4 and 5.

The following table shows the number of employees a company had each year between 2006 and 2010.

Year	*Employees*
2006	13
2007	39
2008	117
2009	351
2010	1,053

4. **If the pattern continues, how many employees will the company have in 2011?**

   A 1,404
   B 1,755
   C 2,106
   D 3,159
   E 9,477

5. **Which is the best estimate of how many employees the company will have in 2013 if the pattern continues?**

   A 10,000
   B 30,000
   C 50,000
   D 70,000
   E 90,000

6. **Which of the following is an example of a rational number that is an integer and a whole number?**

   A 47
   B 0
   C 3/8
   D 0.25
   E −19

7. **Which is equal to 15(24 − 18)?**
   - A $15 \times 24 - 15 \times 18$
   - B $18 \times 15 - 24 \times 15$
   - C $15 \times 24 - 18$
   - D $18(24 - 15)$
   - E $15(18 - 24)$

8. **The answer to (43,987 + 12,302) − 27,546 is between which of the following pairs of numbers?**
   - A 80,000 and 85,000
   - B 55,000 and 60,000
   - C 25,000 and 30,000
   - D 15,000 and 20,000
   - E 0 and 5,000

9. **Which accurately describe $\sqrt{95}$?**
   - A real and rational
   - B real and irrational
   - C rational and whole
   - D integer and rational
   - E natural and irrational

10. **Which is equal to (23 + 17) + 31?**
   - A $(23 + 31) + (17 + 31)$
   - B $23 + 17 + 31 + 17$
   - C $(23 + 17) \times 31$
   - D $23 + (31 + 17)$
   - E $31 + 17 \times 23$

Answers are on page 723.

1. B 2. D 3. C 4. D 5. B 6. A 7. A 8 C 9. B 10. D

Place Value Chart

Hundred Billions	Ten Billions	Billions		Hundred Millions	Ten Millions	Millions		Hundred Thousands	Ten Thousands	Thousands		Hundreds	Tens	Ones	decimal point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
2	1	0	,	9	8	7	,	6	5	4	,	3	2	1	•	2	3	4	5	6

This Chart shows the place value of the number 210,987,654,321.23456

This is how you say it.

Two hundred ten billion, nine hundred eighty seven million, six hundred fifty four thousand, three hundred twenty one, and twenty three thousand four hundred fifty six hundred thousandths.

Math-Aids.Com

OPERATION WORDS

add addition sum total plus more than in addition to increased by enlarged by exceeds by gain rose greater than combine together +	subtract subtraction minus take away decreased by less than diminished by reduced by How many more than? How many fewer than? How many left? fewer than difference dropped loss of −
multiply multiplied by times of by product at groups of each every ×	divided by into quotient per ratio parts dividend divisor distributed among How much for each? How many groups? fractions each every ÷

COMPARISON SYMBOLS

= equals
< is less than
> is greater than
≤ is less than or equal to
≥ is greater than or equal to

OPERATION WORDS

add addition sum total plus more than in addition to increased by enlarged by exceeds by gain rose greater than combine together +	subtract subtraction minus take away decreased by less than diminished by reduced by How many more than? How many fewer than? How many left? fewer than difference dropped loss of −
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COMPARISON SYMBOLS

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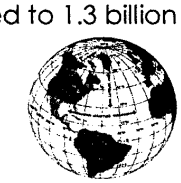
WORLD POPULATION MATH Lesson Ideas

Use the following World Population Chart to practice a variety of math skills with your students. You could use any interesting information that contains numbers in a similar way.

- Have students brainstorm a list of the 10 most populous countries on Earth and find them on a globe or map
- Have students estimate the population of the world, the United States or any country
- Have students read, write, and share their numbers
- Order the estimates
- Determine how far off, or how close, the estimates are
- Practice reading the large numbers once the chart is given out and discuss the information. How is expected population calculated?
- Round off the numbers to the nearest 100 million, million, etc.
- Use the population numbers to review place value:
  Switch the numbers in the thousands place and millions place in the population of China and read the new number. What number is in the ten-thousand place in the population of Iran?
- Write the population numbers in scientific notation
- Find the percent of change between 2000 populations and 2014 populations
- Create word problems such as: How many more people live in China than the USA?
- Graph the populations
- Discuss how large numbers are often rounded and written in words such as 1,330,141,295 may be rounded to 1.3 billion



WORLD POPULATION

TOP TEN COUNTRIES WITH THE HIGHEST POPULATION					
#	**Country**	**2000 Population**	**2010 Population**	**2014 Population**	**2050 Expected Pop.**
1	**China**	1,268,853,362	1,330,141,295	**1,355,692,576**	1,303,723,332
2	**India**	1,004,124,224	1,173,108,018	**1,236,344,631**	1,656,553,632
3	**United States**	282,338,631	310,232,863	**318,892,103**	439,010,253
4	**Indonesia**	213,829,469	242,968,342	**253,609,643**	313,020,847
5	**Brazil**	176,319,621	201,103,330	**202,656,788**	260,692,493
6	**Pakistan**	146,404,914	184,404,791	**196,174,380**	276,428,758
7	**Nigeria**	123,178,818	152,217,341	**177,155,754**	264,262,405
8	**Bangladesh**	130,406,594	156,118,464	**166,280,712**	233,587,279
9	**Russia**	146,709,971	139,390,205	**142,470,272**	109,187,353
10	**Japan**	126,729,223	126,804,433	**127,103,388**	93,673,826

Definitions for Properties of Mathematics

Associative Property of Addition

When three or more numbers are added, the sum is the same regardless of the grouping of the addends. For example $(a + b) + c = a + (b + c)$

Associative Property of Multiplication

When three or more numbers are multiplied, the product is the same regardless of the order of the multiplicands. For example $(a \times b) \times c = a \times (b \times c)$

Commutative Property of Addition

When two numbers are added, the sum is the same regardless of the order of the addends. For example $a + b = b + a$

Commutative Property of Multiplication

When two numbers are multiplied together, the product is the same regardless of the order of the multiplicands. For example $a \times b = b \times a$

Distributive Property

The sum of two numbers times a third number is equal to the sum of each addend times the third number. For example $a \times (b + c) = a \times b + a \times c$

Identity Property of Addition

The sum of any number and zero is the original number. For example $a + 0 = a$.

Identity Property of Multiplication

The product of any number and one is that number. For example $a \times 1 = a$.

Additive Inverse of a Number

The additive inverse of a number, a is -a so that $a + -a = 0$.

Multiplicative Inverse of a Number

The multiplicative inverse of a number, a is $\frac{1}{a}$ so that $a \times \frac{1}{a} = 1$.

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Definitions for Properties of Mathematics

Addition Property of Zero
Adding 0 to any number leaves it unchanged. For example a + 0 = a.

Multiplication Property of Zero
Multiplying any number by 0 yields 0. For example a x 0 = 0.

Property of Equality
The equals sign in an equation is like a scale: both sides, left and right, must be the same in order for the scale to stay in balance and the equation to be true.

Property of Equality for Addition
Property of Equality for Addition says that if a = b, then a + c = b + c.
If you add the same number to both sides of an equation, the equation is still true.

Property of Equality for Subtraction
Property of Equality for Subtraction says that if a = b, then a - c = b - c.
If you subtract the same number from both sides of an equation, the equation is still true.

Property of Equality for Multiplication
Property of Equality for Multiplication says that if a = b, then a x c = b x c.
If you multiply the same number to both sides of an equation, the equation is still true.

Property of Equality for Division
Property of Equality for Division says that if a = b, then a / c = b / c.
If you divide the same number to both sides of an equation, the equation is still true.

Reflexive Property of Equality
Reflexive Property of Equality says that if a = a: anything is congruent to itself.
The equals sign is like a mirror, and the image it "reflects" is the same as the original.

Symmetric Property of Equality
Symmetric Property of Equality says that if a = b, then b = a.

Transitive Property of Equality
Transitive Property of Equality says that if a = b and b = c, then a = c.

Mean, Median, Mode, and Range Definitions

Mean :

The "Mean" is computed by adding all of the numbers in the data together and dividing by the number elements contained in the data set.

Example :

Data Set = 2, 5, 9, 3, 5, 4, 7

Number of Elements in Data Set = 7

Mean = (2 + 5 + 9 + 7 + 5 + 4 + 3) / 7 = 5

Median :

The "Median" of a data set is dependant on whether the number of elements in the data set is odd or even. First reorder the data set from the smallest to the largest then if the number of elements are odd, then the Median is the element in the middle of the data set. If the number of elements are even, then the Median is the average of the two middle terms.

Examples : Odd Number of Elements

Data Set = 2, 5, 9, 3, 5, 4, 7

Reordered = 2, 3, 4, 5, 5, 7, 9

Median = 5

Examples : Even Number of Elements

Data Set = 2, 5, 9, 3, 5, 4

Reordered = 2, 3, 4, 5, 5, 9

Median = (4 + 5) / 2 = 4.5

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Mean, Median, Mode, and Range Definitions

Mode :

The "Mode" for a data set is the element that occurs the most often. It is not uncommon for a data set to have more than one mode. This happens when two or more elements accur with equal frequency in the data set. A data set with two modes is called bimodal. A data set with three modes is called trimodal.

Examples : Single Mode

Data Set	=	2, 5, 9, 3, 5, 4, 7
Mode	=	5

Examples : Bimodal

Data Set	=	2, 5, 2, 3, 5, 4, 7
Modes	=	2 and 5

Examples : Trimodal

Data Set	=	2, 5, 2, 7, 5, 4, 7
Modes	=	2, 5, and 7

Range :

The "Range" for a data set is the difference between the largest value and smallest value contained in the data set. First reorder the data set from smallest to largest then subtract the first element from the last element.

Examples :

Data Set	=	2, 5, 9, 3, 5, 4, 7
Reordered	=	2, 3, 4, 5, 5, 7, 9
Range	=	(9 - 2) = 7

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II. Decimal Numbers and Concepts

1. Develop Number Concepts using Decimal Numbers

a. Understand decimals as part of the place value system
b. Use knowledge of place value to round decimal numbers to the nearest tenths, hundredths, thousandths…. millionths
c. Locate and write decimal numbers on a number line
d. Read and write decimal numbers using place value language
   - *Read 5.2 as 5 and 2 tenths, not 5 point 2*
e. Write decimal numbers as fractions over 10, 100, 1000 ……
f. Compare and order decimal numbers using =, ≠, <, >, ≥, and ≤
g. Understand the decimal point when used with money
   - *6 cents = .06 and 6 dimes = .6*

2. Add and Subtract Decimal Numbers

a. Add and subtract decimal numbers written horizontally by aligning decimal points and adding zeroes when needed
b. Estimate the answer to addition and subtraction problems by looking at whole numbers and decimals and/or rounding decimal numbers
c. Check accuracy of answers using inverse operations
d. Use a calculator to solve and/or check accuracy of computation
e. Apply decimal addition and subtraction skills to solve real world problems, *such as balancing a checkbook and calculating shopping totals and change*

3. Multiply and Divide Decimal Numbers

a. Estimate what the answer to multiplication and division problems will be using rounding and place value knowledge
b. Multiply decimal numbers putting the decimal point in the correct place in the product
c. Use a calculator to multiply decimal numbers and/or check accuracy of computation
d. Divide whole numbers by a decimal number
e. Divide a decimal number by a whole number
f. Divide a decimal by a decimal

g. Use a calculator to divide decimals and/or check the accuracy of computation
h. Use decimal multiplication and division skills to solve real world problems *such as comparison shopping and calculating the cost of multiple items*

4. Develop Algebraic Thinking Skills using Decimals

a. Determine if equations are true or false
- *Does 2.5 + 3.25 = 2.50 + 3.250? Is 5.1 > 5.0999? Is 5.2 – 3 ≥ 5.2 - 2 x 2?*

b. Determine the unknown number that makes an equation true
- *If 5.2(n) = 15.6, what is the value of n? If (10.5 + 5.25) ÷ n = 5.25, what is the value of n?*

c. Follow the order of operations using decimal numbers
d. Identify, describe, and create numeric patterns with decimals
- *If the first song costs 1.50 and each additional song cost 1.25, how much will songs 6 and 7 cost?*

e. Translate word problems using decimals into expressions and equations
f. Use simple formulas

5. Develop Geometry Skills using Decimals

a. Identify 2 and 3 dimensional shapes: *parallelogram, pentagon, cube, rectangular solid, pyramid, cone, and cylinder*
b. Find perimeter, area, and volume using decimal numbers
c. Find the area of a circle
d. Find the circumference of a circle
e. Solve real word problems involving area, perimeter and volume

6. Continue to Develop Measurement and Data Skills using Decimals

a. Describe units of time using decimal numbers
- *1hour and 30 minutes is equivalent to 1.5 hours, 2 days and 12 hours is equivalent to 2.5 days*

b. Describe units of length using decimal numbers
- *1 foot and 6 inches is equal to 1.5 feet*

c. Describe units of capacity using decimal numbers
- *1 pound 4 ounces is equal to 1.25 pounds*

d. Find mean, median and mode using decimal numbers
e. Accurately read charts and graphs that contain decimal numbers

Decimals Assessment

Name____________________________________

Date________________

Write the name of each decimal place indicated.

1) 87.724<u>5</u>64

2) <u>3</u>

3) 69,2<u>0</u>5.90

4) 3.07<u>0</u>285

5) 0.1<u>5</u>32

6) 78<u>5</u>.7

Round each to the place indicated.

7) 5.<u>6</u>44

8) 85.25<u>5</u>080

9) 58.<u>0</u>6

10) 7<u>9</u>.72

11) 40<u>6</u>.323

12) 0.35<u>5</u>3

Evaluate each expression.

13) $7.3 + 7.5 - 5.3$

14) $1.7 + 3.8 + 7.6$

15) $7.04 + 5.5 + 5.8$

16) $6.242 - 0.6 + 0.2$

17) $6 - 3.4 + 0.6$

18) $6.2 + 0.3 - 5.9$

Find each product.

19) 3.1×4.4

20) 3×4.16

21) 0.4×0.5

22) 4.5×5

Find each quotient.

23) $5.8 \div 1.6$

24) $4.1 \div 6.4$

25) $6.6 \div 4.1$

26) $0.6 \div 3.3$

Answers to

1) ten-thousandths	2) ones	3) tens	4) thousandths
5) hundredths	6) ones	7) 5.6	8) 85.255
9) 58.1	10) 80	11) 406	12) 0.355
13) 9.5	14) 13.1	15) 18.34	16) 5.842
17) 3.2	18) 0.6	19) 13.64	20) 12.48
21) 0.2	22) 22.5	23) 3.625	24) 0.640625
25) 1.60975609756	26) 0.181818181818		

Name_________________________ Date________________

Decimal Number Word Problems

1. When Chris used her calculator to divide, she got 0.8571. What is her answer rounded to the nearest hundredth?
   1. .9
   2. .86
   3. .85
   4. .858

2. The rainfall in Dover was .28 inch on Friday, 1 inch on Saturday, and .8 inches on Sunday. What was the total amount of rainfall for these three days?
   1. 2.08 inches
   2. 1.18 inches
   3. .118 inches
   4. 3.7 inches

3. John sold 8 candy bars. Each bar cost 89 cents. He also sold 6 cans of soda for 75 cents each. How much money did he collect?
   1. $7.12
   2. $16.40
   3. $ 4.50
   4. $11.62

4. You worked 7.5 hours on Tuesday and 10.25 hours on Friday. How much longer did you work on Friday?
   1. 17.75 hours
   2. 3.20 hours
   3. 3.25 hours
   4. 2.75 hours

5. You can buy 8 pencils for $2.40. How much does one cost?
   1. $.03
   2. $.25
   3. $.30
   4. $ 3.00

6. A customer paid $3.75 for 3 pounds. How much does one pound cost?
   1. $1.00
   2. $1.30
   3. $1.25
   4. $11.25

7. Order the decimals from smallest to largest: 3.5, 3.45, 4.0, 3.099, 3.61

____

8. The salad bar costs $3.98 a pound. If you bought 1.5 pounds of salad, what would it cost?
    1. $7.96
    2. $2.48
    3. $5.97
    4. $5.48

9. A piece of wood is 72.25 inches long. If it is cut into 5 equal pieces, what will the length of each piece be?
    1. 14.45 inches
    2. 361.25 inches
    3. 2 feet
    4. 62.25 inches

10. If you buy 5 apples @ .45 cents each and 6 peaches @ .60 cents each, how much change would you get from a $20.00 bill?
    1. $18.95
    2. $5.85
    3. $10.50
    4. $14.15

11. Solve:
    .6 X (1.5 + 2) – 2 = ___________________________

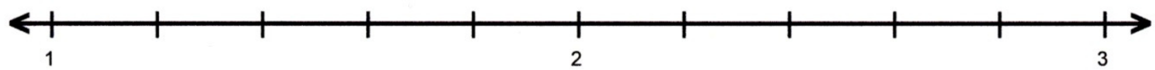
12. What is 2 divided by 4?
    1. 0.5
    2. 2
    3. 5
    4. 2.5

1) 2, 2)1, 3)4, 4)4, 5) 3, 6) 3, 7) 3.099, 3.45, 3.5, 3.61, 4.0, 8)3, 9) 1, 10) 4, 110 .1, 12) 1

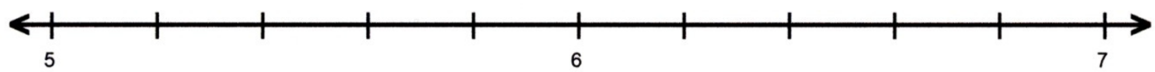
Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

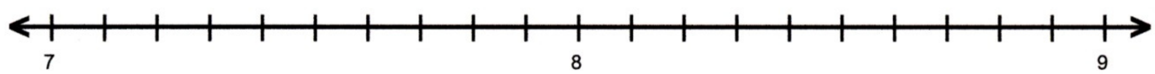
Decimal Numbers on Number Lines



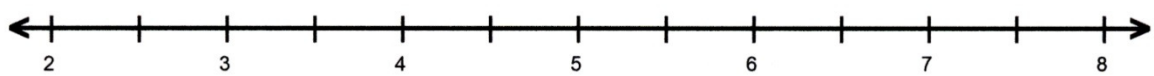
A = 1.8 B = 1.6 C = 1.4 D = 2.2



A = 5.8 B = 5.2 C = 6.4 D = 5.6



A = 7.5 B = 8.6 C = 8.2 D = 8.3



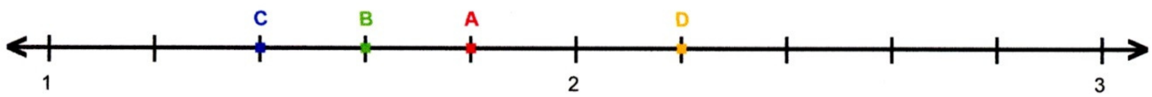
A = 5.5 B = 2.5 C = 3.5 D = 6.5

A = 6.5 B = 5.5 C = 7.5 D = 3.5

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Decimal Numbers on Number Lines



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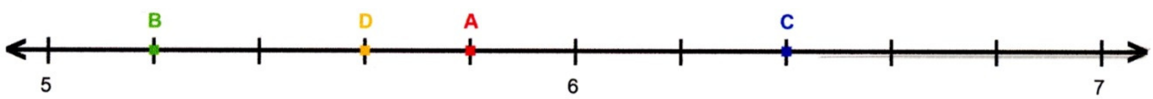
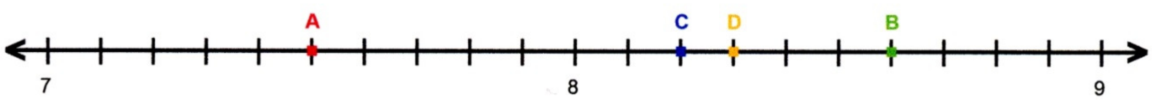


A = 5.8 B = 5.2 C = 6.4 D = 5.6



A = 7.5 B = 8.6 C = 8.2 D = 8.3

A = 5.5 B = 2.5 C = 3.5 D = 6.5

A = 6.5 B = 5.5 C = 7.5 D = 3.5

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

In and Out Boxes

Fill in the Empty Boxes.

1)

In	Out
24.2	
41.4	
42.4	
42.6	

Rule: Subtract 7.6

2)

In	Out
27.5	
28.4	
29.5	
42.3	

Rule: Subtract 4.6

3)

In	Out
21.4	
23.3	
33.6	
36.5	

Rule: Add 1.9

4)

In	Out
26.5	
33.1	
36.5	
48.8	

Rule: Add 2.1

5)

In	28.4	34.8	34.9	49.9
Out				

Rule: Add 1.1

6)

In	24.6	40.5	40.6	43.8
Out				

Rule: Subtract 9.5

7)

In	28.6	41.7	42.8	45.6
Out				

Rule: Subtract 4.9

8)

In	26.5	27.4	34.6	45.5
Out				

Rule: Add 9.7

Write the rule and fill in the empty boxes.

9)

In	Out
25.7	34.6
34.5	43.4
39.4	
43.6	52.5

Rule:

10)

In	Out
21.7	
24.3	33.6
29.4	38.7
32.9	42.2

Rule:

11)

In	Out
21.6	
24.3	28.6
42.8	47.1
46.8	51.1

Rule:

12)

In	Out
20.1	14.5
21.2	15.6
33.2	
34.1	28.5

Rule:

13)

In	25.1	26.8	31.9	39.1
Out		23.5	28.6	35.8

Rule:

14)

In	23.8	27.9	28.8	41.5
Out	21.2		26.2	38.9

Rule:

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

In and Out Boxes

Fill in the Empty Boxes.

1)

In	Out
24.2	16.6
41.4	33.8
42.4	34.8
42.6	35

Rule: Subtract 7.6

2)

In	Out
27.5	22.9
28.4	23.8
29.5	24.9
42.3	37.7

Rule: Subtract 4.6

3)

In	Out
21.4	23.3
23.3	25.2
33.6	35.5
36.5	38.4

Rule: Add 1.9

4)

In	Out
26.5	28.6
33.1	35.2
36.5	38.6
48.8	50.9

Rule: Add 2.1

5)

In	28.4	34.8	34.9	49.9
Out	29.5	35.9	36	51

Rule: Add 1.1

6)

In	24.6	40.5	40.6	43.8
Out	15.1	31	31.1	34.3

Rule: Subtract 9.5

7)

In	28.6	41.7	42.8	45.6
Out	23.7	36.8	37.9	40.7

Rule: Subtract 4.9

8)

In	26.5	27.4	34.6	45.5
Out	36.2	37.1	44.3	55.2

Rule: Add 9.7

Write the rule and fill in the empty boxes.

9)

In	Out
25.7	34.6
34.5	43.4
39.4	48.3
43.6	52.5

Rule: Add 8.9

10)

In	Out
21.7	31
24.3	33.6
29.4	38.7
32.9	42.2

Rule: Add 9.3

11)

In	Out
21.6	25.9
24.3	28.6
42.8	47.1
46.8	51.1

Rule: Add 4.3

12)

In	Out
20.1	14.5
21.2	15.6
33.2	27.6
34.1	28.5

Rule: Subtract 5.6

13)

In	25.1	26.8	31.9	39.1
Out	21.8	23.5	28.6	35.8

Rule: Subtract 3.3

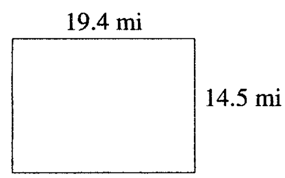
14)

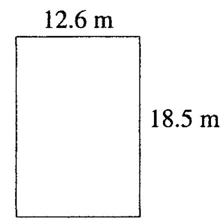
In	23.8	27.9	28.8	41.5
Out	21.2	25.3	26.2	38.9

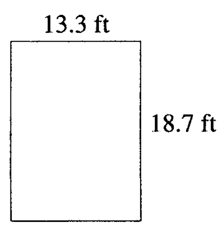
Rule: Subtract 2.6

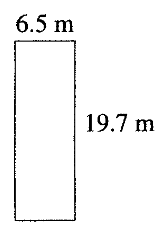
Area and Perimeter of Rectangles (A)

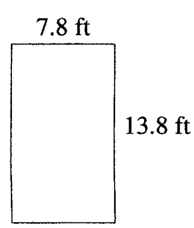
Find the area and perimeter of each rectangle.

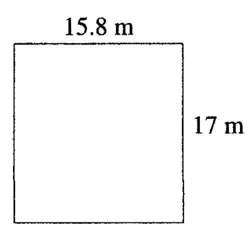






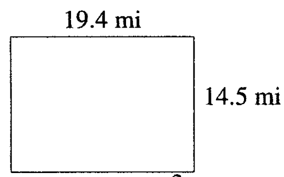




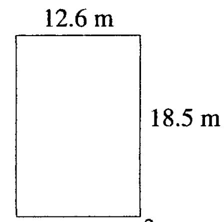


Area and Perimeter of Rectangles (A) Answers

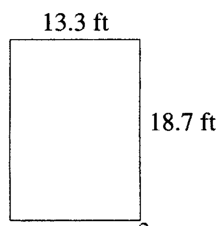
Find the area and perimeter of each rectangle.



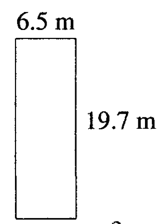
A = 281.30 mi^2
P = 67.8 mi



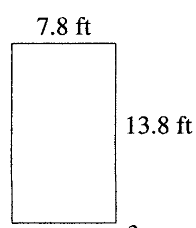
A = 233.10 m^2
P = 62.2 m



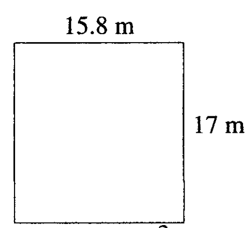
A = 248.71 ft^2
P = 64.0 ft



A = 128.05 m^2
P = 52.4 m



A = 107.64 ft^2
P = 43.2 ft



A = 268.6 m^2
P = 65.6 m

III. Fractions

1. Develop an Understanding of Fractions

a. Read, write and draw fractions, proper fractions, improper fractions, and mixed numbers
b. Understand that $\frac{x}{y}$ means division
c. Locate and place fractions and mixed numbers on a number line
d. Understand that two fractions are equivalent if they are the same size or lie on the same point on a number line
e. Translate words into fractions
   - *6 employees out of 18 were absent, what fraction of the employees were absent?*
f. Recognize and generate equivalent fractions
   - $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{10}{20}$
g. Compare and order fractions using =, ≠, <, >, ≥, and ≤
h. Express whole numbers as fractions: $3 = \frac{3}{1}$
i. Simplify fractions: $\frac{4}{8} = \frac{1}{2}$
j. Convert mixed numbers to improper fractions and improper fractions to mixed numbers: $2\frac{3}{4} = \frac{11}{4}$
k. Demonstrate an understanding of and apply knowledge of the value of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{1}{3}$
l. Round fractions and mixed numbers to the nearest whole number
m. Convert fractions to decimals
n. Use a calculator to simplify fractions and convert fractions to decimals
o. Find the least common multiple of two whole numbers
   - *The smallest positive number that is a multiple of two or more numbers*
p. Find the greatest common factor of two whole numbers
   - *The highest number that divides exactly into two or more numbers*

2. Add and Subtract Fractions and Mixed Numbers and Apply

a. Add and subtract fractions and mixed numbers with the same denominator and simplify answers
b. Find a common denominator or a least common denominator to write equivalent fractions

c. Add and subtract fractions and mixed numbers with unlike denominators
d. Make reasonable estimates of sums and differences using knowledge of fractions and mixed numbers
e. Use a calculator to add and subtract fractions and mixed numbers and to check accuracy of computation done without a calculator
f. Solve one step and multi-step real world word problems involving fractions and mixed numbers using addition and subtraction skills and assess the reasonableness of the answers

3. **Multiply and Divide Fractions and Mixed Numbers and Apply**

a. Multiply a whole number by a fraction and simplify the answer
b. Multiply a whole number by a mixed number and simplify
c. Multiply 2 or more fractions, simplifying fractional factors by using the greatest common factor when possible, and simplify
d. Multiply mixed numbers and simplify
e. Divide a whole number by a fraction
f. Divide a fraction by a fraction
g. Divide using fractions and mixed numbers
h. Use a calculator to multiply or divide fractions and mixed numbers or to check accuracy of answers
i. Make reasonable estimates of products and quotients using knowledge of fractions and mixed numbers
j. Solve real world word problems using fractions and mixed numbers and assess the reasonableness of the answer

4. Apply Computational Skills Using Fractions and Mixed Numbers to Solve Real World Problems:

a. Find sale price: *find $\frac{1}{2}$ off the regular price, find $\frac{1}{3}$ off the regular price*
b. Find the fraction of a number: *find $\frac{1}{3}$ of your weekly salary to save for rent*
c. Double, triple and half a recipe
d. Order meat and cheese from the deli
e. Relate fractions to time and measurement
   - *$\frac{1}{2}$ hour is 30 minutes, $\frac{1}{4}$ of a year is 3 months, $\frac{1}{2}$ of a foot is 6 inches, $\frac{3}{4}$ of a dollar is .75 cents, etc.*

f. Measure and use a map scale
   - $\frac{1}{2}$ *inch =100 miles, how many miles is 3* $\frac{1}{2}$ *inches?*

g. Calculate doses of medicine
   - *Take 1* $\frac{1}{4}$ *teaspoons every 4 hours, how much will you take in a 24 hour time span?*

h. Calculate prices
   - *Buy one get one half off, how much does each item cost?*
     Is buy one, get one half off a better deal than buy three and get one free?

5. Continue to Develop Algebraic Thinking Skills Using Fractions and Mixed Numbers

a. Determine if equations are true or false
   - *Does* $\frac{1}{2} + \frac{1}{2} = 2 - 1$? *Is* $\frac{5}{6} < \frac{4}{9}$?
   - *If* $n = 1\frac{1}{2}$ *does* $10 \times n + n = 30$

b. Determine the unknown number that makes the equation true
   - *If* $5\frac{1}{4} + n = 10$, *what is the value of n?*

c. Follow the order of operations using fractions and mixed numbers

d. Identify, describe, and create numeric patterns using fractions and mixed numbers
   - *1* $\frac{1}{2}$, *3, 4* $\frac{1}{2}$, *6,* ___, ___, ___, ___

6. Continue to Develop Geometry Skills using Fractions and Mixed Numbers

a. Find perimeter, area, and volume using fractions and mixed numbers

b. Find the circumference and area of a circle using $\frac{22}{7}$ for PI

7. Continue to Develop Measurement and Data Skills using Fractions and Mixed Numbers

a. Using a ruler or tape measure, measure to the neatest 16th of an inch
   - *Understand that $\frac{8}{16}$ of a foot is equivalent to $\frac{1}{2}$ foot, $\frac{4}{16}$ of a foot is equivalent to $\frac{1}{4}$ of a foot, etc.*

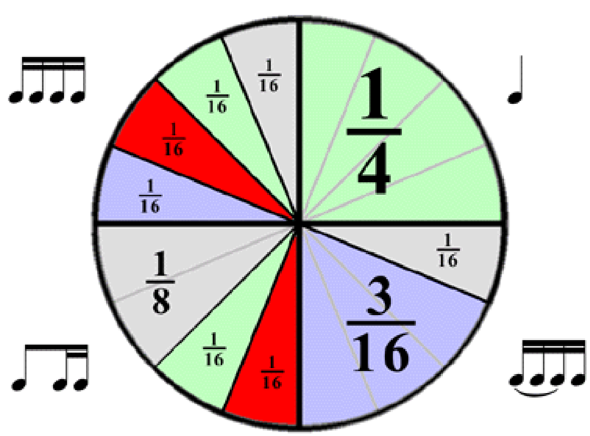
b. Apply knowledge of fractions and measurement of capacity
   - *8 ounces is equivalent to $\frac{1}{2}$ of a cup, $\frac{1}{2}$ of a gallon is equivalent to 2 quarts, etc.*

c. Apply knowledge of fractions and measurement of weight
   - *1 pound and 8 ounces is equivalent to 1 $\frac{1}{2}$ pounds, etc.*

d. Apply knowledge of fractions and time
   - *30 minutes is equivalent to $\frac{1}{2}$ hour, 4 months is $\frac{2}{3}$ of a year, etc.*

e. Accurately read and interpret charts and graphs using fractions

******Refer to the Measurement, Algebra, and Geometry sections for more information*



Fractions Assessment Name________________________________

Date______________

Simplify each. Write your answer as a mixed number when possible.

1) $\frac{2}{4}$

2) $\frac{12}{15}$

3) $\frac{4}{12}$

4) $\frac{6}{9}$

5) $\frac{4}{8}$

6) $\frac{2}{6}$

7) $\frac{8}{6}$

8) $\frac{27}{18}$

9) $\frac{10}{4}$

10) $\frac{12}{8}$

11) $\frac{9}{6}$

12) $\frac{18}{15}$

Evaluate each expression.

13) $\frac{6}{5} - \frac{2}{5}$

14) $\frac{2}{3} + 1\frac{1}{4}$

15) $2\frac{1}{6} + \frac{1}{6}$

16) $2 + \frac{9}{8}$

17) $3\frac{1}{4} + \frac{1}{3}$

18) $4\frac{2}{5} - 2\frac{1}{2}$

Find the Greatest Common Factor of each.

19) 8, 12

20) 40, 32

21) 25, 35

22) 20, 36

Find the Least Common Multiple of each.

23) 3, 8

24) 9, 6

25) 15, 9

26) 20, 15

Find each product.

27) $\left(2\frac{1}{4}\right)(2)$

28) $\left(1\frac{2}{3}\right)\left(\frac{3}{2}\right)$

29) $\left(\frac{2}{3}\right)\left(\frac{3}{2}\right)$

30) $\left(1\frac{3}{4}\right)\left(\frac{1}{2}\right)$

Find each quotient.

31) $2\frac{1}{6} \div 4\frac{4}{7}$

32) $8\frac{4}{7} \div 4\frac{5}{6}$

33) $3\frac{2}{3} \div \frac{5}{7}$

34) $3\frac{1}{3} \div 6\frac{6}{7}$

35) A recipe for a cake calls for $4\frac{3}{7}$ cups of flour. Mark has already put in $3\frac{1}{8}$ cups. How many more cups does he need to put in?

36) Sarawong is cooking cupcakes. The recipe calls for $3\frac{3}{8}$ cups of sugar. He accidentally put in $5\frac{9}{10}$ cups. How many extra cups did he put in?

37) Your cousin gave you $16.17 with which to buy a present. This covered $\frac{7}{8}$ of the cost. How much did the present cost?

38) A mean ogre stole 10 of your muffins. That was $\frac{2}{9}$ of all of them! How many are left?

39) A colony of ants carried away 28 of your muffins. That was $\frac{7}{8}$ of all of them! How many are left?

40) If the weight of a package is multiplied by $\frac{3}{10}$ the result is 23.1 pounds. Find the weight of the package.

Answers to

1) $\frac{1}{2}$	2) $\frac{4}{5}$	3) $\frac{1}{3}$	4) $\frac{2}{3}$
5) $\frac{1}{2}$	6) $\frac{1}{3}$	7) $1\frac{1}{3}$	8) $1\frac{1}{2}$
9) $2\frac{1}{2}$	10) $1\frac{1}{2}$	11) $1\frac{1}{2}$	12) $1\frac{1}{5}$
13) $\frac{4}{5}$	14) $1\frac{11}{12}$	15) $2\frac{1}{3}$	16) $3\frac{1}{8}$
17) $3\frac{7}{12}$	18) $1\frac{9}{10}$	19) 4	20) 8
21) 5	22) 4	23) 24	24) 18
25) 45	26) 60	27) $4\frac{1}{2}$	28) $2\frac{1}{2}$
29) 1	30) $\frac{7}{8}$	31) $\frac{91}{192}$	32) $1\frac{157}{203}$
33) $5\frac{2}{15}$	34) $\frac{35}{72}$	35) $1\frac{17}{56}$	36) $2\frac{21}{40}$
37) \$18.48	38) 35	39) 4	40) 77

Name____________________ Date____________

Fraction Word Problems

1. Maria worked $7\frac{1}{2}$ hours on Friday, $8\frac{1}{2}$ hours on Saturday and $10\frac{1}{2}$ hours on Sunday. How many hours did she work in all? ___________

2. Maria's check was $180.00. She plans on putting $\frac{1}{4}$ of this amount in the bank. How much money will she put in the bank? ___________

3. $3\frac{1}{2}$ inches of rain fell on Monday and $2\frac{1}{4}$ inches of rain fell on Tuesday. How much more rain fell on Monday? ______________

4. 12,000 people are registered to vote. During the last election $\frac{3}{4}$ of registered voters voted. How many people voted? ______________
   How many people did not vote? ______________

5. John has a 12 foot long board. How many $1\frac{1}{2}$ foot pieces can he cut?
   ____

6. Peri needs to put in 40 hours this week. She worked $8\frac{1}{2}$ hours on Monday, $6\frac{1}{2}$ hours on Tuesday, 8 hours on Wednesday and $6\frac{1}{3}$ hours on Thursday. How many hours does she need to work on Friday to get to 40? ____________________

7. Maria needs $1\frac{3}{4}$ cups of flour to make one batch of oatmeal cookies. How much flour would she need if she tripled the recipe? ___________

8. Cory walks $1\frac{1}{2}$ miles every day. How many miles does he walk in one week?
   ____

DMR

Solve the following word problems:

Last week Tom worked 7 ½ hours on Monday, 9 hours on Tuesday, 7 ¼ on Wednesday, 9 on Thursday and 9 ¾ on Friday. He gets paid $ 10.00 per hour and time and a half for any hours he works over 40 hours.

1. How many hours did Tom work last week? ________________
2. How many overtime hours did he work? __________________
3. What is his hourly pay for overtime? ______________________
4. How much did Tom earn last week (gross pay)? ___________
5. 1/4 of Tom's gross pay is taken out of his check to pay taxes, social security, Insurance, and retirement benefits. How much money is deducted from his check? ________________ (round to the nearest penny)
6. How many minutes in ¾ of an hour? _________________
7. How may inches in 1 ½ feet? _______________________
8. What is ¼ or $10.00? ________________________________

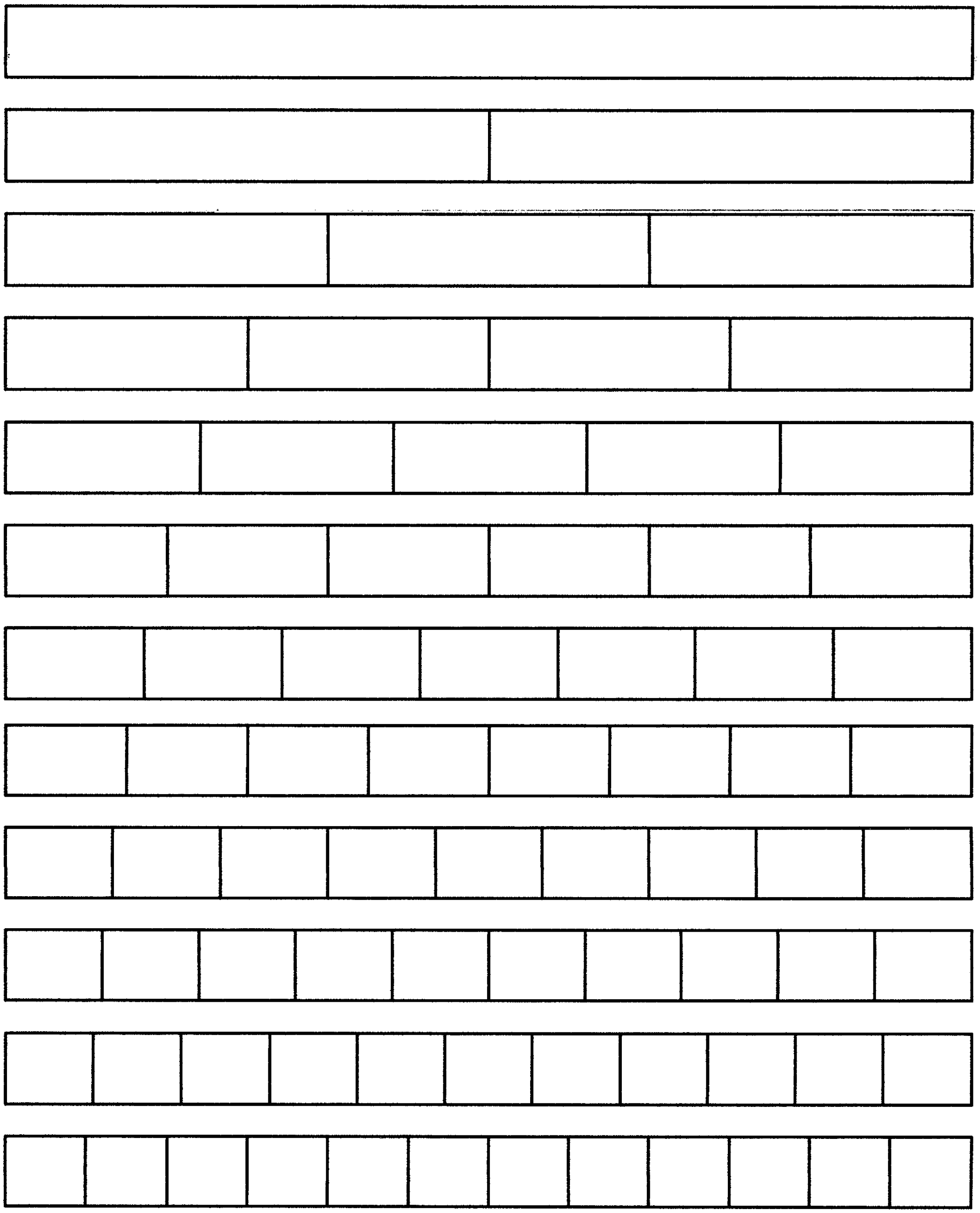
Name____________________ Date___________

Fraction Word Problems Answers:

1. 26 ½
2. $45.00
3. 1 ¼
4. 9,000
5. 8
6. 10 2/3
7. 5 ¼
8. 10 ½

1. 42 ½
2. 2 ½
3. $15.00
4. $437.50
5. $94.38
6. 45
7. 18
8. $2.50

Black Line Fraction Strips



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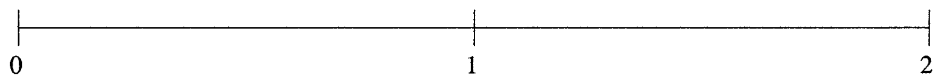
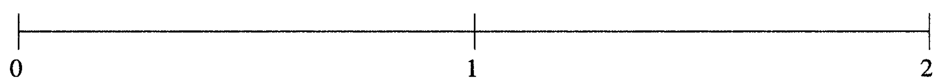
Ordering Fractions (A)

Order each set of fractions using the number line.

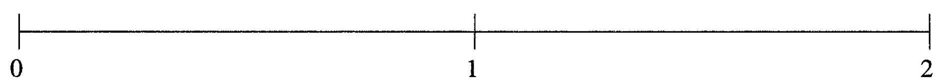
$\frac{1}{2}, \frac{4}{5}, \frac{1}{5}, 1\frac{3}{10}, 1\frac{9}{10}$



$1\frac{1}{2}, 1\frac{4}{5}, \frac{2}{5}, \frac{7}{10}, 1\frac{1}{5}$



$\frac{1}{2}, \frac{4}{5}, \frac{1}{5}, 1\frac{3}{5}, 1\frac{1}{10}$



Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Equivalent Fractions

1) $\frac{5}{10} = \frac{10}{\quad} = \frac{15}{\quad} = \frac{20}{\quad} = \frac{25}{\quad} = \frac{\quad}{60} = \frac{\quad}{70}$

2) $\frac{3}{10} = \frac{\quad}{20} = \frac{9}{\quad} = \frac{\quad}{40} = \frac{15}{\quad} = \frac{18}{\quad} = \frac{\quad}{70}$

3) $\frac{1}{2} = \frac{\quad}{4} = \frac{3}{\quad} = \frac{4}{\quad} = \frac{\quad}{10} = \frac{6}{\quad} = \frac{\quad}{14}$

4) $\frac{2}{5} = \frac{\quad}{10} = \frac{\quad}{15} = \frac{8}{\quad} = \frac{10}{\quad} = \frac{\quad}{30} = \frac{14}{\quad}$

5) $\frac{3}{5} = \frac{\quad}{10} = \frac{\quad}{15} = \frac{12}{\quad} = \frac{15}{\quad} = \frac{18}{\quad} = \frac{21}{\quad}$

6) $\frac{1}{4} = \frac{2}{\quad} = \frac{3}{\quad} = \frac{4}{\quad} = \frac{\quad}{20} = \frac{6}{\quad} = \frac{7}{\quad}$

7) $\frac{3}{5} = \frac{6}{\quad} = \frac{\quad}{15} = \frac{12}{\quad} = \frac{\quad}{25} = \frac{\quad}{30} = \frac{\quad}{35}$

8) $\frac{2}{3} = \frac{\quad}{6} = \frac{6}{\quad} = \frac{\quad}{12} = \frac{10}{\quad} = \frac{\quad}{18} = \frac{14}{\quad}$

9) $\frac{3}{4} = \frac{\quad}{8} = \frac{\quad}{12} = \frac{12}{\quad} = \frac{15}{\quad} = \frac{\quad}{24} = \frac{\quad}{28}$

10) $\frac{1}{3} = \frac{2}{\quad} = \frac{\quad}{9} = \frac{4}{\quad} = \frac{\quad}{15} = \frac{\quad}{18} = \frac{\quad}{21}$

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

Equivalent Fractions

1) $\frac{5}{10} = \frac{10}{20} = \frac{15}{30} = \frac{20}{40} = \frac{25}{50} = \frac{30}{60} = \frac{35}{70}$

2) $\frac{3}{10} = \frac{6}{20} = \frac{9}{30} = \frac{12}{40} = \frac{15}{50} = \frac{18}{60} = \frac{21}{70}$

3) $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14}$

4) $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25} = \frac{12}{30} = \frac{14}{35}$

5) $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30} = \frac{21}{35}$

6) $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24} = \frac{7}{28}$

7) $\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25} = \frac{18}{30} = \frac{21}{35}$

8) $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \frac{10}{15} = \frac{12}{18} = \frac{14}{21}$

9) $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24} = \frac{21}{28}$

10) $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18} = \frac{7}{21}$

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

Working with fractions and whole numbers.

1) Find $\frac{1}{3}$ of 36 =

2) Find $\frac{4}{5}$ of 220 =

3) Find $\frac{2}{6}$ of 132 =

4) Find $\frac{4}{5}$ of 40 =

5) Find $\frac{4}{6}$ of 240 =

6) Find $\frac{1}{4}$ of 12 =

7) Find $\frac{4}{8}$ of 224 =

8) Find $\frac{3}{6}$ of 36 =

9) Find $\frac{1}{3}$ of 36 =

10) Find $\frac{4}{8}$ of 256 =

11) Find $\frac{1}{8}$ of 72 =

12) Find $\frac{1}{5}$ of 35 =

13) Find $\frac{1}{3}$ of 12 =

14) Find $\frac{2}{5}$ of 60 =

15) Find $\frac{4}{10}$ of 360 =

16) Find $\frac{1}{3}$ of 18 =

17) Find $\frac{5}{6}$ of 300 =

18) Find $\frac{5}{10}$ of 400 =

19) Find $\frac{1}{10}$ of 50 =

20) Find $\frac{6}{10}$ of 300 =

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Working with fractions and whole numbers.

1) Find $\frac{1}{3}$ of 36 =
12

2) Find $\frac{4}{5}$ of 220 =
176

3) Find $\frac{2}{6}$ of 132 =
44

4) Find $\frac{4}{5}$ of 40 =
32

5) Find $\frac{4}{6}$ of 240 =
160

6) Find $\frac{1}{4}$ of 12 =
3

7) Find $\frac{4}{8}$ of 224 =
112

8) Find $\frac{3}{6}$ of 36 =
18

9) Find $\frac{1}{3}$ of 36 =
12

10) Find $\frac{4}{8}$ of 256 =
128

11) Find $\frac{1}{8}$ of 72 =
9

12) Find $\frac{1}{5}$ of 35 =
7

13) Find $\frac{1}{3}$ of 12 =
4

14) Find $\frac{2}{5}$ of 60 =
24

15) Find $\frac{4}{10}$ of 360 =
144

16) Find $\frac{1}{3}$ of 18 =
6

17) Find $\frac{5}{6}$ of 300 =
250

18) Find $\frac{5}{10}$ of 400 =
200

19) Find $\frac{1}{10}$ of 50 =
5

20) Find $\frac{6}{10}$ of 300 =
180

Order of Operations (A)

Perform the operations in the correct order.

1. $12-\left(\frac{8}{5}+3\div\frac{2}{3}\right)$

6. $\frac{11}{2}+\frac{9}{2}-(3-2)$

11. $4-\frac{4}{3}\times\frac{5}{4}+\frac{11}{6}$

2. $\left(\frac{9}{2}+\frac{5}{2}\right)\div\frac{11}{2}\div\frac{1}{5}$

7. $10+4-\frac{3}{2}-\frac{9}{2}$

12. $\left(\frac{11}{3}-\frac{7}{3}+2\right)\div\frac{2}{5}$

3. $5^{\frac{2}{3}+1+\frac{1}{3}}$

8. $\frac{4}{3}^{2}\times 5\times 1$

13. $1-1+8-\frac{2}{5}$

4. $\frac{1}{2}\div\frac{9}{4}\times\left(11-\frac{4}{3}\right)$

9. $2\times\frac{1}{3}\div 2\times\frac{6}{5}$

14. $3-2+2-1$

5. $\frac{1}{2}^{3}+\frac{3}{2}\div\frac{2}{3}$

10. $(1+2)^{\frac{8}{3}\times\frac{3}{4}}$

15. $2\div(8\times 8-2)$

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Order of Operations (A) Answers

Perform the operations in the correct order.

1. $12 - \left(\frac{8}{5} + 3 \div \frac{2}{3}\right)$
$= \frac{59}{10}$

2. $\left(\frac{9}{2} + \frac{5}{2}\right) \div \frac{11}{2} \div \frac{1}{5}$
$= \frac{70}{11}$

3. $5^{\frac{2}{3}+1+\frac{1}{3}}$
$= 25$

4. $\frac{1}{2} \div \frac{9}{4} \times \left(11 - \frac{4}{3}\right)$
$= \frac{58}{27}$

5. $\frac{1}{2}^{3} + \frac{3}{2} \div \frac{2}{3}$
$= \frac{19}{8}$

6. $\frac{11}{2} + \frac{9}{2} - (3 - 2)$
$= 9$

7. $10 + 4 - \frac{3}{2} - \frac{9}{2}$
$= 8$

8. $\frac{4}{3}^{2} \times 5 \times 1$
$= \frac{80}{9}$

9. $2 \times \frac{1}{3} \div 2 \times \frac{6}{5}$
$= \frac{2}{5}$

10. $(1 + 2)^{\frac{8}{3} \times \frac{3}{4}}$
$= 9$

11. $4 - \frac{4}{3} \times \frac{5}{4} + \frac{11}{6}$
$= \frac{25}{6}$

12. $\left(\frac{11}{3} - \frac{7}{3} + 2\right) \div \frac{2}{5}$
$= \frac{25}{3}$

13. $1 - 1 + 8 - \frac{2}{5}$
$= \frac{38}{5}$

14. $3 - 2 + 2 - 1$
$= 2$

15. $2 \div (8 \times 8 - 2)$
$= \frac{1}{31}$

Ratios & Proportional Relationships

1. Understand Ratio Concepts

a. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities
   - *The ratio of women to men in class is 3:1, because for every 3 women in class there is 1 man. For every 1 man in the class there are 3 women*

b. Write ratios using the word "to", with a colon, and as a fraction
   - *3 to 1, 3:1,* $\frac{3}{1}$

c. Set up a proportion to fit a situation
   - *There are 18 marbles in a bag, 4 are red, 5 are green, 6 are blue, and 3 are black. What is the ratio of green marbles to red marbles? What is the ratio of black marbles to all marbles?*

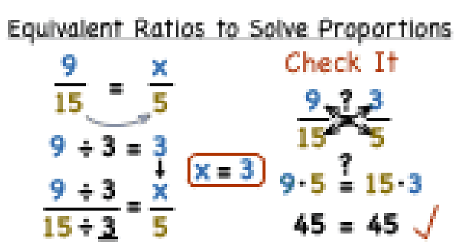
d. Write equivalent ratios

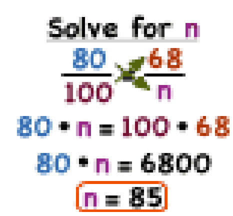
e. Determine if 2 ratios are proportional

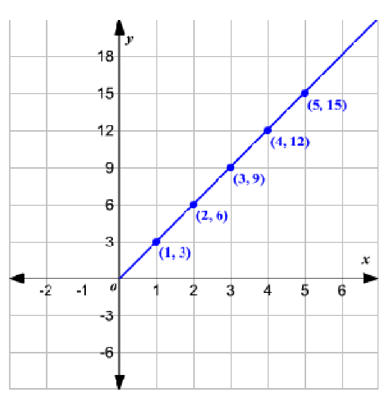
f. Use a graph to represent equivalent ratios

g. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on a coordinate plane. Use tables to compare ratios

h. Cross multiply and divide to find an unknown in a proportion and understand that the cross products of a proportion are equal







2. Use Ratio and Proportion to Solve Real World Problems

a. Understand the concept of unit rate and use ratio and proportion reasoning skills to solve unit rate problems
   - *It cost $75.00 to buy 6 pizzas, the unit rate (price for 1) is $12.50.*
   - *If a box of 18 pencils costs $3.98, what is the unit price per pencil?*

b. Write a proportion to solve a problem
   - *There are 21 grams of protein in a 3 oz. serving of tuna. If you ate 12oz. of tuna last week, how many grams of protein did you consume?* $\frac{21}{3} = \frac{x}{12}$

c. Use ratio and proportion to solve scale problems
   - *The scale on a highway map is 1 inch: 30 miles. What is the distance in miles if you measure 2.5 inches between your starting and ending locations?*

d. Use ratio reasoning to convert measurement units
   - *There are 3 feet in a yard. How many feet are there in 5 yards, 8 yards, 12 yards?* $\frac{3}{1} = \frac{x}{12}$ *and/or create a table*

e. Represent proportional relationships by equations
   - *If total cost (t) is proportional to the number (n) of items purchased at a constant price (p), the relationship between the total cost and the number of items can be expressed as t = p x n*

f. Solve multi-step real world problems using ratio and proportion

g. Use proportional relationships to solve percent problems

Ratio and Proportion Assessment

Name________________________________

Date______________

State if each pair of ratios forms a proportion.

1) $\frac{12}{30}$ and $\frac{4}{6}$

2) $\frac{4}{3}$ and $\frac{32}{18}$

3) $\frac{5}{3}$ and $\frac{25}{15}$

4) $\frac{20}{24}$ and $\frac{5}{6}$

Solve each proportion.

5) $\frac{m}{4} = \frac{6}{5}$

6) $\frac{3}{7} = \frac{a}{8}$

7) $\frac{4}{2} = \frac{x}{4}$

8) $\frac{r}{6} = \frac{6}{5}$

Answer each question and round your answer to the nearest whole number.

9) Julio bought 12 packages of cherry tomatoes for $36. How many packages of cherry tomatoes can Matt buy if he has $18?

10) Five packages of grape tomatoes cost $13. How many packages of grape tomatoes can you buy for $26?

11) Jessica bought 12 packages of fresh chives for $24. How many packages can Julia buy if she has $12?

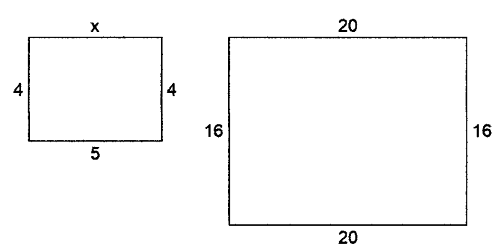
12) 18 bags of carrots cost $60. How many bags can you buy for $20?

13) If you can buy 48 yellow potatoes for $24, then how many can you buy with $4?

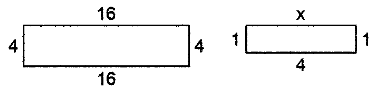
14) If you can buy 42 mangos for $35, then how many can you buy with $5?

Each pair of figures is similar. Find the missing side.

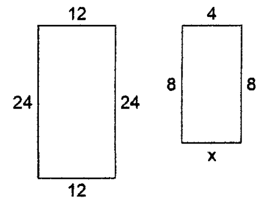
15)



16)



17)



Answer each question and round your answer to the nearest whole number.

18) A woman that is 6 ft tall casts a shadow that is 4 ft long. Find the length of the shadow that a 9 ft adult elephant casts.

19) A model car is 20 in long. If it was built with a scale of 4 in : 3 ft, then how long is the real car?

20) A model train has a scale of 1 in : 3 ft. If the real train is 24 ft tall, then how tall is the model train?

21) Greenwood and Abbots Rise are 108 mi from each other. How far apart would the cities be on a map that has a scale of 1 in : 27 mi?

22) If a 6 ft tall lawn ornament casts a 24 ft long shadow, then how tall is a car that casts a 16 ft shadow?

23) Georgetown and Oak Grove are 40 km from each other. How far apart would the cities be on a map that has a scale of 5 cm : 20 km?

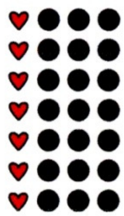
Answers to

1) No	2) No	3) Yes	4) Yes
5) $\{4.8\}$	6) $\{3.42\}$	7) $\{8\}$	8) $\{7.2\}$
9) 6	10) 10	11) 6	12) 6
13) 8	14) 6	15) 5	16) 4
17) 4	18) 6 ft	19) 15 ft	20) 8 in
21) 4 in	22) 4 ft	23) 10 cm	

Name : ____________________ Score : __________

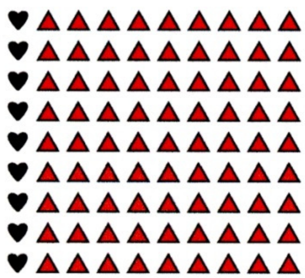
Teacher : ____________________ Date : __________

Find the Ratios



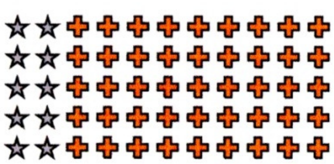
What is the ratio of ♥ to ● ? = ____ : ____ = ____ : ____ (Simplified)

What is the ratio of ● to (♥ + ●) ? = ____ : ____ = ____ : ____



What is the ratio of ♥ to ▲ ? = ____ : ____ = ____ : ____ (Simplified)

What is the ratio of ▲ to (♥ + ▲) ? = ____ : ____ = ____ : ____



What is the ratio of ☆ to ✚ ? = ____ : ____ = ____ : ____ (Simplified)

What is the ratio of ✚ to (☆ + ✚) ? = ____ : ____ = ____ : ____

What is the ratio of ✚ to ☆ ? = ____ : ____ = ____ : ____ (Simplified)

What is the ratio of ☆ to (✚ + ☆) ? = ____ : ____ = ____ : ____

Name : ______________________________ Score : ____________

Teacher : ____________________________ Date : ____________

Find the Ratios

= 7 : 21 = 1 : 3

= 21 : 28 = 3 : 4

= 9 : 81 = 1 : 9

= 81 : 90 = 9 : 10

= 10 : 45 = 2 : 9

= 45 : 55 = 9 : 11

= 12 : 54 = 2 : 9

= 54 : 66 = 9 : 11

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Equivalent Ratios

Write two equivalent ratios.

1)

9		
2		

2)

3		
5		

3)

12		
7		

4)

11		
3		

5)

10		
3		

6)

4		
9		

Determine whether the ratios are equivalent.

7) $\frac{3}{11}$ and $\frac{15}{55}$ ______

8) $\frac{3}{10}$ and $\frac{12}{40}$ ______

9) $\frac{8}{5}$ and $\frac{4}{5}$ ______

10) $\frac{7}{11}$ and $\frac{42}{66}$ ______

11) $\frac{5}{6}$ and $\frac{3}{8}$ ______

12) $\frac{11}{8}$ and $\frac{9}{10}$ ______

Use equivalent ratios to find the unknown value.

13) $\frac{9}{y} = \frac{3}{4}$ y = _____

14) $\frac{36}{y} = \frac{12}{11}$ y = _____

15) $\frac{7}{3} = \frac{f}{12}$ f = _____

16) $\frac{b}{44} = \frac{6}{11}$ b = _____

17) $\frac{b}{10} = \frac{11}{5}$ b = _____

18) $\frac{11}{12} = \frac{22}{h}$ h = _____

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Equivalent Ratios

Write two equivalent ratios.

1)

9	18	27
2	4	6

2)

3	6	9
5	10	15

3)

12	24	36
7	14	21

4)

11	22	33
3	6	9

5)

10	20	30
3	6	9

6)

4	8	12
9	18	27

Determine whether the ratios are equivalent.

7) $\frac{3}{11}$ and $\frac{15}{55}$ Yes

8) $\frac{3}{10}$ and $\frac{12}{40}$ Yes

9) $\frac{8}{5}$ and $\frac{4}{5}$ No

10) $\frac{7}{11}$ and $\frac{42}{66}$ Yes

11) $\frac{5}{6}$ and $\frac{3}{8}$ No

12) $\frac{11}{8}$ and $\frac{9}{10}$ No

Use equivalent ratios to find the unknown value.

13) $\frac{9}{y} = \frac{3}{4}$ y = 12

14) $\frac{36}{y} = \frac{12}{11}$ y = 33

15) $\frac{7}{3} = \frac{f}{12}$ f = 28

16) $\frac{b}{44} = \frac{6}{11}$ b = 24

17) $\frac{b}{10} = \frac{11}{5}$ b = 22

18) $\frac{11}{12} = \frac{22}{h}$ h = 24

Using Ratio and Proportion to Solve Many Types of Problems

Many kinds of problems can be solved by using proportions. Using "the box" to solve proportion problems enables students to label the problem information thus eliminating a big source of confusion. For each problem the process is the same: multiply the two diagonal numbers and divide that product by the only number that remains in the grid. The answer to this division will fit in the missing grid square, completing the proportion. To check any proportion problem, the product of each diagonal ***should be equal.***

Sugar (pounds) 2	Sugar (pounds) x
Fruit (quarts) 3	Fruit (quarts) 9

A "Standard" Proportion Problem

Example: Pam uses 2 pounds of sugar for every 3 quarts of fruit in her jam recipe. If she has 9 quarts of fruit, how many pounds of sugar will she need?

Method: $2 \times 9 = 18$ $18 \div 3 = 6$ $x = 6$ pounds of sugar

Part (is) 36	% 20
Whole (of) x	100

Percentage

Example: 36 is 20% of what number?

Notice "the part" is associated with "is."

"The whole" is associated with "of."

Method: $36 \times 100 = 3{,}600$ $3{,}600 \div 20 = 180$ $x=180$

Change 700	% x
Original 2,500	100

Percentage of Increase or Decrease

Example: May's property taxes went up from $2,500 to $3,200. By what percent did they increase?

First find the CHANGE (difference).

$$\begin{array}{r} 3{,}200 \\ \underline{-\ 2{,}500} \\ 700 \end{array}$$

Remember to use the ORIGINAL in the lower left corner.

Method: $700 \times 100 = 70{,}000$ $70{,}000 \div 2{,}500 = 28$ $x = 28\%$

Linda's height 5 feet	Flagpole's height x
Linda's shadow 7 feet	Flagpole's shadow 35 feet

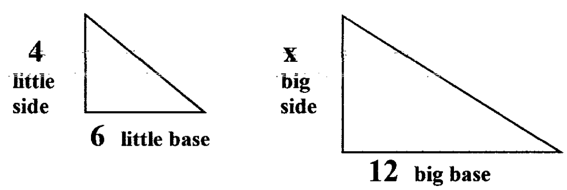
"Flagpole Problem"

Example: Linda is 5 feet tall and casts a shadow that's 7 feet long. If a flagpole next to Linda casts a shadow that is 35 feet long, how tall is the flagpole?
(Hint: Go outside and demonstrate this on a sunny day!)
Method: $5 \times 35 = 175$ $175 \div 7 = 25$ $x = 25$ feet

Little side 4	Big side x
Little base 6	Big base 12

Similar Triangles

If these two triangles are similar, what is the length of side x?
Method: $4 \times 12 = 48$ $48 \div 6 = 8$ Side $x = 6$



Ounces 32	Ounces 224
Quarts 1	Quarts x

Changing Units of Measurement

Example: How many quarts are there in 224 ounces?
Use the unit conversion for two of the grid squares.
e.g. 32 ounces = 1 quart
Method: $1 \times 224 = 224$ $224 \div 32 = 7$ $x = 7$ quarts

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Word Problems

1) Gas mileage is the number of miles you can drive on a a gallon of gasoline. A test of a new car results in 530 miles on 10 gallons of gas. How far could you drive on 70 gallons of gas? What is the car's gas mileage? __________

2) An ice cream factory makes 240 quarts of ice cream in 5 hours. How many quarts could be made in 12 hours? What was that rate per day? __________

3) You can buy 3 apples at the Quick Market for $1.08. You can buy 5 of the same apples at Stop and Save for $2.50. Which place is the better buy? __________

4) A jet travels 460 miles in 2 hours. At this rate, how far could the jet fly in 8 hours? What is the rate of speed of the jet? __________

5) You can buy 5 cans for green beans at the Village Market for $3.60. You can buy 10 of the same cans of beans at Sam's Club for $5.30. Which place is the better buy? __________

6) A ferris wheel can accommodate 35 people in 30 minutes. How many people could ride the ferris wheel in 4 hours? What was that rate per hour? __________

7) The bakers at Healthy Bakery can make 360 bagels in 5 hours. How many bagels can they bake in 19 hours? What was that rate per hour? __________

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Word Problems

1) Gas mileage is the number of miles you can drive on a a gallon of gasoline. A test of a new car results in 530 miles on 10 gallons of gas. How far could you drive on 70 gallons of gas? What is the car's gas mileage?

3710 miles
53 mpg

2) An ice cream factory makes 240 quarts of ice cream in 5 hours. How many quarts could be made in 12 hours? What was that rate per day?

576 quarts
1152 quarts/day

3) You can buy 3 apples at the Quick Market for $1.08. You can buy 5 of the same apples at Stop and Save for $2.50. Which place is the better buy?

Quick Market

4) A jet travels 460 miles in 2 hours. At this rate, how far could the jet fly in 8 hours? What is the rate of speed of the jet?

1840 miles
230 mph

5) You can buy 5 cans for green beans at the Village Market for $3.60. You can buy 10 of the same cans of beans at Sam's Club for $5.30. Which place is the better buy?

Sam's Club

6) A ferris wheel can accommodate 35 people in 30 minutes. How many people could ride the ferris wheel in 4 hours? What was that rate per hour?

280 people
70 people/hour

7) The bakers at Healthy Bakery can make 360 bagels in 5 hours. How many bagels can they bake in 19 hours? What was that rate per hour?

1368 bagels
72 bagels/hour

Tune in to Learning

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Ratios, Averages, & Exponents

1. 

Video: Ratios TV chef Curtis Aikens puts ratios to work in the kitchen.

2. 

Video: Averages Ice skaters Tai Babalonia and Randy Gardner demonstrate how to find an average.

3. 

Video: Phone Plans TV411's math-savvy women estimate and calculate their way to an affordable cell phone plan.

4. 

Video: Pyramid Schemes A federal con-buster reveals the math behind common money scams.

5. 

Video: Road Trip Map NFL members demonstrate how to figure out time and distance on a map when planning a trip.

6. 

Video: Unit Prices Laverne (Liz Torres) explains how unit price labels point the way to the best deal.

Web Lessons

1. Working with Ratios
2. Understanding Mean, Median and Mode
3. Keeping Up With Exponential Growth
4. Get Going with Rate

Print Downloads

1. Think Math: Exponential Growth
2. Think Math: Data Analysis
3. Think Math: Ratios
4. Think Math: Rate

TV 411 is an excellent website designed for adult education students. Short and interesting videos as well as interactive web lessons and worksheets are available for free. The videos really help students see how math is applied to real life in a fun way. Highly recommended!

Ratios and Proportions

If you enjoy cooking, as Curtis Aikens does, you probably know quite a bit of math. Every time you make dressing for one portion of salad, for example, there's more to think about than just vinegar and oil. You're also dealing with **ratios**.

A **ratio** (RAY-she-o) shows a relationship between two numbers or quantities. If your dressing calls for 2 tablespoons of oil and 1 tablespoon of vinegar, the ratio of oil to vinegar is **2 to 1**. This means that for every 2 parts of oil, the dressing has 1 part of vinegar.

The ratio 2 to 1 can be written in two other ways:

- With a colon, 2:1
- As a fraction, $\frac{2}{1}$

The colon is always read as "to," as in "two to one."

TRY IT

Write the following ratios with a colon, and then as a fraction.

1. 5 to 3 ___________ and ___________
2. 21 to 4 ___________ and ___________

Back to our salad dressing. Using 2 tablespoons of oil and 1 tablespoon of vinegar makes dressing for only *your* salad. What if you're making salad for **three** people? **That means you'll need three times as much of each ingredient, since you are cooking for three times as many people as the recipe calls for.** But, you'll want the relationship of the ingredients (2:1) to stay the same.

So here's where your math skills come in. You **multiply** the amount you have of each ingredient by 3 — the number of total salad eaters. Instead of 2 tablespoons of oil and 1 tablespoon of vinegar (for 1 person), you'll need 6 tablespoons of oil and 3 tablespoons of vinegar for 3 people. Even though you are using more oil and more vinegar, the **ratio** of the ingredients stays the same: 6 parts oil to 3 parts vinegar is the same ratio as 2 parts oil to 1 part vinegar. Since you multiplied each ingredient by the same number, the **ratio** of oil to vinegar hasn't changed.

And now you've moved into a new math concept called **proportion** (pro-POR-shun).

A **proportion** is a statement that two ratios are equal: **2:1 = 6:3**, **2 to 1 equals 6 to 3**, or $\frac{2}{1} = \frac{6}{3}$. For our salad dressing, you've kept the same ratio of oil to vinegar — you've just *tripled* (or multiplied by 3) the quantity of the original recipe.

YOUR TURN

Write the ratios. Then set up the proportion.

1. To make chili, you use 3 hot chili peppers to 2 pounds of chopped meat. The ratio of how many hot peppers you use to how much meat you use is ___________. For a Kwanzaa celebration, you want to make **four times** as much chili. The ratio of how many hot peppers you use to how much meat you use is ___________. To show these two ratios are the same, write them as a proportion: _____________.

2. For lemon sauce, the recipe calls for 6 lemons to 2 tablespoons of fresh chopped tarragon. The **ratio** of how many lemons you use to how much tarragon you use is _______. But if you want to want to double the recipe, you multiply the ingredients by 2. What's the new ratio of lemons and tarragon? ________________ Now write the two ratios as a proportion. ________________

MORE PRACTICE

Say you're making a pot of coffee, and you're following the directions on the can:

1. What's the ratio of coffee to water? ________

2. If you use 5 tablespoons of coffee, how many fl. oz. of water will you need to keep the ratio of coffee to water the same? Write your answer as a ratio. _______ (Hint: The directions call for 1 tablespoon of coffee for every 6 fl. oz. of water. If you use 5 tablespoons of coffee, you've multiplied the amount of coffee by 5. What do you need to multiply the amount of water by to keep the ratio of coffee to water the same?)

3. Since the two ratios are the same, they are in proportion. Write the proportion.

   ________________________________.

4. You're having a party. There are 17 men who've accepted the invitation, but only 8 women. What is the ratio of men to women?

   ____

tips Always set up the ratio in the order in which it is written. Here, it is men to women — the number for men first, then the number for women.

5. A punch recipe calls for 3 cups of juice to every 4 cups of water. What is the ratio of juice to water in the punch? ________ You need to double (multiply by 2) the amount the recipe calls for. How many cups of juice and water should you mix? Write your answer as a ratio. ________ Now re-write the answer as a proportion.

   ____

BUT WHAT IF...

Suppose you need **less** of a recipe, not more. You still have to keep the ingredients in proportion. To do that, you **divide** the ingredients by the same number. Dividing all ingredients by the same number keeps the amounts of all of the ingredients in the same proportion as the recipe calls for. Say you were planning to make a pot of coffee, calling for 10 tablespoons of coffee and 60 fluid ounces of water. If you decide to make half as much, just **divide** your ingredients by 2 to keep everything in proportion. Instead of 10 tablespoons of coffee, use 5; instead of 60 ounces of water, use 30: and your coffee will be good to the last drop.

10:60 = 5:30

TRY IT

A recipe calls for 4 cups of cornbread mix and 2 eggs. But you realize you have only one egg left in the fridge. If you want to keep the ratio of cornbread mix to eggs the same as the recipe calls for, how many cups of cornbread mix should you combine with your one egg? __________ Try re-writing the answer as a proportion. ____________

Try It: 2 cups, for the ratio of 2:1 or $\frac{2}{1}$, 4:2 = 2:1 or $\frac{4}{2} = \frac{2}{1}$

More Practice: **1.** 1:6 or $\frac{1}{6}$; **2.** 5:30 or $\frac{5}{30}$; **3.** 1:6 = 5:30 or $\frac{1}{6} = \frac{5}{30}$; **4.** 17:8 or $\frac{17}{8}$; **5.** 3:4 or $\frac{3}{4}$, 6:8 or $\frac{6}{8}$, 3:4 = 6:8 or $\frac{3}{4} = \frac{6}{8}$

ANSWERS: Try It: **1.** 5:3, $\frac{5}{3}$; **2.** 21:4, $\frac{21}{4}$. **Your Turn:** **1.** 3:2 or $\frac{3}{2}$, 12:8 or $\frac{12}{8}$, 3:2 = 12:8 or $\frac{3}{2} = \frac{12}{8}$; **2.** 6:2 or $\frac{6}{2}$, 12:4 or $\frac{12}{4}$, $\frac{6}{2} = \frac{12}{4}$ or 6:2 = 12:4

Percent

1. Demonstrate an understanding of Percent and Apply

a. Understand percent as a rate over 100, *the word percent means "for each hundred"*

b. Draw a representation of common percentages

c. Estimate the percent of a shaded picture and a given number

d. Convert percents to decimals and fractions

e. Find the percent of a number by changing the percent to a decimal and multiplying, by using proportions, and with a calculator

f. Find what percent one number is of another
   - *In a company with 500 employees, 50 called in sick on Super Bowl Sunday. What percent of the employees called in sick on Super Bowl Sunday?* $\frac{50}{500} = \frac{1}{10}$ *= .1 = 10% or* $\frac{50}{500} = \frac{x}{100}$

g. Find a number when given a percent
   - *20% of the employees at Concord Manufacturing called in sick on Super Bowl Sunday. If 80 employees called in sick, how many employees does Concord Manufacturing have in all?*
     $\frac{80}{.2}$ *= 400 or* $\frac{20}{100} = \frac{80}{x}$ *X = 400*

h. Find the rate of change (*the rate of change compares a new amount to an original amount)*
   - *According to the new Census, the population of Dover, NH is 30,500. In the previous Census the population was 29,000. By what percent did the population increase?*
     30,000 – 29,000 = 1,500, $\frac{1500}{30000}$ *= 0.05 = 5%*
     The population increased by 5%

i. Memorize common equivalents: 50% = $\frac{1}{2}$ = .5, 25% = $\frac{1}{4}$ = .25,
   75% = $\frac{3}{4}$ = .75, 10% = $\frac{1}{10}$ = .1

2. Solve Problems using Percent

a. Find simple interest: *I=PRT, Interest = Principle x Rate x Time*

b. Apply percent skills to find discount and sale prices

c. Apply percent skills to calculate taxes

d. Apply percent skills to calculate fees and commissions

e. Apply percent skills to calculate loan interest

Percent Assessment

Name____________________________

Date______________

Write each as a percent. Round to the nearest tenth of a percent.

1) 0.9

2) 0.024

3) 0.14

4) 0.7

Write each as a decimal. Round to the hundredths place.

5) 90%

6) 342%

7) 13%

8) 99%

9) 7%

10) 5%

Write each as a percent. Round to the nearest tenth of a percent.

11) $\frac{3}{5}$

12) $\frac{1}{2}$

13) $\frac{3}{10}$

14) $\frac{3}{4}$

Find the selling price of each item.

15) Original price of socks: $21.50
Discount: 25%

16) Original price of a sled: $30.00
Discount: 10%

17) Original price of a tie: $14.50
Discount: 40%

18) Original price of a book: $35.99
Discount: 20%
Tax: 5%

19) Original price of a microscope: $119.95
Discount: 20%
Tax: 3%

20) Original price of a hat: $38.99
Discount: 50%
Tax: 2%

Find each percent change. Round to the nearest tenth of a percent. State if it is an increase or decrease.

21) From 16 to 18

22) From 19 to 8

Use simple interest to find the ending balance.

23) $1,250 at 13% for 2 years

24) $50,100 at 14% for 2 years

Answers to

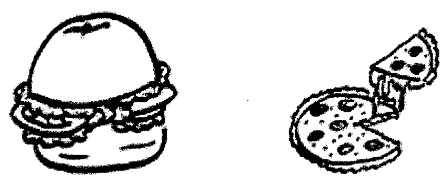
1) 90%	2) 2.4%	3) 14%	4) 70%
5) 0.9	6) 3.42	7) 0.13	8) 0.99
9) 0.07	10) 0.05	11) 60%	12) 50%
13) 30%	14) 75%	15) $16.13	16) $27.00
17) $8.70	18) $30.23	19) $98.84	20) $19.88
21) 12.5% increase	22) 57.9% decrease	23) $1,575.00	24) $64,128.00

New Hampshire Meals Tax

New Hampshire does not impose any form of general sales tax upon the sale or use of tangible personal property within the state. New Hampshire does, however, levy a tax on meals, room occupancies, and motor vehicle rentals.

In New Hampshire any food or beverage that is prepared and served by a restaurant, whether served for consumption on or off the restaurant premises, is considered to be a meal.

The New Hampshire meals tax rate is 9%. The rooms tax is imposed on any occupancy in a hotel or any similar establishment offering sleeping accommodations in the State of New Hampshire. The tax rate is currently 9% of the rent for each occupancy. A motor vehicle rental tax is imposed under the meals and room tax classification at a rate of 9% on the gross rental receipts of each rental.



1. If you order a pizza for $15.00, what will you pay in taxes? ________
2. What will the total cost be, pizza and tax? ______________________
3. If you order $10.00 worth of food at Wendy's, how much will you pay in tax? ______________________________
4. What will your total cost be at Wendy's? ______________________
5. If you and a friend spend $22.50 for a pizza and drinks, what will the total cost be including taxes? ____________
6. How much will each friend owe for the pizza and drinks if the bill is split evenly? _________________________
7. 10 friends go out to eat and the bill is $125.00. They decide to leave a 20% tip. How much will the tip be? ___________________________
   How much will the tax be? _____________
   What will the total be for meal, tax, and tip? __________________

DMR

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Common Percent Table

Fraction	Decimal	Percent
$\frac{1}{20}$	0.05	
$\frac{1}{10}$		10 %
$\frac{1}{5}$	0.2	
$\frac{1}{4}$		25 %
$\frac{3}{10}$		30 %
$\frac{1}{3}$		$33\frac{1}{3}$ %
$\frac{2}{5}$	0.4	
$\frac{1}{2}$	0.5	
	0.6	60 %
	$0.\overline{6}$	$66\frac{2}{3}$ %
	0.7	70 %
	0.75	75 %
$\frac{4}{5}$	0.8	
$\frac{9}{10}$		90 %

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Common Percent Table

Fraction	Decimal	Percent
$\frac{1}{20}$	0.05	5 %
$\frac{1}{10}$	0.1	10 %
$\frac{1}{5}$	0.2	20 %
$\frac{1}{4}$	0.25	25 %
$\frac{3}{10}$	0.3	30 %
$\frac{1}{3}$	$0.\bar{3}$	$33\frac{1}{3}$ %
$\frac{2}{5}$	0.4	40 %
$\frac{1}{2}$	0.5	50 %
$\frac{3}{5}$	0.6	60 %
$\frac{2}{3}$	$0.\bar{6}$	$66\frac{2}{3}$ %
$\frac{7}{10}$	0.7	70 %
$\frac{3}{4}$	0.75	75 %
$\frac{4}{5}$	0.8	80 %
$\frac{9}{10}$	0.9	90 %

Tractors, Discounts, and Taxes

A Problem Solving Exercise

Name: ____________________ **Date:** ____________________

The Story

Jim's farm is located between three towns. Each town has a farm equipment dealer selling the new LandMaster 3000-SX tractor he wants to buy. He made a call to each dealer to find out what he could get it for.

The dealer in Midtown said, "Our price is $8,195, and Midtown sales tax is 5%."

The dealer in Riverside said, "It's in stock for $8,849, but Riverside's sales tax on farm equipment is only 3%, so you should buy it here. Also, all LandMaster tractors are 7% off this week!"

The dealer in Andover said, "We have the 3000-SX for $9,339, but for you, Jim, I'm going to knock 15% off that price. The sales tax here is 6.5%."

The Problem

Which dealer will give Jim the best final price for the tractor?

Hints

- ❑ Calculate each offer to include any discount and tax.
- ❑ Try organizing the information in a table.

Extra

Before Jim left, he calculated the final price from each dealer. However he incorrectly computed the sales tax using the sticker price, not the discounted price. He was happily surprised when he paid less than he thought he would, but he could have done better by going to another dealer. Which dealer did he go to, and how much more did he pay for the tractor than if he had gone to the dealer with the true lowest total price?

Tractors, Discounts, and Taxes

A Problem Solving Exercise

Solution

The **Andover dealer** has the best final price.

The best way to start this problem is to create a table to organize all the pieces of data. If students have trouble translating the scenario into data, the table will easily reveal where they went wrong. The following table shows how we organized it.

Dealer	Price	Discount %	Discount	Tax %	Tax	Final Price
Midtown	$8,195.00	0%	0	5%	$409.75	$8,604.75
Riverside	$8,849.00	7%	$619.43	3%	$246.89	$8,476.46
Andover	$9,339.00	15%	$1,400.85	6.5%	$515.98	$8,454.13

The table reveals that the dealer with the highest price and highest tax rate also offers the lowest final price due to the big discount. The margin is very small so any mistake in computation has a good chance of causing the wrong dealer to be chosen, as the extra problem demonstrates.

Extra

He went to the Riverside dealer and overpaid by $40.91.

It is common for people to forget that tax is only charged on the price after a discount is applied. Here is what the calculations look like when the tax is incorrectly computed on the sticker price.

Dealer	Price	Discount %	Discount	Tax %	Tax	Final Price
Midtown	$8,195.00	0%	0	5%	$409.75	$8,604.75
Riverside	$8,849.00	7%	$619.43	3%	$265.46	$8,495.04
Andover	$9,339.00	15%	$1,400.85	6.5%	$607.04	$8,545.19

Using this information and that from the solution table, we can see that Jim would have gone to the Riverside dealer instead of Andover. He also paid $18.58 less than he thought he was, but he would have been even happier if he could have saved the other $40.

Lunch at Benny's

A Problem Solving Exercise

Name: ______________________ **Date:** ______________________

The Scenario

Mr. Jones brought his three children to Benny's Diner to have lunch today.

James, who is twelve years old, ordered a bacon cheeseburger plate. It had a menu price of $6.50. Mr. Jones ordered a chef's salad which was $2.70 more. Sarah, age seven, and Samantha, age six, both ordered grilled cheese sandwiches. Each of them also ordered a drink which cost $1.20 each.

When the bill came, Mr. Jones noticed that the total of the prices of the items (before tax) was exactly $30.

The Problem

What was the menu price of one grilled cheese sandwich?

Hints

- ❑ Try to find out the prices of the other items first.
- ❑ A good strategy might be to work backwards from the total after you determine the other prices.

Extra

What will Mr. Jones pay for lunch if the tax rate is 8%?

Lunch at Benny's

A Problem Solving Exercise

Solution

This problem requires the use of addition, subtraction, multiplication and division of decimals before the cost of one grilled cheese sandwich can be revealed. Addition is needed to identify the cost of the salad, multiplication for the total of the drinks, subtraction to find out the cost of both sandwiches and division to get the final answer. At any step along the way, students might forget that a certain step is needed. Fortunately for them, any misstep will result in a sandwich price which doesn't make sense. Based on the price of the bacon cheeseburger, a grilled cheese sandwich should be somewhere in the three to six dollar range. If their result comes out to be over the price of the salad, they should know that something is wrong, such as forgetting to divide by two.

We know from the problem description what the bacon cheeseburger cost and what each of the drinks cost. We can also do some decimal addition and establish what the salad cost. We don't know what the grilled cheese sandwiches cost, but we do have the total of all the items. We can start by adding up the known values:

$$\$6.50 + (\$6.50 + \$2.70) + (4 \times \$1.20) = \$20.50$$

Since the total of the items was $30, we can subtract the total of the known items and come out with the difference:

$$\$30.00 - \$20.50 = \$9.50$$

There were two grilled cheese sandwiches, so we have to divide this result by two.

$$\$9.50 / 2 = \$4.75$$

So the price of one grilled cheese sandwich is $4.75.

We can verify this by adding up all the items again:

$$\$6.50 + (\$6.50 + \$2.70) + (4 \times \$1.20) + (2 \times \$4.75) = \$30.00$$

Solution to Extra

We can multiply $30 by .08 to find out the amount of tax on the bill, or we can multiply it by 1.08 to get the total including tax. Since the problem asked for the total amount paid, we should use the second approach:

$$\$30.00 \times 1.08 = \$32.40$$

The total paid by Mr. Jones is $32.40.

VI. Data Analysis

1. **Represent and Interpret Data**

   a. Draw a picture graph and a bar graph, with a single-unit scale, to represent a data set with several categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph
   b. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one and two step "how many more" and "how many less" problems using information presented in scaled bar graph
      - *Draw a bar graph in which each square in the graph might represent 5 dogs*
   c. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units….whole numbers, halves, or quarters

2. **Apply Math and Reasoning Skills to Analyze Data**

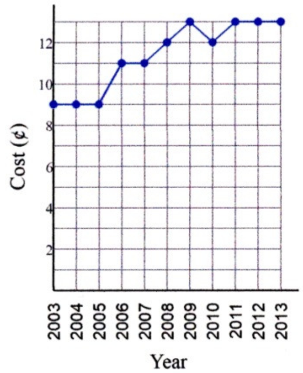
   a. Analyze and interpret data presented in a table
      - *A table displays information in rows and columns*
   b. Analyze and interpret data presented in a bar graph
      - *A bar graph organizes information along a vertical and horizontal axis*
   c. Analyze and interpret data presented in a line graph
      - line graphs are used to show trends, patterns, or changes over time
   d. Analyze and interpret data presented in a circle or pie graph
      - *These graphs represent a whole amount, and each section represents a percentage of that whole*
   e. Find the mean, median, mode, and range when presented with graphic information

Data Analysis Name____________________________________

Date________________

Find the mode, median, and mean for each data set.

1) Cost of Electricity, per kWh



2) Average Lifespan

Animal	Years
Guinea Pig	8
Horse	40
Parrot	80

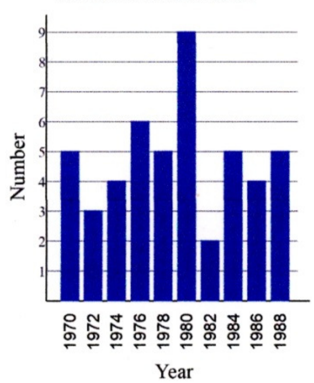
Animal	Years
Newt	7
Pionus Parrot	15

Animal	Years
Rattlesnake	22
Cat	25

Animal	Years
Canada Goose	33
Hare	10

Animal	Years
American Toad	15
Bottlenose Dolphin	20

3) Atlantic Hurricanes



Find the lower quartile, upper quartile, and interquartile range for each data set.

4) Average Time to Maturity

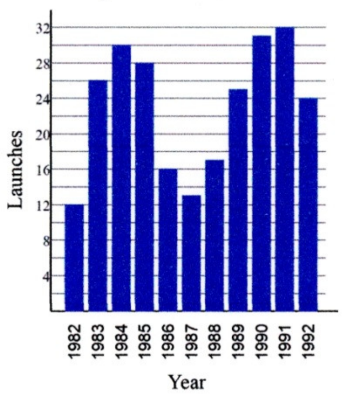
Plant	Days
Radish	22
Bok Choi	45
Roma II Bush Bean	53

Plant	Days
Kale	60
Collards	62
Cherry Tomato	65

Plant	Days
Soy Bean	70
Broccoli	71
Honeydew	80

Plant	Days
Seedless Watermelon	85
Garlic	120

5) US Spacecraft Launches

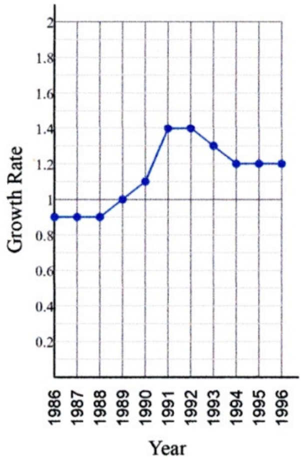


6) Life Expectancy

Country	Years
Congo (DRC)	49.5
Guinea	55
Malawi	58
Rwanda	60
Russia	70.5
Belarus	72.5

Country	Years
Cape Verde	74.5
Tunisia	74.6
Portugal	80
Iceland	83.3
Singapore	84

7) US Population Growth



Answers to

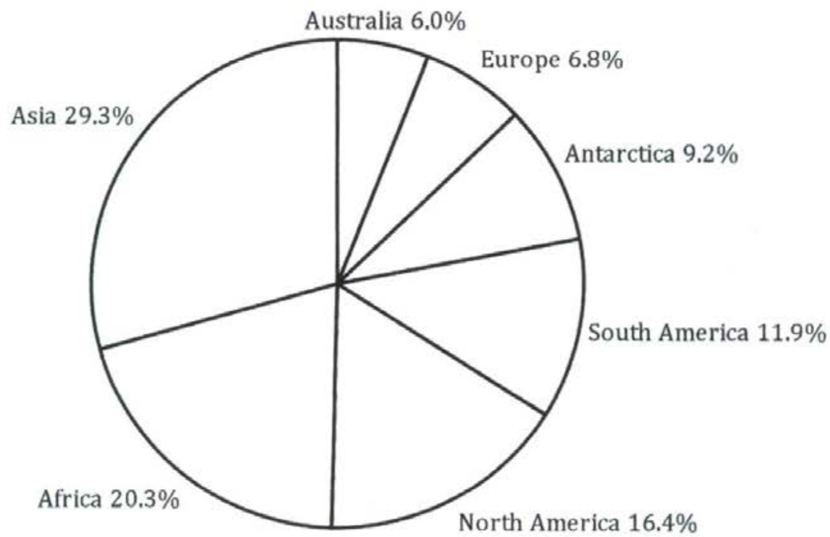
1) Mode = 13, Median = 12 and Mean = 11.36
2) Mode = 15, Median = 20 and Mean = 25
3) Mode = 5, Median = 5 and Mean = 4.8
4) $Q_1 = 53$, $Q_3 = 80$ and IQR = 27
5) $Q_1 = 16$, $Q_3 = 30$ and IQR = 14
6) $Q_1 = 58$, $Q_3 = 80$ and IQR = 22
7) $Q_1 = 0.9$, $Q_3 = 1.3$ and IQR = 0.4

Interpreting Circle Graphs (B)

Answer the questions about the circle graph.

Land Area of Continents

As a percentage of the total land area of the Earth.



Source of data: http://en.wikipedia.org/wiki/Continents

What are the largest and smallest continents?

What percentage of the world's land area is made up of the Americas?

What is larger: Africa or Australia, Europe and Antarctica put together?

If the land area of Africa is about 30 million square kilometers, what is the approximate land area of the Earth?

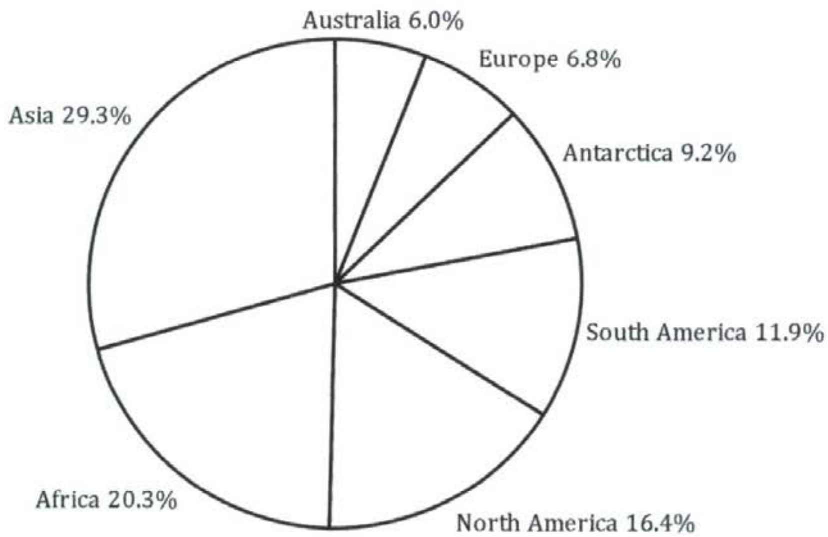
Is the size of the continent related to the number of people who live on that continent? Explain your answer.

Interpreting Circle Graphs (B) Answers

Answer the questions about the circle graph.

Land Area of Continents

As a percentage of the total land area of the Earth.



Source of data: http://en.wikipedia.org/wiki/Continents

What are the largest and smallest continents?

Largest is Asia and the smallest is Australia

What percentage of the world's land area is made up of the Americas?

28.30%

What is larger: Africa or Australia, Europe and Antarctica put together?

Australia, Europe and Antarctica together (22.0%) is larger than Africa (20.3%)

If the land area of Africa is about 30 million square kilometers, what is the approximate land area of the Earth?

Africa is about 1/5 of the land area, so multiply 30 million by 5 to get 150 million sq. km.

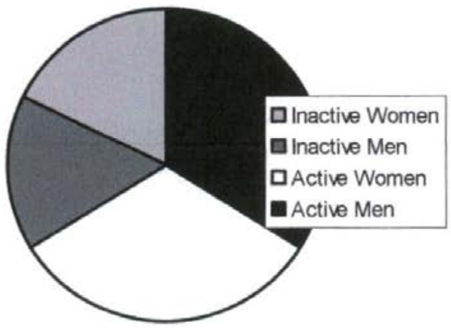
Is the size of the continent related to the number of people who live on that continent? Explain your answer.

No. Asia and Africa fit this model, but the other continents don't.

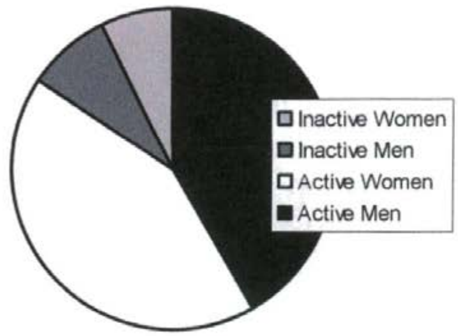
Interpreting Circle Graphs (G)

Answer the questions about the circle graph.

Canadian Exercise Rates 2012



Greek Exercise Rates 2012



Source of data: http://www.theguardian.com/news/datablog/2012/jul/18/physical-inactivity-country-laziest

Which country has a greater percent of people who exercise?

About what percent of the Canadian population is inactive?

About what percent of the Greek population is active?

In which country are the men more inactive than the women?

Why do you think Greeks are more active than Canadians?

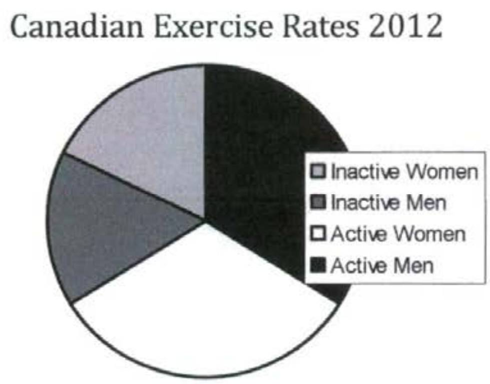
There are about 11 million people in Greece. About how many Greeks are active?

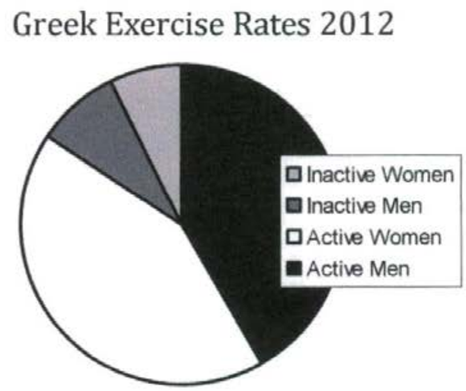
There are about 35 million people in Canada. About how many Canadian men are inactive?

How would the graph for your school look?

Interpreting Circle Graphs (G) Answers

Answer the questions about the circle graph.





Source of data: http://www.theguardian.com/news/datablog/2012/jul/18/physical-inactivity-country-laziest

Which country has a greater percent of people who exercise?

Greece

About what percent of the Canadian population is inactive?

Actual 34%

About what percent of the Greek population is active?

Actual 84%

In which country are the men more inactive than the women?

Greece.

Why do you think Greeks are more active than Canadians?

Many answers possible.

There are about 11 million people in Greece. About how many Greeks are active?

11 * 0.84 = 9.24 million. 84% can be derived from a previous question.

There are about 35 million people in Canada. About how many Canadian men are inactive?

Inactive men: about 16%, so 35 * 0.16 = 5.6 million men are inactive.

How would the graph for your school look?

Various answers possible. Hopefully, more active than both these graphs :-)

Name: ______________________ Class: ____________________ Date: ____________________

Fatality Rates with Snake Bites

Data was collected on all recorded cases of bites from each of these different species of venomous snakes. The death rate percentage was calculated for each snake.

Type of Snake	Death Rate (%)
Black Mamba	75
Bushmaster	80
Copperhead	1
Eastern coral snake	15
European viper	5
Asp Viper	20
Indian krait	77
King cobra	33
Death adder	50
Tiger Snake	60

The purpose of this study is to make a comparison of the different types of venomous snakes. In this case, a bar graph would be the most appropriate type to use. Below, make a bar graph of the venomous snake death rate data. Remember, to follow all the rules of graph construction you learned!

Name: ____________________ Class: ________________ Date: ________________

3. Which snake is the deadliest, according to this data?

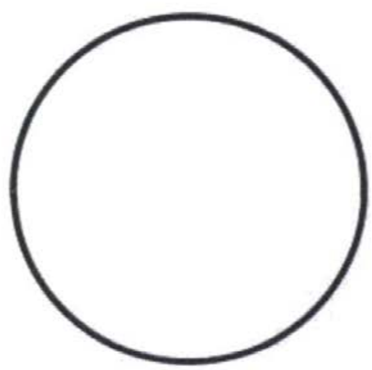
4. Why are **bar graphs** a good option for displaying data that is for comparison?

Elements of the Human Body

The human body contains a consistent mix of only handful of the known elements. The chart below represents the percentage by mass of each of these elements. Note: Trace elements that account for less than 0.1% of the human body mass have been excluded from this data.

Element	Percent by Mass	Element	Percent by Mass
Oxygen	65	Phosphorus	1.0
Carbon	18	Potassium	0.4
Hydrogen	10	Sulfur	0.3
Nitrogen	3	Sodium	0.2
Calcium	1.5	Magnesium	0.1

Data like this that adds up to a full 100% can be conveniently displayed using a **pie chart**. To make one of these charts, start with a circle and create a segment for the largest percentage first. Then, begin making smaller segments to account for each of the other data points. Label each portion of the pie chart.



5. Why are pie charts a good way to display data that adds up to 100%?

VII. Statistics and Probability

1. Develop Understanding of Statistical Variability

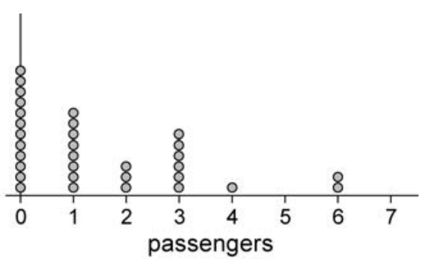
a. Recognize a statistical question. A statistical question is one that has several possible answers (variability), rather than a single answer
   - *Example, "How old am I?" is not a statistical question because it has just one answer, but "How old are the students in my school?" is a statistical question because there are several possible answers for students' ages*

b. Understand that a set of data collected to answer a statistical question has a distribution, such as a bell curve, which can be described by its center, spread, and overall shape

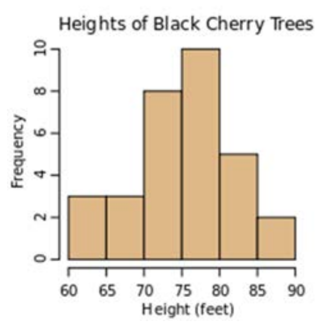
c. Recognize that a measure of center (mean, median, or mode) for a numerical data set summarizes all of its values with a single number, while a measure of variation (range, standard deviation, or mean absolute deviation) describes how its values vary with a single number

2. Summarize and Describe Distributions

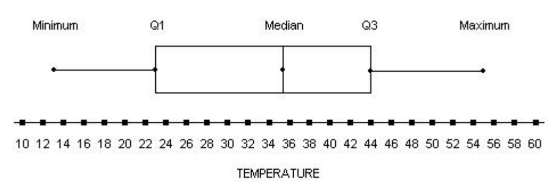
a. Display numerical data in plots on a number line, including dot plots, histograms, and box plots



Sample Dot Plot
Represents each data point with a dot



Sample Histogram bars
Represents frequency for each class of data



Sample Box Plot
Represents spread of the data with quartiles, median, and max/min of data

b. Summarize numerical data sets in relation to their context, such as by:
   - *Reporting the number of observations in the data set*
   - *Describing the data set, including how it was measured and its units of measurement*
   - *Calculating the measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any outliers in the data set.*
     Median: the middle value in an ordered data set
     Mean: the average value of the data set
     Interquartile range: the distance between quartile 1 and quartile 3 (represents the middle 50% of the data)
     Mean absolute deviation: represents the average distance from the mean for the data points
     - *This is calculated by the following steps:*
       subtract each data item from the mean to find its deviation
       add all deviations
       divide by the number of data items to find the average
   - *Being able to choose measurements of center and variability as well as appropriate visual displays for the data set*

3. Use Random Sampling to Draw Inferences about a Population

a. Understand that statistics can be used to gain information about a population by examining a sample of the population
   - *A sample has to be representative of a population in order to make generalizations about a population*
   - *Random sampling is the best way to produce representative samples and to be able to support valid inferences*

b. Use data from a random sample to draw inferences about a population to answer a particular question about the population
   - *Example: In a poll of 50 people at a State House rally, 67% of the people there stated they are in favor of a particular candidate for President. Should the local newspaper report that 67% of the voters in New Hampshire are in favor of this particular candidate? Why or why not? If not, what other methods of sampling would produce more valid results?*

c. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions

- *For example, estimate the mean word length in a book by randomly sampling words from the book. Gauge how far off the estimate or prediction might be*

4. Draw Informal Comparative Inferences about Two Populations

a. Compare and contrast two similar sets of data. Note the differences in the mean as well as in the variability
   - *Example: The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team. The mean height is also about twice the variability (mean absolute deviation) on either team; additionally, when the data from each sample is represented on a dot plot, the separation between the two distributions of heights is noticeable*

b. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations (compare the two populations)
   - *Example: Decide whether the words in one chapter of a science book are generally longer or shorter than the words in another chapter of a lower level science book*

5. Investigate Chance Processes and Develop, Use, and Evaluate Probability Models

a. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring
   - *Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely (50-50 chance), and a probability near 1 indicates a likely event*

b. Calculate experimental or empirical probability of a chance event. Collect data on the event by performing experiments (such as rolling a die) and observe the frequency over a large number of outcomes. Predict the approximate frequency given the probability of the event.
   - *If you know the probability of rolling a 1 on a standard die is $\frac{1}{6}$, predict the number of times a 1 will be rolled in 100 rolls of the die*
   - *When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times*

c. Determine the theoretical (predicted) probability of an event and use it to find probabilities of events. Compare theoretical probabilities to observed frequencies from experiments; if there are differences between the two results, explain possible sources of the discrepancy

d. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events
    - *Example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected (find a sample lesson in the resource section)*

e. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process
    - *Find the approximate probability that a spinning penny will land heads up. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*
    - *Find the approximate probability that a tossed paper cup will land open-end down. What are the possible outcomes for this experiment? Do they appear to be equally likely?*

6. Find Probabilities of Compound Events using Organized Lists, Tables, Tree Diagrams, and Simulation
    - *Compound events are the result of combining two or more events, such as rolling a 2 or a 6 on a die, and sample space is the set of all possible outcomes for an event. For example: Rolling 1 standard die produces the sample space {1,2,3,4,5,6}*

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs
    - *For example: P (Rolling a number 5 or greater on a standard die) is found by combining the probabilities for rolling a 5 or rolling a 6. So the result is* $\frac{1}{6}+\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$*, since 2 of the six outcomes are 5 or greater*

b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language, such as “rolling double sixes”, identify the outcomes in the sample space which compose the event

- *Example: “rolling double sixes” means rolling a 6 and a 6 when rolling two standard dice. Creating a table of possible outcomes for two standard dice is a good way to visualize the results and determine the probability of this compound event:*

Dice #1

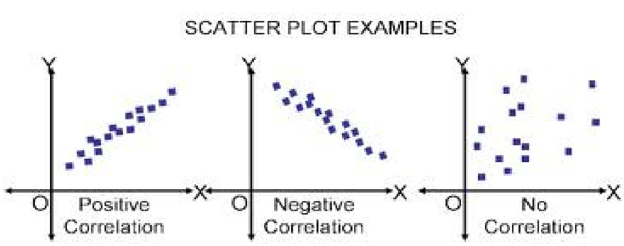
Dice #2	1	2	3	4	5	6
1	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(6,1)
2	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(6,2)
3	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(6,3)
4	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(6,4)
5	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(6,5)
6	(1,6)	(2,6)	(3,6)	(4,6)	(5,6)	(6,6)

Using the table allows the student to see that double sixes only occur 1 time in 36 outcomes, so the probability is $\frac{1}{36}$.

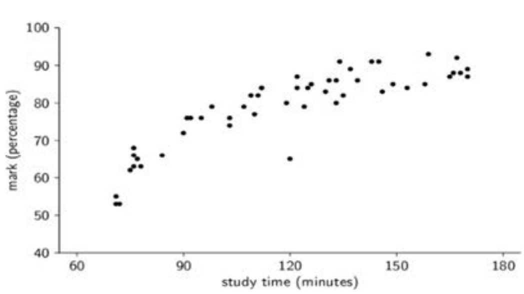
c. Design and use a simulation to generate frequencies for compound events
   - *Example: Suppose each box of a popular brand of cereal contains a pen as a prize. The pens come in four colors, blue, red, green and yellow. Each color of pen is equally likely to appear in any box of cereal. Design and carry out a simulation to help you answer each of the following questions.*
     The lesson can be found in with the resources at the end of this section

7. Investigate Patterns of Association in 2-variable (Bivariate) Data

a. Construct and interpret scatter plots for bivariate measurement data *(such as height/weight growth charts or time/distance graphs)* to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

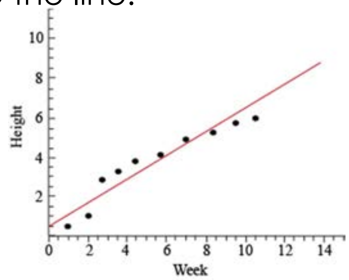


Here, the first two graphs display clustering of data points. The first graph has a positive, linear association. The second graph has a negative, linear association. The far right graph has no visible pattern to the data points, so no correlation is observed.

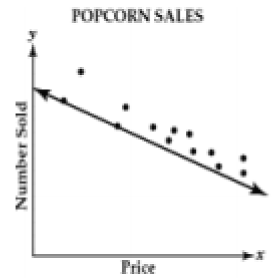


Here, the graph has clustering that appears to be positive and linear association. Observe the outlier of the data that stands alone below the clustered points.

b. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, draw a straight line closely representing the data, and determine whether the line "fits" the data well or not by judging the closeness of the data points to the line.



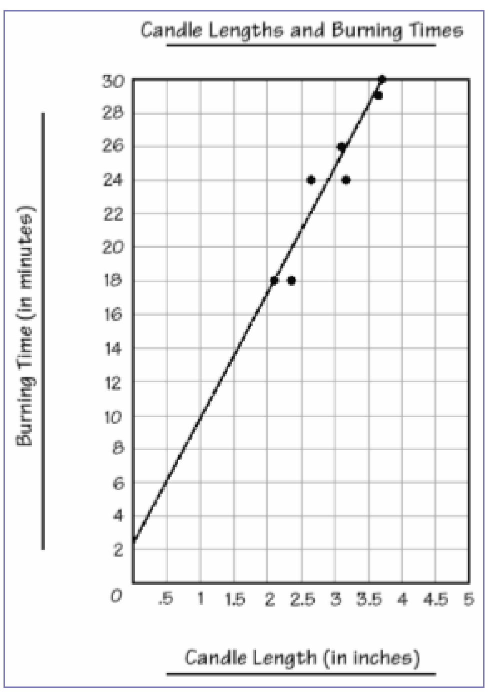
This graph shows a line that "fits" the data well. There is an even distribution of points on either side of the line.



This graph shows a line that doesn't fit the data well. Most of the points are strictly on one side of the line.

c. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

- *Example: In the linear model for a chemistry experiment shown below, the slope is found to be 7 min/in, and the y-intercept is 3 min. The slope is interpreted to mean that it takes 7 minutes for a candle to burn 1 inch.*



To calculate slope, choose two points on the line. For example, the y-intercept is at (0, 3) and the point (1,10) is also on the line. The slope is found by finding $\frac{change\ in\ y}{change\ in\ x}, \frac{10-3}{1-0} = \frac{7}{1} = 7$

d. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table

- *Example: The two-way table shown below displays data for two variables- ice cream flavor and person's age. These variables are categorized as shown in the table.*

What flavor of ice cream would you pick?			
	Chocolate	Vanilla	Neither
Children	40	22	15
Teens	12	16	45
Adults	55	54	10
Total	107	92	70

e. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects

- *Example: Collect data from students in your class on whether or not they like to cook and whether they participate actively in a sport. Is there evidence that those who like to cook also tend to play sports?*

	Like to Cook?			
Like Sports?		Yes	No	Total
	Yes	16	3	19
	No	4	1	5
	Total	20	4	24

f. Use relative frequencies calculated for rows or columns to describe possible association between the two variables

- *Example: Of the 20 people who like to cook, 16 of them like sports, so we could conclude that there is an association between the variables of cooking and sports for this data.*

Also, observe that this set of data shows 19 of the 24 people asked like to cook, and 20 of the 24 people asked like sports. Overall, this set of data indicates a connection between cooking and sports.

Statistics   Name________________________

Date______________

Find the mean for each data set.

1) Average Lifespan

Animal	Years
Ant (Worker)	1.5
Leopard Frog	6
Superb Parrot	36
Quail	6
Catfish	60
Bat	24

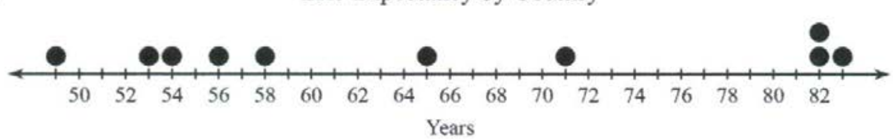
Animal	Years
Fox	14
Mosquitofish	2
Toad	36
Rhinoceros	40
Ferret	12

2) Sales Tax by State

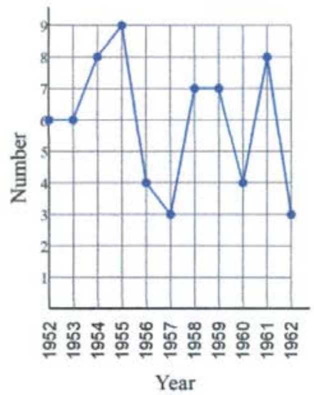
Percent	Frequency
0	2
4	2
4.3	1
5.6	1
6	4
7	1

Find the median for each data set.

3) Life Expectancy by Country

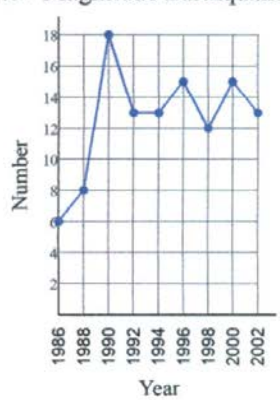


4) Atlantic Hurricanes



Find the mode for each data set.

5) 7.0+ Magnitude Earthquakes



6) 2012 Summer Olympics

Country	Medals
Spain	17
Russia	81
Norway	4
Iran	12
Australia	35

Country	Medals
Germany	44
Switzerland	4
Kenya	11
Canada	18

Find the range for each data set.

7) Average Lifespan

Animal	Years
American Alligator	56
Tiger Salamander	11
Golden Hamster	4
Hog	18
Box Turtle	123

Animal	Years
Fence Lizard	4
Gerbil	5
Parakeet	18
Tree Frog	14

Animal	Years
Mountain Lion	20
Boa Constrictor	23
Nutria	15
Dog, large	10

Animal	Years
Humming Bird	8
Wombat	15
Bear	40
Woodchuck	15

8) Mountain Heights

Name	Feet
Istor-o-Nal	24,288
Chomo Lonzo	25,604
Diran	23,839
Chongtar	23,999

Name	Feet
Kirat Chuli	24,153
Annapurna I	26,545
Distaghil Sar	25,866
Chogolisa	25,148

Name	Feet
Churen Himal	24,229
Annapurna Dakshin	23,684
Noijin Kangsang	23,642
Yutmaru Sar	23,894

Name	Feet
Khartaphu	23,665
Baintha Brakk	23,901
Siguang Ri	23,980

Find the mean absolute deviation for each data set.

9) Average Time to Maturity

Plant	Days
Tomatillo	100
Lima Bean	75
Eggplant	74
Leek	100
Radicchio	90
Parsnip	105

Plant	Days
Iceburg Lettuce	85
Brussel Sprouts	90
Honeydew	80
Kale	60
Cucumber	58

10) Large US Cities

City	Population
Virginia Beach	437,994
New York	8,175,133
Orlando	238,300
Glendale	226,721
St. Louis	319,294
Boston	617,594
Atlanta	420,003
Gilbert	208,453
Irvine	212,375

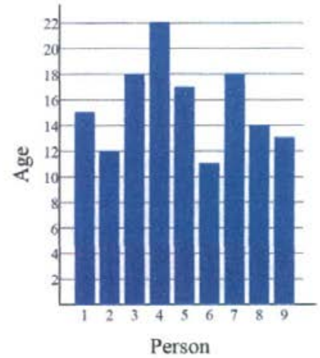
Find the interquartile range for each data set.

11) Life Expectancy

Country	Years
Ukraine	68.25
Nepal	69
Vietnam	75
Chad	51
Philippines	73

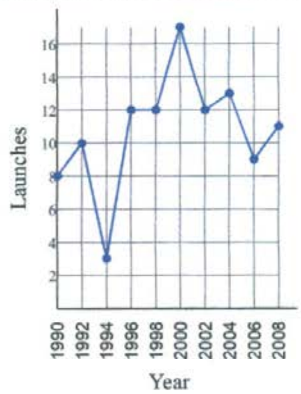
Country	Years
Rep. of Congo	58
Niger	56
Suriname	74.5
Czech Republic	78
Turkmenistan	66.5

12) Age at First Job



Find the mode, median, mean, and range for each data set.

13) European Spacecraft Launches



14) Length of Book Titles

# Words	Frequency
1	2
2	4
3	2
4	2

Answers to

1) 21.59	2) 4.45	3) 61.5	4) 6
5) 13	6) 4	7) 119	8) 2,903
9) 12.69	10) 1,548,650.12	11) 16.5	12) 5.5

13) Mode = 12, Median = 11.5, Mean = 10.7 and Range = 14

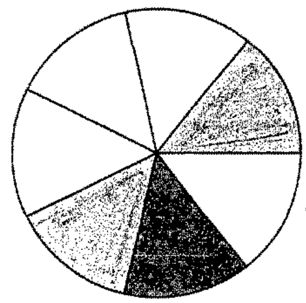
14) Mode = 2, Median = 2, Mean = 2.4 and Range = 3

Basic Probability

Name:

Use each diagram to solve the problems.

1) How many pieces are there total in the spinner?

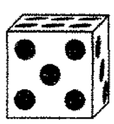


2) If you spun the spinner 1 time, what is the probability it would land on a gray piece?

3) If you spun the spinner 1 time, what is the probability it would land on a black piece?

4) If you spun the spinner 1 time, what is the probability it would land on a white piece?

5) If you spun the spinner 1 time, what is the probability of landing on either a white piece or a black piece?

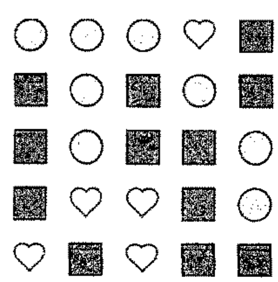


6) If you were to roll the dice one time what is the probability it will land on a 3?

7) If you were to roll the dice one time what is the probability it will NOT land on a 2?

8) If you were to roll the dice one time, what is the probability of it landing on an even number?

9) How many shapes are there total in the array?



10) If you were to select 1 shape at random from the array, what is the probability it will be a circle?

11) If you were to select 1 shape at random from the array, what shape do you have the greatest probability of selecting?

12) Which shape has a 32% chance (8 out of 25) of being selected?

Answers

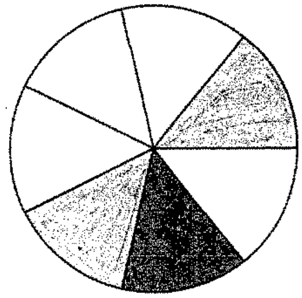
1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________

1-10	92	83	75	67	58	50	42	33	25	17
11-12	8	0								

Basic Probability Name: **Answer Key**

Use each diagram to solve the problems.

1) How many pieces are there total in the spinner?

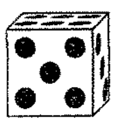


2) If you spun the spinner 1 time, what is the probability it would land on a gray piece?

3) If you spun the spinner 1 time, what is the probability it would land on a black piece?

4) If you spun the spinner 1 time, what is the probability it would land on a white piece?

5) If you spun the spinner 1 time, what is the probability of landing on either a white piece or a black piece?



6) If you were to roll the dice one time what is the probability it will land on a 3?

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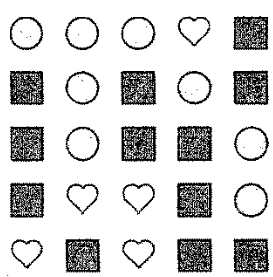
9) How many shapes are there total in the array?



10) If you were to select 1 shape at random from the array, what is the probability it will be a circle?

11) If you were to select 1 shape at random from the array, what shape do you have the greatest probability of selecting?

12) Which shape has a 32% chance (8 out of 25) of being selected?

Answers

1. 7
2. 2 out of 7
3. 1 out of 7
4. 4 out of 7
5. 5 out of 7
6. 1 out of 6
7. 5 out of 6
8. 3 out of 6
9. 25
10. 8 out of 25
11. square
12. circle

1-10	92	83	75	67	58	50	42	33	25	17
11-12	8	0								

Definitions for Probability

Probability

Probability is the likelihood of the occurance of an event. The probability of event A is written P(A). Probabilities are always numbers between 0 and 1, inclusive.

The four basic rules of probability:

1) For any event A, $0 \leq P(A) \leq 1$.
2) P(impossible event) = 0.
   Also written P(empty set) = 0 or $P(\emptyset) = 0$.
3) P(sure event) = 1.
   Also written P(S) = 1, where S is the sample set.
4) P(not A) = 1 - P(A).
   Also written P(complement of A) = 1 - P(A) or
   $P(A^C) = 1 - P(A)$ or $P(\bar{A}) = 1 - P(A)$.

Experiment

In the study of probability, the name given to any controlled and repeatable process.

Event

A set of possible outcomes resulting from a particular experiment.

Outcome

A single specific possible result of an experiment.

Experiment	Outcomes
Tossing a coin	Heads, Tails
Rolling a six sided die	1,2,3,4,5,6

Probability : Independent Events

Independent Events

Independent Events are not affected by previous Events.

A coin does not "know" it came up heads before ...

... each toss of a coin is a perfect isolated event.

When rolling a pair of dice, one die does not affect the outcome of the other die ...

... each die is an isolated event.

$$\textbf{Probability of an event happening} = \frac{\textbf{Number of ways it can happen}}{\textbf{Total number of outcomes}}$$

Probability of getting a "Head" when tossing a coin?

$$P(\text{Head}) = \frac{\text{"Head"}}{\text{"Head and Tail"}} = \frac{1}{2}$$

Probability of rolling a "4" on a die?

$$P(4) = \frac{\text{"4"}}{\text{"1","2","3","4","5","6"}} = \frac{1}{6}$$

Probability : Independent Events

Two or More Events

You can calculate the probability of two or more Events by multipling the individual probabilities.

So, for Independent Events:

P(A and B) = P(A) x P(B)

Example: Probability of 3 Heads in a Row

For each toss of a coin a "Head" has a probability of 0.5 :

0.5

0.5 x 0.5 = 0.25

0.5 x 0.5 x 0.5 = 0.125

So the Probability of getting three Heads in a Row is 0.125.

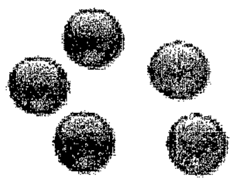
Conditional Probability : Dependent Events

Dependent Events

Dependent Events are affected by previous events.

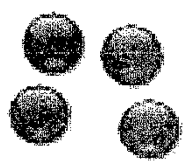
Example: Marbles in a Bag

There are 3 blue and 2 red marbles in a bag.
What is the probability of drawing a blue marble on the first and second draw?



$$P(\text{Blue 1st Draw}) = \frac{3}{5}$$

after the first draw you have changed the chances for the next draw



$$P(\text{Blue 2nd Draw}) = \frac{2}{4} = \frac{1}{2}$$

The probability of

$$P(\text{Blue 1st Draw and Blue 2nd Draw}) = P(\text{Blue 1st Draw}) \times P(\text{Blue 2nd Draw})$$

$$P(\text{Blue 1st Draw and Blue 2nd Draw}) = \frac{3}{5} \times \frac{1}{2}$$

$$P(\text{Blue 1st Draw and Blue 2nd Draw}) = \frac{3}{10}$$

Replacement

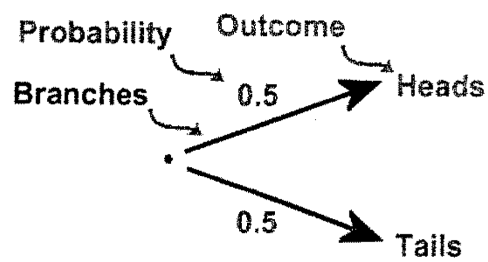
Note: if you had replaced the marbles in the bag each time, then the chances would not have changed and the events would be independent:

- With Replacement: the Events are Independent (the chances don't change)
- Without Replacement: the Events are Dependent (the chances change)

Probability Tree Diagrams

Calculating probability is confusing at times, especially for multiple events.
Tree diagrams give you a visual and more simple way to solve complex probability problems.
The diagrams are composed of three items: Branches, Probabilites, and Outcomes.

Example: Probability of tossing a coin.

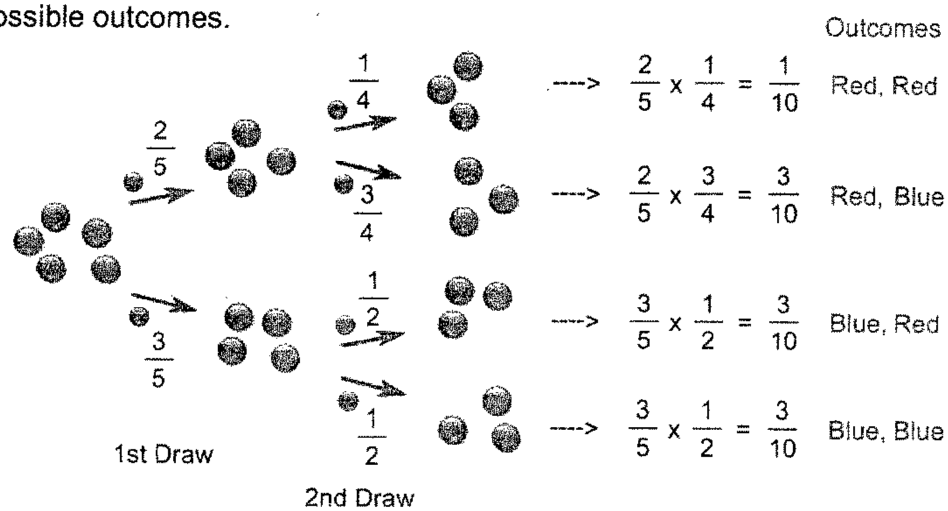


There are two Branches (Heads and Tails)

- The probability for each Branch is written on the Branch (0.5)
- The Outcome is written at the end of the Branch.

Now let us graph the previous example of the 3 blue and 2 red marbles in a bag.
What is the probability of drawing two red marbles from the bag with no replacements?

Graph out all possible outcomes.



You may find the probability of any outcome by multiplying the probabilities along any path.

Statistics Help Sheet

Name:

For these examples we will be using two different sets of numbers.

Set A:
13, 10, 4, 12, 20, 14, 18

Set B:
10, 23, 2, 7, 9, 15, 7, 20

When dealing with statistics it is often easier to put the data in order (smallest to largest).

4, 10, 12, 13, 14, 18, 20

2, 7, 7, 9, 10, 15, 20, 23

Mean (aka. Average):

To find the 'Mean' of a set of numbers take the sum of all the numbers and divide it by the quantity of numbers.

$4 + 10 + 12 + 13 + 14 + 18 + 20 = 91$

$91 \div 7 =$ **13**

$2 + 7 + 7 + 9 + 10 + 15 + 20 + 23 = 93$

$93 \div 8 =$ **11.6**

Median:

The 'Median' of a set of numbers is the value that is in the center. In set A, the median is **13**.

4, 10, 12, 13, 14, 18, 20

2, 7, 7, 9, 10, 15, 20, 23

Since set B has no number in the middle the median is the average of the two center numbers (9 & 10).
Set B's median is **9.5** ($19 \div 2$) .

Range:

The 'Range' of a set of numbers is the difference between the largest and smallest amount.

4, 10, 12, 13, 14, 18, 20

$20 - 4 =$ **16**

2, 7, 7, 9, 10, 15, 20, 23

$23 - 2 =$ **21**

Quartiles

To find the quartiles of a set, split the set into quarters (4ths). Set B's quartiles are between numbers, so the average of the numbers is used.

4, 10, 12, 13, 14, 18, 20

Q1: **10**
Q2: **13**
Q3: **18**

2, 7, 7, 9, 10, 15, 20, 23

Q1: $14 \div 2 =$ **7**
Q2: $19 \div 2 =$ **9.5**
Q3: $35 \div 2 =$ **17.5**

Interquartile Range

The 'Interquartile Range' is the difference between the first quarter and the third quarter (see above).

4, 10, 12, 13, 14, 18, 20

$18 - 10 =$ **8**

2, 7, 7, 9, 10, 15, 20, 23

$17.5 - 7 =$ **10.5**

Mean Absolute Deviation

The 'Mean Absolute Deviation' is the mean of the numbers distance from the mean.

Number	Distance from Mean (13)
4	9
10	3
12	1
13	0
14	1
18	5
20	7

Number	Distance from Mean (11.6)
2	9.6
7	4.6
7	4.6
9	2.6
10	1.6
15	3.4
20	8.4
23	11.4

$9 + 3 + 1 + 0 + 1 + 5 + 7 = 26$

$26 \div 7 =$ **3.7**

$9.6 + 4.6 + 4.6 + 2.6 + 1.6 + 3.4 + 8.4 + 11.4 = 46.2$

$46.2 \div 8 =$ **5.8**

Examining Data Sets Name:

Find the Mean, Median, Interquartile Range and Mean Absolute Deviation of the set of numbers. If possible round to the nearest tenth.

Ex) 3, 4, 8, 7, 2
2, 3, 4, 7, 8
Q1 = 2.5
Q3 = 7.5

Mean = 4.8
Median = 4
I.Q.R. = 5

Number	2	3	4	7	8
Distance	2.8	1.8	0.8	2.2	3.2

M.A.D. = 2.2

1) 7, 9, 1, 4, 9

2) 3, 1, 2, 3, 7, 6

3) 9, 5, 6, 3, 1, 8

4) 8, 8, 9, 9, 7, 9, 6

5) 1, 1, 4, 8, 9, 3, 8

6) 6, 7, 2, 1, 5, 2, 1, 6

7) 7, 6, 8, 6, 7, 5, 7, 3

Answers

Ex. 4.8 4 5 2.2

1. ___ ___ ___ ___

2. ___ ___ ___ ___

3. ___ ___ ___ ___

4. ___ ___ ___ ___

5. ___ ___ ___ ___

6. ___ ___ ___ ___

7. ___ ___ ___ ___

Find the Mean, Median, Interquartile Range and Mean Absolute Deviation of the set of numbers. If possible round to the nearest tenth.

Ex) 3, 4, 8, 7, 2
2, 3, 4, 7, 8
Q1 = 2.5
Q3 = 7.5

Mean = 4.8
Median = 4
I.Q.R. = 5

Number	2	3	4	7	8
Distance	2.8	1.8	0.8	2.2	3.2

M.A.D. = 2.2

1) 7, 9, 1, 4, 9
1, 4, 7, 9, 9
Q1 = 2.5
Q3 = 9

Mean = 6
Median = 7
I.Q.R. = 6.5

Number	1	4	7	9	9
Distance	5	2	1	3	3

M.A.D. = 2.8

2) 3, 1, 2, 3, 7, 6
1, 2, 3, 3, 6, 7
Q1 = 2
Q3 = 6

Mean = 3.7
Median = 3
I.Q.R. = 4

Number	1	2	3	3	6	7
Distance	2.7	1.7	0.7	0.7	2.3	3.3

M.A.D. = 1.9

3) 9, 5, 6, 3, 1, 8
1, 3, 5, 6, 8, 9
Q1 = 3
Q3 = 8

Mean = 5.3
Median = 5.5
I.Q.R. = 5

Number	1	3	5	6	8	9
Distance	4.3	2.3	0.3	0.7	2.7	3.7

M.A.D. = 2.3

4) 8, 8, 9, 9, 7, 9, 6
6, 7, 8, 8, 9, 9, 9
Q1 = 7
Q3 = 9

Mean = 8
Median = 8
I.Q.R. = 2

Number	6	7	8	8	9	9	9
Distance	2	1	0	0	1	1	1

M.A.D. = 0.9

5) 1, 1, 4, 8, 9, 3, 8
1, 1, 3, 4, 8, 8, 9
Q1 = 1
Q3 = 8

Mean = 4.9
Median = 4
I.Q.R. = 7

Number	1	1	3	4	8	8	9
Distance	3.9	3.9	1.9	0.9	3.1	3.1	4.1

M.A.D. = 3

6) 6, 7, 2, 1, 5, 2, 1, 6
1, 1, 2, 2, 5, 6, 6, 7
Q1 = 1.5
Q3 = 6

Mean = 3.8
Median = 3.5
I.Q.R. = 4.5

Number	1	1	2	2	5	6	6	7
Distance	2.8	2.8	1.8	1.8	1.2	2.2	2.2	3.2

M.A.D. = 2.3

7) 7, 6, 8, 6, 7, 5, 7, 3
3, 5, 6, 6, 7, 7, 7, 8
Q1 = 5.5
Q3 = 7

Mean = 6.1
Median = 6.5
I.Q.R. = 1.5

Number	3	5	6	6	7	7	7	8
Distance	3.1	1.1	0.1	0.1	0.9	0.9	0.9	1.9

M.A.D. = 1.1

Answers

Ex.	4.8	4	5	2.2
1.	6	7	6.5	2.8
2.	3.7	3	4	1.9
3.	5.3	5.5	5	2.3
4.	8	8	2	0.9
5.	4.9	4	7	3
6.	3.8	3.5	4.5	2.3
7.	6.1	6.5	1.5	1.1

6-SP.2,5d Electoral College

Task

Unlike many elections for public office where a person is elected strictly based on the results of a popular vote (i.e., the candidate who earns the most votes in the election wins), in the United States, the election for President of the United States is determined by a process called the Electoral College. According to the National Archives, the process was established in the United States Constitution "as a compromise between election of the President by a vote in Congress and election of the President by a popular vote of qualified citizens." (*http://www.archives.gov/federal-register/electoral-college/about.html* accessed September 4, 2012).

Each state receives an allocation of electoral votes in the process, and this allocation is determined by the number of members in the state's delegation to the US Congress. This number is the sum of the number of US Senators that represent the state (always 2, per the Constitution) and the number of Representatives that represent the state in the US House of Representatives (a number that is directly related to the state's population of qualified citizens as determined by the US Census). Therefore the larger a state's population of qualified citizens, the more electoral votes it has. Note: the District of Columbia (which is not a state) is granted 3 electoral votes in the process through the 23rd Amendment to the Constitution.

The following table shows the allocation of electoral votes for each state and the District of Columbia for the 2012, 2016, and 2020 presidential elections. (*http://www.archives.gov/federal-register/electoral-college/allocation.html* accessed September 4, 2012).

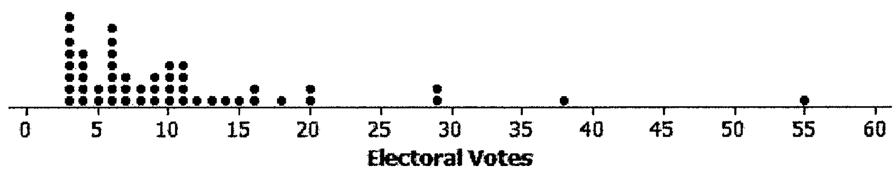
Illustrative Mathematics

State	Electoral Votes	State	Electoral Votes	State	Electoral Votes
Alabama	9	Kentucky	8	North Dakota	3
Alaska	3	Louisiana	8	Ohio	18
Arizona	11	Maine	4	Oklahoma	7
Arkansas	6	Maryland	10	Oregon	7
California	55	Massachusetts	11	Pennsylvania	20
Colorado	9	Michigan	16	Rhode Island	4
Connecticut	7	Minnesota	10	South Carolina	9
Delaware	3	Mississippi	6	South Dakota	3
District of Columbia	3	Missouri	10	Tennessee	11
Florida	29	Montana	3	Texas	38
Georgia	16	Nebraska	5	Utah	6
Hawaii	4	Nevada	6	Vermont	3
Idaho	4	New Hampshire	4	Virginia	13
Illinois	20	New Jersey	14	Washington	12
Indiana	11	New Mexico	5	West Virginia	5
Iowa	6	New York	29	Wisconsin	10
Kansas	6	North Carolina	15	Wyoming	3

a. Which state has the most electoral votes? How many votes does it have?

b. Based on the given information, which state has the second highest population of qualified citizens?

c. Here is a dotplot of the distribution.



i. What is the shape of this distribution: skewed left, symmetric, or skewed right?

ii. Imagine that someone you are speaking with is unfamiliar with these shape terms. Describe clearly and in the context of this data set what the shape description you have chosen means in terms of the distribution.

d. Does the dotplot lead you to think that any states are outliers in terms of their number of electoral votes? Explain your reasoning, and if you do believe that there are outlier values, identify the corresponding states.

e. What measure of center (mean or median) would you recommend for describing this data set? Why did you choose this measure?

f. Determine the value of the median for this data set (electoral votes).

IM Commentary

In addition to providing a task that relates to other disciplines (history, civics, current events, etc.), this task is intended to demonstrate that a graph can summarize a distribution as well as provide useful information about specific observations. With the table provided, the graph and values have context. The purpose of this task is to help students understand that a distribution can be described in terms of shape and center, and also to provide practice in selecting and calculating measures of center.

This task was designed so that it does not require the use of technology. If students have access to technology, you can also consider having students calculate the value of the mean and then comparing the values of the mean and the median for this data set. You could then facilitate a discussion of the effect of outliers on the value of the mean, which would support the choice of the median to describe the center for this data set.

Solution

1. 55 is the maximum value in the distribution, and that is California's value.
2. As explained in the introduction, the larger a state's population of qualified citizens, the more electoral votes it has. Thus, the second highest number of electoral votes would be associated with the state with the second highest number of qualified citizens. 38 is the second highest value in the distribution, and that is the electoral vote count for Texas.
3.
   i. This distribution is skewed right.
   ii. Said another way, most of the states have a small number of electoral votes (nearly 2/3 of the states have between 3 and 10 votes) while a very few states have a large number of electoral votes (for example, only 4 states have more than 20 votes; 1 state has 38, 1 state has 55).

4. Students should explain their choices carefully. California at 55 electoral votes and Texas at 38 votes should be listed as outliers based on the visible gaps in the dotplot and/or based on a numerical argument that their values are very far away from the cluster of the other observations. Many students will also say that New York and Florida are outliers, because of the gap of 9 electoral votes between these observations and the "3 to 20" votes cluster.
5. Because the data distribution is skewed to the right and there are outliers, the median would be a better choice to describe center for this data set.
6. Median = 8. Since there are 51 observations, the median would be the 26th observation when the observations are arranged in order from smallest to largest. Since the data are not presented in ordered form, students should NOT simply pick the 26th observation in the list (Missouri = 10). Note: particularly if the median is computed without the assistance of software or calculator, students may realize that they need only count up to (or down to) the 26th ordered observation and that they do not need to order the entire dataset.

8.SP.1 Hand span and height

Task

Do taller people tend to have bigger hands? To investigate this question, each student in your class should measure his or her hand span (in cm) and height (in inches). Record these values in the table below.

Student	Hand Span (cm)	Height (inches)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

a. Create a clearly labeled graph that displays the relationship between height and hand span.

b. Based on the graph, how would you answer the question about whether taller people tend to have bigger hands?

c. Based on your graph, would you describe the relationship between hand span and height as linear or nonlinear? Explain your choice.

IM Commentary

- For purposes of consistency, it is recommended that one person be in charge of taking the measurements. This role may be taken by the instructor, or one of the students in class.

- For consistency, measure hand span of right hand for all students.
- To measure hand span, spread out fingers as much as possible, and then measure the distance (cm) between tip of thumb to tip of little finger.
- Height should be measured without shoes and without anything on the head that would inflate height (besides hair). For those wearing religious headwear, be careful not to measure the height of the headwear.
- Notice that hand span is measured in centimeter which is a finer unit of measurement, to account for the fact that a difference in hand span is on a relatively finer scale compared to difference in height, which can be measured in inches (and is typically measured in feet and inches).
- Typically, the association between hand span and height has observed to be positive, moderately strong, and linear, with relatively few outliers. That is, people with larger hand spans tend to be taller. Also, depending, there may be some noticeable separation of males and females, with heights and hand spans of females being towards the left bottom corner of the scatterplot, and those for the males being towards the right top corner. Of course, there may be outliers in this case, too.

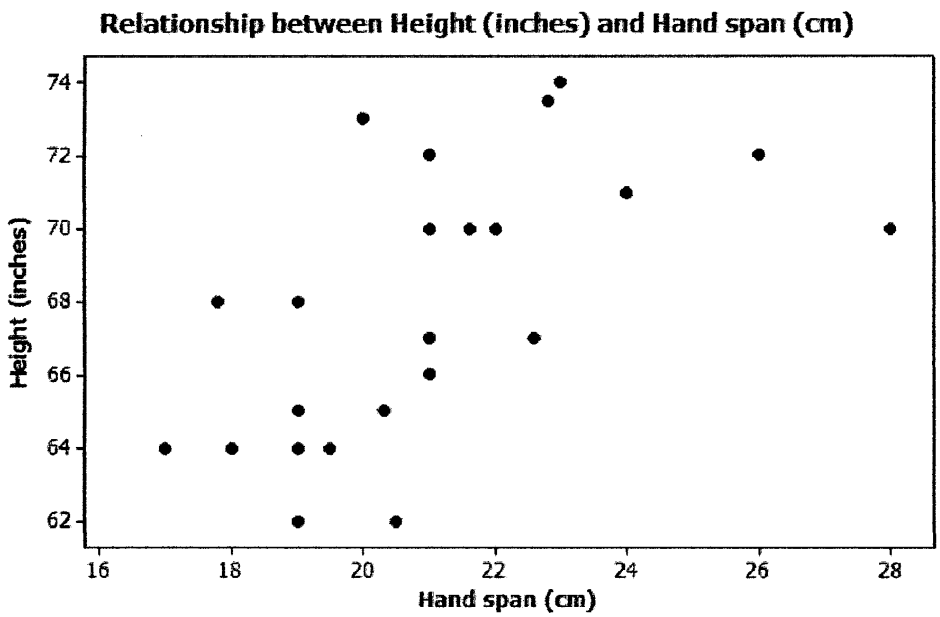
Solution

Solutions will vary depending on the actual data values collected by the class. Below is a solution based on the following hypothetical data set:

Student	Hand Span (cm)	Height (inches)
1	17.0	64.0
2	21.0	67.0
3	20.3	65.0
4	26.0	72.0
5	24.0	71.0
6	22.0	70.0
7	21.0	66.0
8	19.0	62.0

9	20.0	73.0
10	19.0	65.0
11	17.8	68.0
12	20.5	62.0
13	21.0	70.0
14	22.8	73.5
15	22.6	67.0
16	21.0	72.0
17	23.0	74.0
18	21.6	70.0
19	21.0	72.0
20	28.0	70.0
21	18.0	64.0
22	19.0	68.0
23	19.5	64.0
24	19.0	64.0

a. Here is a scatterplot showing the relationship between height and hand span.



b. Each dot represents a student, and the position of the dot with regard to the horizontal axis represents the student's hand span (cm), whereas the position of the dot with regard to the vertical axis represents the student's height. We can see that dots with lower hand span values also tend to have lower values for height; similarly dots with higher hand span values also tend to have higher values for height. Overall, we can see that there is an upward trend in the scatterplot. This shows that taller people tend to have bigger hand spans.

c. The overall form of the relationship between height and hand span appears to be linear, except for the student with a hand span of 28cm and height of 70inches. We can say this because a line seems to be the most appropriate pattern to represent how height is changing with hand span.

VIII. Measurement

1. Measure and Estimate Length in Standard Units

a. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes
b. Estimate lengths using inches, feet, centimeters, and meters
c. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit

2. Compare, Contrast, and Convert Customary (American) Units of Length

1 foot = 12 inches   1 yard = 3 feet   1 mile =5,280 feet   1 mile = 1,760 yards

a. Use multiplication and division to convert one customary unit of length to another
b. Generate conversion tables for customary units of length
c. Change length from a mixed unit to a single unit
    - *Does 5 feet 4 inches = 64 inches?*
d. Change length from a single unit to a mixed unit
    - *Does 64 inches = 5 feet 4 inches?*
e. Compare and contrast units
    - *Is 4 feet > 2 yards?   Is 2 yards < 48 inches?*

3. Solve Real-World Multistep Word Problems Involving Customary Units of Length

a. Use all four operations to solve word problems involving units of length
    - *When adding and subtracting, first add and subtract like units, then convert the units to simplest form   5 feet 8 inches + 3 feet 10 inches = 8 feet 18 inches = 9 feet 6 inches or 9.5 feet*
b. Solve perimeter problems using customary units of length
c. Solve area problems using customary units of length
d. Solve volume problems using customary units of length

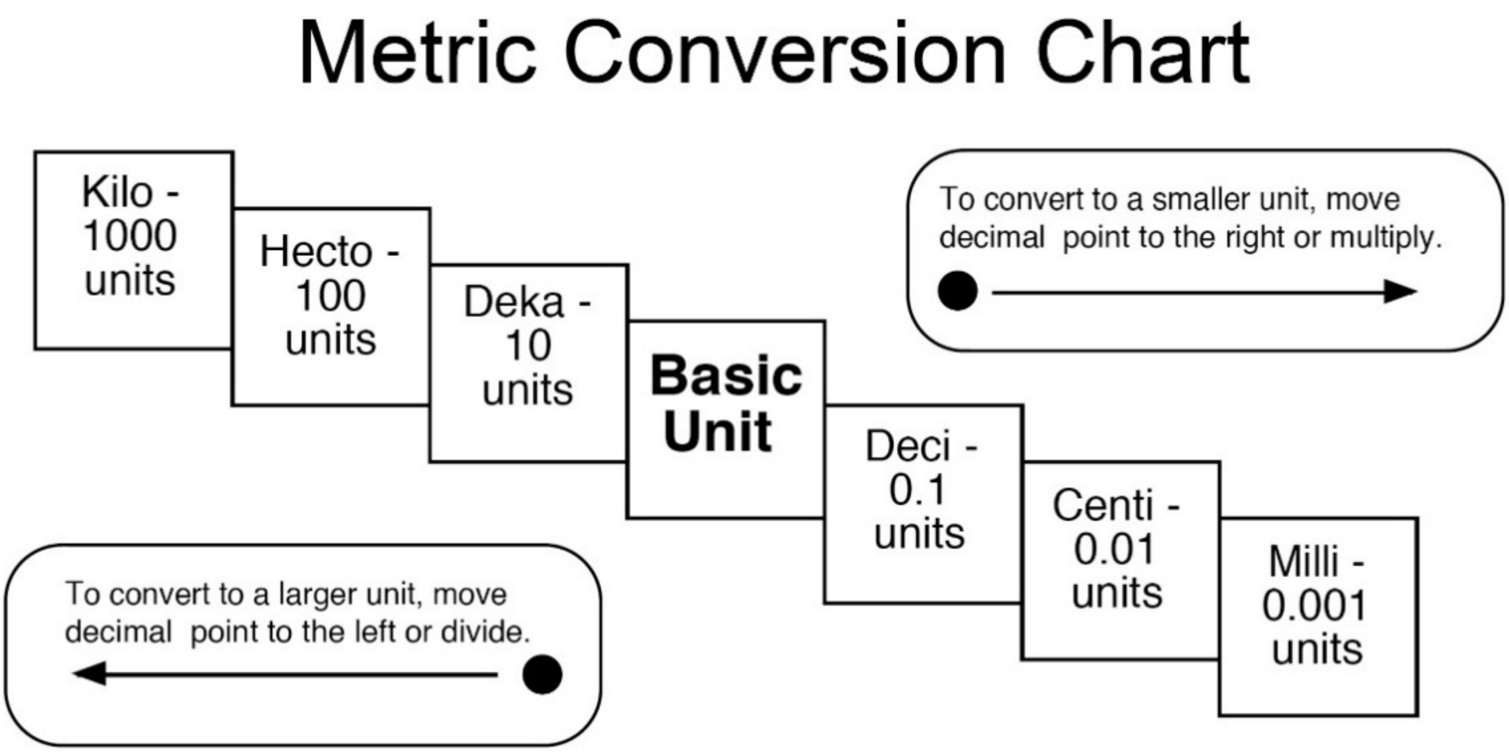
4. Compare, Contrast, and Convert Metric (International) Units of Length

a. Understand that a meter is the main metric unit of length and is about the size of a yard *(a yard is 0.9144 meters)*

b. Understand that the metric system is based on place value and to convert between units in the metric system, you multiply or divide by a power of 10 *(see the chart below)*
c. Use multiplication and division to convert one metric unit of length to another
d. Generate conversion tables for metric units of length
e. Compare and contrast metric units of length
   - *A kilometer is 1000 times larger than a meter, a millimeter is 1000 times smaller than a meter*

5. Solve Real-World Multistep Word Problems Involving Metric Units of Length

a. Use all four operations to solve word problems involving metric units of length
b. Understand and solve perimeter problems involving metric units of length
c. Understand and solve area problems involving metric units of length
d. Understand and solve volume problems involving metric units of length



Prefix	Meaning	Length	Mass	Capacity
kilo-	thousand (1,000)	kilometer	kilogram	kiloliter
hecto-	hundred (100)	hectometer	hectogram	hectoliter
deka-	ten (10)	dekameter	dekagram	dekaliter
*base unit	ones (1)	meter	gram	liter
deci-	tenths (0.1)	decimeter	decigram	deciliter
centi-	hundredths (0.01)	centimeter	centigram	centiliter
milli-	thousandths (0.001)	millimeter	milligram	milliliter

6. Measure and Estimate Customary Capacity

a. Understand that capacity is the amount a container can hold
b. Understand that customary capacity is measured in fluid ounces, cups, pints, quarts, and gallons
c. Measure using fluid ounces and cups
d. Estimate how many fluid ounces, cups, pints, quarts, or gallons a container can hold

7. Compare, Contrast, and Convert Customary Capacity

1 cup (c) = 8 fluid ounces (fl. oz.), 1 pint (pt.) = 2 cups, 1 quart (qt.) = 2 pints

1 quart = 4 cups, 1 gallon (gal.) = 4 quarts

a. Use multiplication and division to convert customary capacity
b. Generate conversion tables for customary capacity
c. Compare and contrast units of capacity

8. Solve Real-World Multistep Word Problems Involving Customary Capacity

a. Use all four operations to solve real-life, multistep word problems using customary capacity

9. Compare, Contrast, and Convert Customary Units of Weight

16 ounces (oz.) = 1 pound (lb.), 2000 pounds = 1 ton

a. Use multiplication and division to convert customary units of weight
b. Generate conversion tables for customary units of weight
c. Compare and contrast customary units of weight

10. Solve Real-World Multi-step Word Problems Involving Customary Units of Weight

a. Use all four operations to solve real-life, multistep word problems using customary units of weight

11. Compare, Contrast, and Convert Customary Units of Time

60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 7 days = 1 week,

52 weeks = 1 year, 365 days = 1 year

a. Use multiplication and division to convert units of time

b. Generate conversion tables for units of time

c. Compare and contrast units of time

12. Solve Real-World Multi-step Word Problems Involving Units of Time

a. Use all four operations to solve real-life, multistep word problems using units of time
b. Solve elapsed time problems
   - *If you start work at 7:45, take off a halt hour for lunch, and punch out at 3:15, how many hours did you work?*

Converting Tables Name:

Fill in the blanks in each of the conversion tables.

	Pounds	Ounces
1)		80
2)		128
3)	2	
4)		16
5)	4	

	Yards	Feet
6)		3
7)	4	
8)		6
9)	10	
10)		15

	Minutes	Hours
11)		9
12)	240	
13)	180	
14)		7
15)	60	

	Cups	Pints
16)	14	
17)		5
18)		6
19)		4
20)	4	

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
10. ____________
11. ____________
12. ____________
13. ____________
14. ____________
15. ____________
16. ____________
17. ____________
18. ____________
19. ____________
20. ____________

1-10	95	90	85	80	75	70	65	60	55	50
11-20	45	40	35	30	25	20	15	10	5	0

Converting Tables

Name: **Answer Key**

Fill in the blanks in each of the conversion tables.

	Pounds	Ounces
1)	5	80
2)	8	128
3)	2	32
4)	1	16
5)	4	64

	Yards	Feet
6)	1	3
7)	4	12
8)	2	6
9)	10	30
10)	5	15

	Minutes	Hours
11)	540	9
12)	240	4
13)	180	3
14)	420	7
15)	60	1

	Cups	Pints
16)	14	7
17)	10	5
18)	12	6
19)	8	4
20)	4	2

Answers

1. 5
2. 8
3. 32
4. 1
5. 64
6. 1
7. 12
8. 2
9. 30
10. 5
11. 540
12. 4
13. 3
14. 420
15. 1
16. 7
17. 10
18. 12
19. 8
20. 2

1-10	95	90	85	80	75	70	65	60	55	50
11-20	45	40	35	30	25	20	15	10	5	0

CONVERTING UNITS

This activity gets students thinking about the various standard units of measure and how they compare.

1. Copy the measurements below onto paper or card stock and cut into pieces.
2. Copy the grid on the next page.
3. Have students work alone or in pairs to decide where to place the pieces on the grid. They can guess at first and then check their answers by calculating, or they can calculate as they go.
4. When they are done, they can check their answers with the key below.

60	$1\frac{2}{3}$	1,209,600
20,160	14	2
48	24	12
3	3200	1/10

PURPOSE

To practice converting standard units of measure.

YOU NEED:

- ☆ *a copy of the chart on the next page*
- ☆ *the numbers below, copied and cut into pieces*

Number of players
1 - 2

Answers:

60 in. = 5 ft = $1\frac{2}{3}$ yd

1,209,600 sec = 20,160 min = 336 hr = 14 days = 2 weeks

384 ounces = 48 cups = 24 pints = 12 quarts = 3 gallons

3200 ounces = 200 pounds = 1/10 ton

length	_______in	5 ft	_______yd		
weight	_______oz	200 lbs	_______tons		
liquid measure	384 oz	_______cups	_______pints	_______qts	_______gal
time	_______sec	_______min	336 hours	_______days	_______wks

____

Converting Lengths (A)

Convert the lengths in the problems below.

____ feet = 9 yards

____ inches = 8 feet

5 feet = ____ inches

3 feet = ____ inches

288 inches = ____ yards

2 yards = ____ feet

____ yards = 108 inches

15 feet = ____ yards

4 yards = ____ inches

3 feet = ____ inches

How many feet are in 6 yards? ____________

How many inches are in 7 feet? ____________

How many inches are in 4 feet? ____________

How many feet are in 8 yards? ____________

How many inches are in 5 feet? ____________

Underline the longer length:	14 feet	3 yards
Underline the shorter length:	144 inches	7 yards

Converting Lengths (A) Answers

Convert the lengths in the problems below.

____ feet = 9 yards
27 feet

____ inches = 8 feet
96 inches

5 feet = ____ inches
60 inches

3 feet = ____ inches
36 inches

288 inches = ____ yards
8 yards

2 yards = ____ feet
6 feet

____ yards = 108 inches
3 yards

15 feet = ____ yards
5 yards

4 yards = ____ inches
144 inches

3 feet = ____ inches
36 inches

How many feet are in 6 yards? 18 feet

How many inches are in 7 feet? 84 inches

How many inches are in 4 feet? 48 inches

How many feet are in 8 yards? 24 feet

How many inches are in 5 feet? 60 inches

Underline the longer length: <u>14 feet</u> 3 yards

Underline the shorter length: <u>144 inches</u> 7 yards

Converting Liquid Measures (A)

Convert the liquid measures in the problems below.

28 quarts = ____ gallons

____ quarts = 38 pints

____ gallons = 60 quarts

5 gallons = ____ pints

____ quarts = 10 gallons

____ quarts = 9 gallons

____ pints = 8 gallons

10 gallons = ____ quarts

17 quarts = ____ pints

____ quarts = 22 pints

How many pints are in 4 gallons? ________________

How many pints are in 5 gallons? ________________

How many quarts are in 14 gallons? ________________

How many pints are in 6 quarts? ________________

How many quarts are in 16 gallons? ________________

Underline the greater volume: 72 quarts 23 gallons

Underline the lesser volume: 45 quarts 11 gallons

Converting Liquid Measures (A) Answers

Convert the liquid measures in the problems below.

28 quarts = ____ gallons
7 gallons

____ quarts = 38 pints
19 quarts

____ gallons = 60 quarts
15 gallons

5 gallons = ____ pints
40 pints

____ quarts = 10 gallons
40 quarts

____ quarts = 9 gallons
36 quarts

____ pints = 8 gallons
64 pints

10 gallons = ____ quarts
40 quarts

17 quarts = ____ pints
34 pints

____ quarts = 22 pints
11 quarts

How many pints are in 4 gallons? 32 pints

How many pints are in 5 gallons? 40 pints

How many quarts are in 14 gallons? 56 quarts

How many pints are in 6 quarts? 12 pints

How many quarts are in 16 gallons? 64 quarts

Underline the greater volume: 72 quarts <u>23 gallons</u>

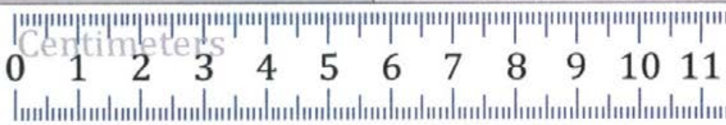
Underline the lesser volume: 45 quarts <u>11 gallons</u>

Metric System

Commonly Used Metric Relationships

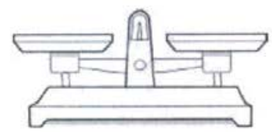
Length

1 mm	=	0.1 cm	1 mm	=	0.001 m	1 m	=	0.001 km
1 cm	=	10 mm	1 m	=	1,000 mm	1 km	=	1,000 m
						1 cm	=	0.01 m
						1 m	=	100 cm



Mass

1 mg	=	0.001 g	1 g	=	0.001 kg
1 g	=	1,000 mg	1 kg	=	1,000 g



Volume

1 mL	=	0.001 L	1 mL	=	1 cm^3
1 L	=	1,000 mL	1 L	=	1,000 cm^3

Metric System Facts

Prefixes always have the same value no matter what the unit.

Really Small			*In the Middle*			*Really Big*		
pico	10^{-12}	trillionth	centi	10^{-2}	1/100	kilo	10^{3}	thousand
nano	10^{-9}	billionth	deci	10^{-1}	1/10	mega	10^{6}	million
micro	10^{-6}	millionth	--	10^{0}	1	giga	10^{9}	billion
milli	10^{-3}	thousandth	deka	10^{1}	10	tera	10^{12}	trillion
			hecto	10^{2}	100			

E.g. A gigameter is 1 billion meters; a gigabyte is 1 billion bytes

Other Metric Relationships to know

1 L of water has a mass of 1 kg.

1 cm^3 of water is 1 mL and has a mass of 1 g.

1 hectare is a square with sides measuring 100 m.

1 tonne (Metric ton) is 1,000 kg and can also be called a megagram.

1 m^3 of water has a mass of 1 tonne.

Math-Drills.com

King	Henry	Died	Unusually	Drinking	Chocolate	Milk
1,000	100	10	1	.10 or 1/10	.01 or 1/100	.001 or 1/1,000
Kilo	Hecto	Deca	Unit (Meter, Liter, or Gram)	Deci	Centi	Milli

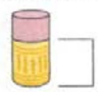
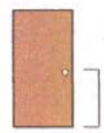
Estimating Distance (Metric) Name:

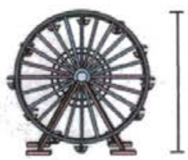
Determine which letter best represents the length / height.

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______

Millimeter (mm)	**Centimeter (cm)**	**Meter (m)**	**Kilometer (km)**
A millimeter is about the thickness of a credit card.	10 mm = 1 cm. The metal portion of a pencil is about 1 cm. A ruler is about 30 centimeters.	100 cm = 1 m From the floor to a door knob is about 1 meter.	1,000 m = 1 km Most major roads are at least a kilometer long.

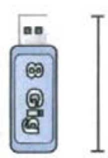




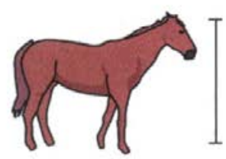
1) Ferris Wheel
A. 30 centimeters
B. 5 meters
C. 50 kilometers
D. 23 meters

2) Screw
A. 20 centimeters
B. 25 centimeters
C. 3 centimeters
D. 1 meter

3) Can of Beans
A. 120 centimeters
B. 2 meters
C. 2 kilometers
D. 10 centimeters

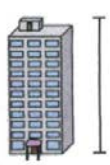


4) Flash Drive
A. 30 centimeters
B. 60 centimeters
C. 6 centimeters
D. 15 centimeters

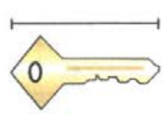


5) Adult Horse
A. 90 centimeters
B. 2 meters
C. 25 millimeters
D. 30 centimeters

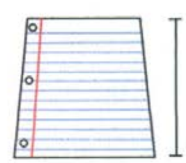
6) Recliner
A. 60 centimeters
B. 1 meter
C. 120 centimeters
D. 10 millimeters



7) 11 Story Building
A. 1 meter
B. 3 kilometers
C. 335 centimeters
D. 30 meters



8) Key
A. 5 centimeters
B. 2 meters
C. 15 centimeters
D. 150 centimeters



9) Notebook Paper
A. 15 centimeters
B. 1 meters
C. 60 centimeters
D. 25 centimeters

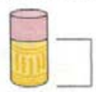
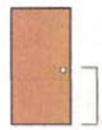
1-9	89	78	67	56	44	33	22	11	0

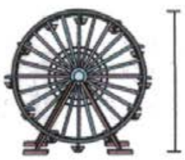
Estimating Distance (Metric)

Name: **Answer Key**

Determine which letter best represents the length / height.

Millimeter (mm)	Centimeter (cm)	Meter (m)	Kilometer (km)
A millimeter is about the thickness of a credit card.	10 mm = 1 cm. The metal portion of a pencil is about 1 cm. A ruler is about 30 centimeters.	100 cm = 1 m From the floor to a door knob is about 1 meter.	1,000 m = 1 km Most major roads are at least a kilometer long.

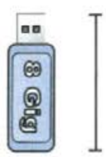




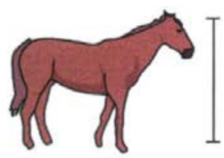
1) Ferris Wheel
A. 30 centimeters
B. 5 meters
C. 50 kilometers
D. 23 meters

2) Screw
A. 20 centimeters
B. 25 centimeters
C. 3 centimeters
D. 1 meter

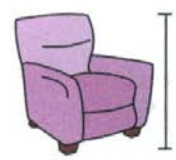
3) Can of Beans
A. 120 centimeters
B. 2 meters
C. 2 kilometers
D. 10 centimeters



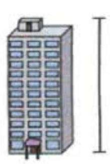
4) Flash Drive
A. 30 centimeters
B. 60 centimeters
C. 6 centimeters
D. 15 centimeters



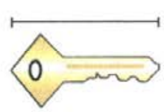
5) Adult Horse
A. 90 centimeters
B. 2 meters
C. 25 millimeters
D. 30 centimeters



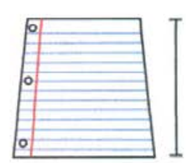
6) Recliner
A. 60 centimeters
B. 1 meter
C. 120 centimeters
D. 10 millimeters



7) 11 Story Building
A. 1 meter
B. 3 kilometers
C. 335 centimeters
D. 30 meters



8) Key
A. 5 centimeters
B. 2 meters
C. 15 centimeters
D. 150 centimeters



9) Notebook Paper
A. 15 centimeters
B. 1 meters
C. 60 centimeters
D. 25 centimeters

Answers

1. D
2. C
3. D
4. C
5. B
6. C
7. D
8. A
9. D

1-9	89	78	67	56	44	33	22	11	0

Converting m, cm and mm (A)

Convert each measurement to the unit indicated.

63.7 mm to cm	0.0000757 m to mm
76.8 mm to cm	3,980 cm to m
16.6 cm to mm	35.7 cm to m
5.52 m to cm	53.4 cm to mm
6.54 m to mm	40.4 mm to cm
0.134 mm to cm	2.43 mm to cm
580 mm to cm	2.39 cm to mm
18.6 mm to m	6.39 m to cm
77.1 mm to m	0.00858 m to cm
885 mm to cm	229 cm to m

Converting m, cm and mm (A) Answers

Convert each measurement to the unit indicated.

63.7 mm to cm
6.37 cm

76.8 mm to cm
7.68 cm

16.6 cm to mm
166 mm

5.52 m to cm
552 cm

6.54 m to mm
6,540 mm

0.134 mm to cm
0.0134 cm

580 mm to cm
58 cm

18.6 mm to m
0.0186 m

77.1 mm to m
0.0771 m

885 mm to cm
88.5 cm

0.0000757 m to mm
0.0757 mm

3,980 cm to m
39.8 m

35.7 cm to m
0.357 m

53.4 cm to mm
534 mm

40.4 mm to cm
4.04 cm

2.43 mm to cm
0.243 cm

2.39 cm to mm
23.9 mm

6.39 m to cm
639 cm

0.00858 m to cm
0.858 cm

229 cm to m
2.29 m

IX. Geometric Measurement

1. Recognize Perimeter as an Attribute of Plane Figures and Distinguish between Linear and Area Measures

a. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters

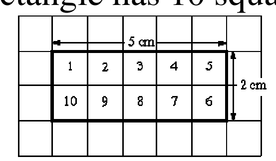
2. Understand Concepts of Area and Relate Area to Multiplication and Addition

a. Recognize area as an attribute of plane figures and understand concepts of area measurement
   - *A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area*
   - *A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units*

3. Measure Areas by Counting Unit Squares

- *square in., square ft., square yds., square cm., square m.*

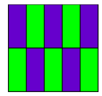
This rectangle has 10 square units



4. Relate Area to the Operations of Multiplication and Addition

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths
b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning

- When you fit individual tiles together with no gaps or overlaps to fill a flat space like a ceiling, wall, or floor, you have a tiling



d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems

5. **Understand Concepts of Volume and Relate Volume to Multiplication and Addition**
   a. Recognize volume as an attribute of solid figures and understand concepts of volume measurement
      - *A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume*
      - *A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units*

6. **Measure Volumes by Counting Unit Cubes, using cubic cm, cubic in, cubic ft., and Improvised Units**

7. **Relate Volume to the Operations of Multiplication and Addition and Solve Real World and Mathematical Problems Involving Volume**

   a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base
   b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems
   c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems

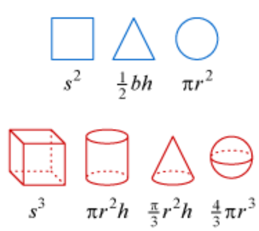
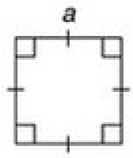
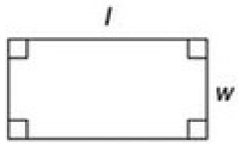
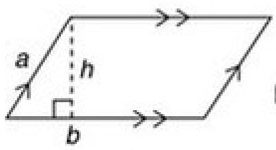
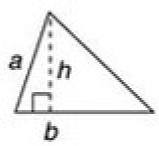
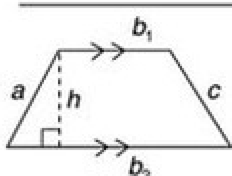
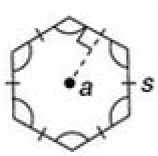
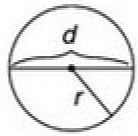


Figure	Name	Perimeter/ Circumference	Area
 (a)	square	$4a$	a^2
 (b)	rectangle	$2l + 2w$ or $2(l + w)$	lw
 (c)	parallelogram	$2a + 2b$ or $2(a + b)$	bh
 (d)	triangle	$a + b + c$	$1/2bh$
 (e)	trapezoid	$a + b_1 + c + b_2$	$1/2(b_1+b_2)h$
 (f)	regular polygon	ns n = number of sides	$1/2ap$ p = perimeter a = apothem
 (g)	circle	πd or $2\pi r$	πr^2

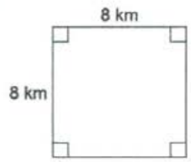
Geometric Measurement Assessment

Name________________________________

Date______________

Find the area of each.

1)



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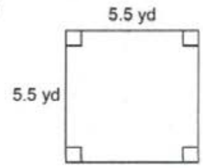

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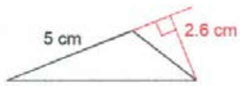

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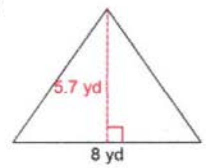
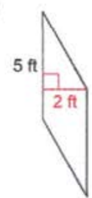


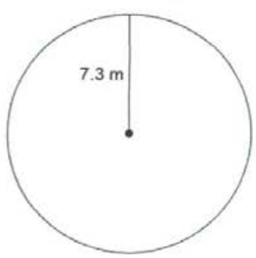
5)

6)



Find the area of each. Round your answer to the nearest tenth.

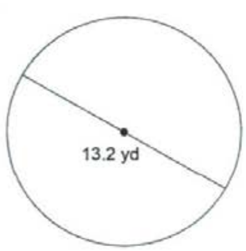
7)



8)

Find the circumference of each circle. Round your answer to the nearest tenth.

9)



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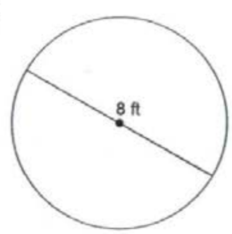

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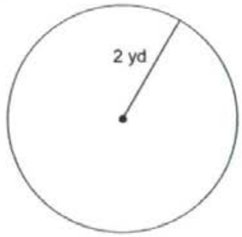

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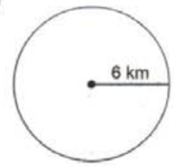
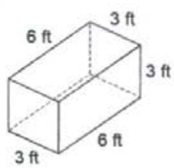


Find the volume of each figure. Round to the nearest tenth.

13)



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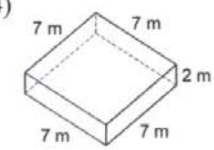

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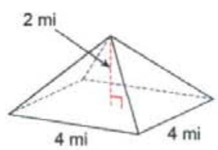

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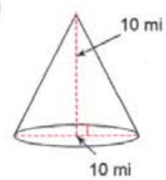

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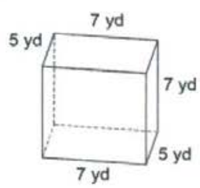

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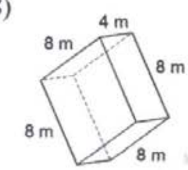


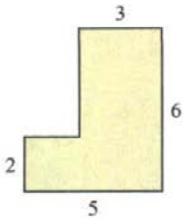
Answers to

1) 64 km^2	2) 30.25 yd^2	3) 6.5 cm^2	4) 22.8 yd^2
5) 22.04 km^2	6) 10 ft^2	7) 167.4 m^2	8) 201.1 mi^2
9) 41.5 yd	10) 25.1 ft	11) 12.6 yd	12) 37.7 km
13) 54 ft^3	14) 98 m^3	15) 10.7 mi^3	16) 261.8 mi^3
17) 245 yd^3	18) 256 m^3		

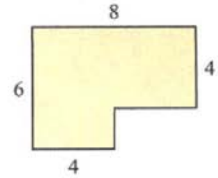
Determining Rectilinear Area   Name:

Find the total area of each shape. Measurement is in millimeters (mm). Not to scale.

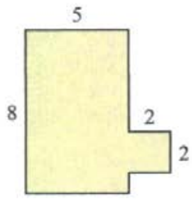
1)



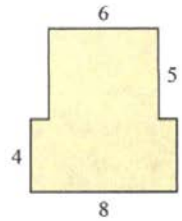
2)



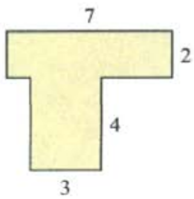
3)



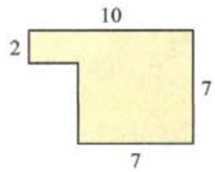
4)



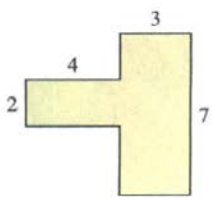
5)



6)



7)



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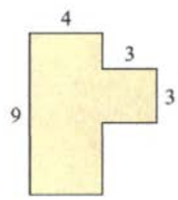

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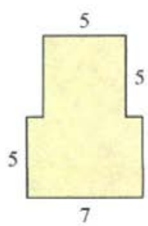

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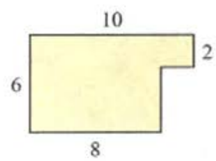


Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______

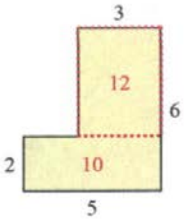
1-10	90	80	70	60	50	40	30	20	10	0

Determining Rectilinear Area

Name: **Answer Key**

Find the total area of each shape. Measurement is in millimeters (mm). Not to scale.

1)



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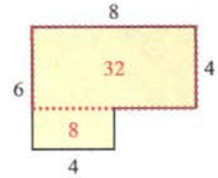

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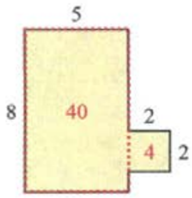

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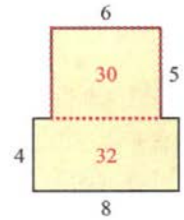

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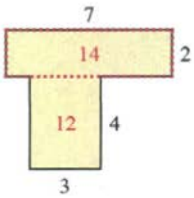

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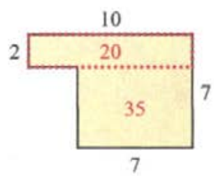

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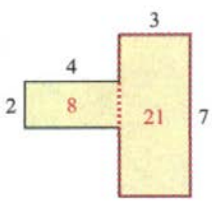

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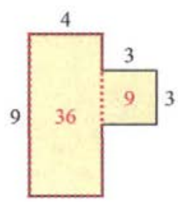

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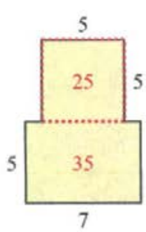

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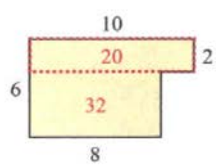


Answers

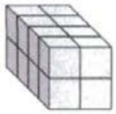
1. $22\ mm^2$
2. $40\ mm^2$
3. $44\ mm^2$
4. $62\ mm^2$
5. $26\ mm^2$
6. $55\ mm^2$
7. $29\ mm^2$
8. $45\ mm^2$
9. $60\ mm^2$
10. $52\ mm^2$

1-10	90	80	70	60	50	40	30	20	10	0

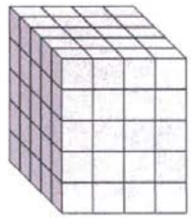
Finding Volume with Unit Cubes Name:

Find the length, width and height of the rectangular prism. Then find the volume.

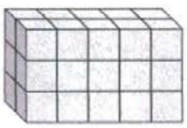
Ex)



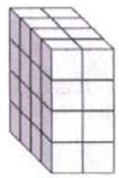
1)



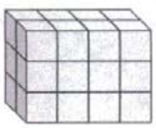
2)



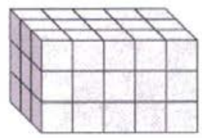
3)



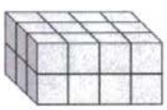
4)



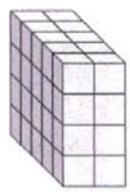
5)



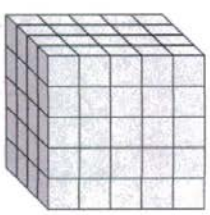
6)



7)



8)



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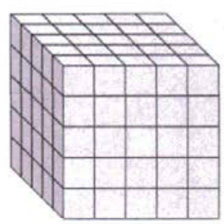

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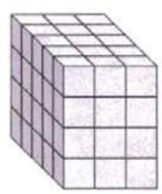


Answers

	L	W	H	V
Ex.	4	2	2	16
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10.				
11.				

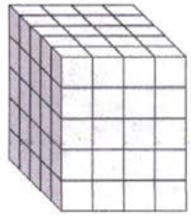
Finding Volume with Unit Cubes

Name: **Answer Key**

Find the length, width and height of the rectangular prism. Then find the volume.

Ex)

1)



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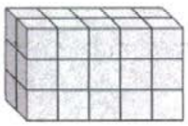

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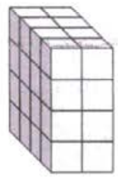

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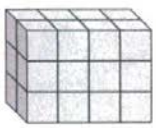

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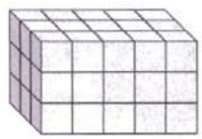

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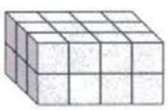

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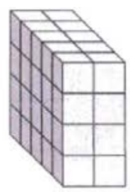

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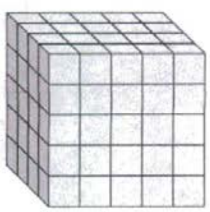

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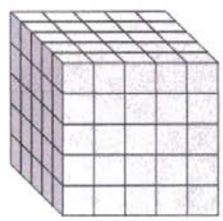

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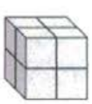

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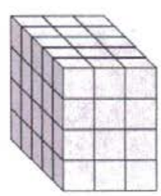


Answers

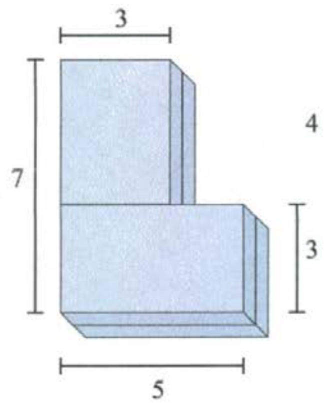
	L	W	H	V
Ex.	4	2	2	16
1.	5	4	5	100
2.	2	5	3	30
3.	4	2	4	32
4.	2	4	3	24
5.	3	5	3	45
6.	3	4	2	24
7.	5	2	4	40
8.	4	5	5	100
9.	5	5	5	125
10.	2	2	2	8
11.	5	3	4	60

1-10	91	82	73	64	55	45	36	27	18	9
11	0									

Finding Total Volume Name:

Find the total volume of each figure shown. Measured in cm (not to scale).

1)



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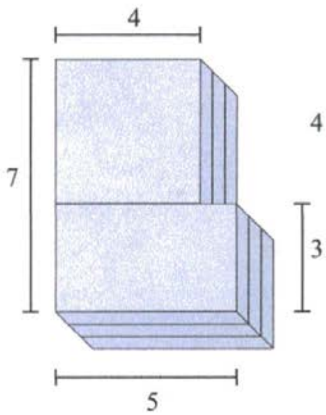

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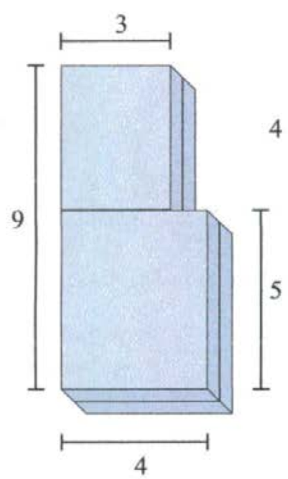

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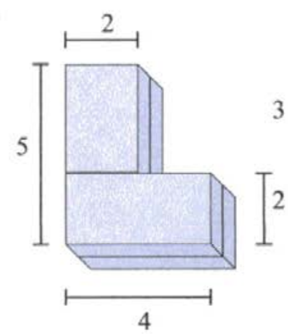

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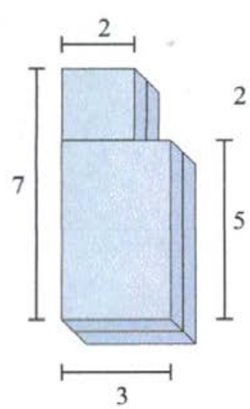

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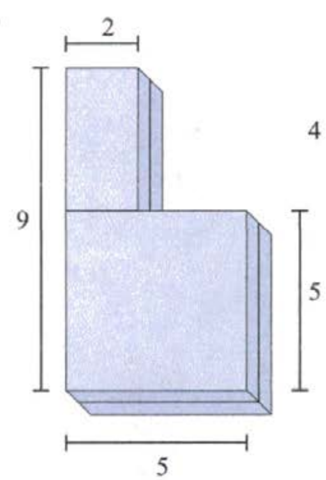


Answers

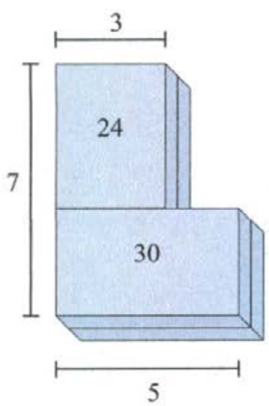
1. ________
2. ________
3. ________
4. ________
5. ________
6. ________

Finding Total Volume

Name: **Answer Key**

Find the total volume of each figure shown. Measured in cm (not to scale).

1)



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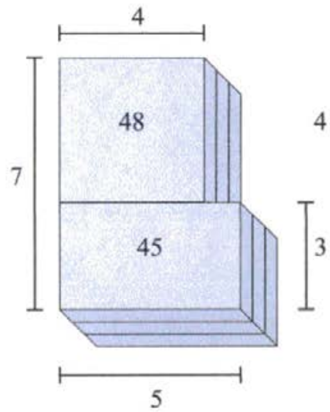

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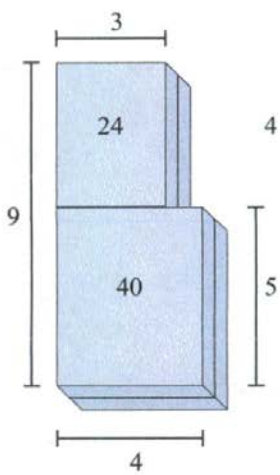

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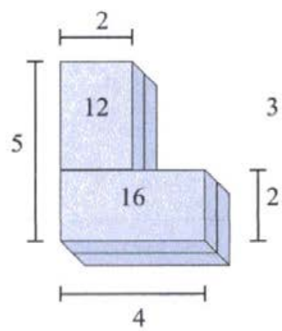

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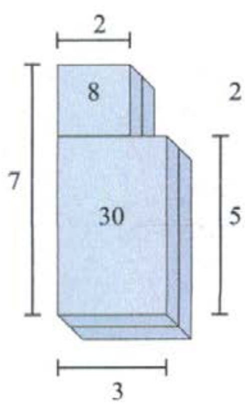

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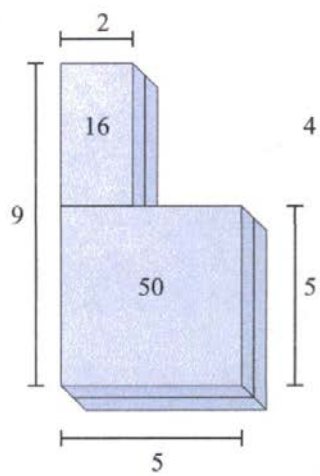


Answers

1. 54 cm^3
2. 93 cm^3
3. 64 cm^3
4. 28 cm^3
5. 38 cm^3
6. 66 cm^3

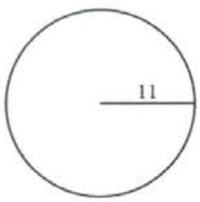
Finding the Area and Circumference of a Circle

Name:

Find the area and circumference of each circle. Circles are not to scale.

59.69	87.96	283.53	38.48	69.12
91.11	31.42	380.13	78.54	660.52
615.75	94.25	21.99	78.54	65.97
18.85	706.86	490.87	28.27	346.36

1)



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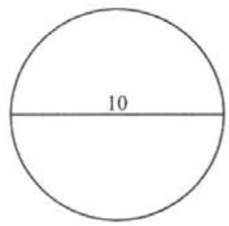

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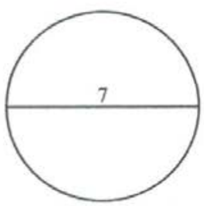

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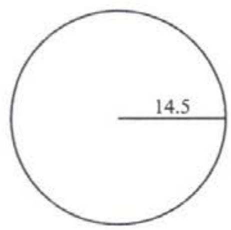

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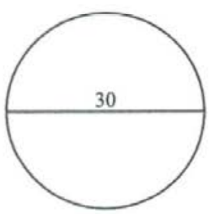

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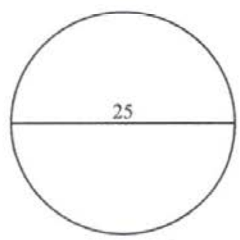

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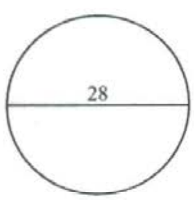

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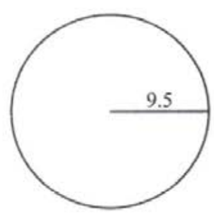

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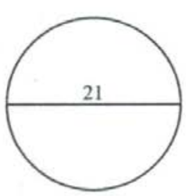

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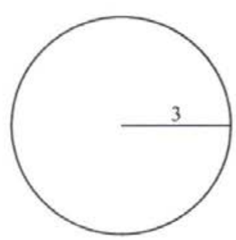


Answers

1a. ____________

1c. ____________

2a. ____________

2c. ____________

3a. ____________

3c. ____________

4a. ____________

4c. ____________

5a. ____________

5c. ____________

6a. ____________

6c. ____________

7a. ____________

7c. ____________

8a. ____________

8c. ____________

9a. ____________

9c. ____________

10a. ____________

10c. ____________

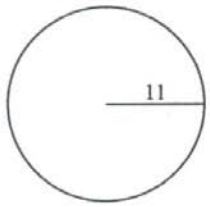
1-10	94	88	81	75	69	63	56	50	44	38
11-16	31	25	19	13	6	0				

Finding the Area and Circumference of a Circle

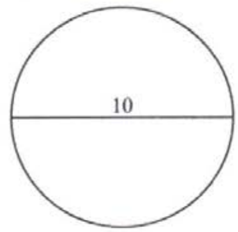
Name: **Answer Key**

Find the area and circumference of each circle. Circles are not to scale.

1)



2)



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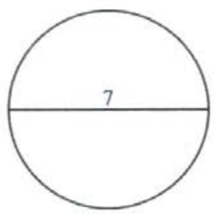

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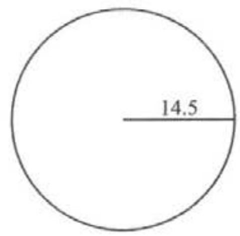

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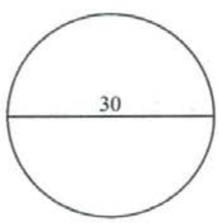

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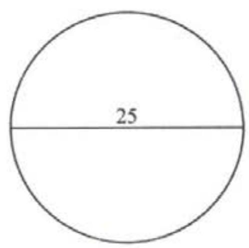

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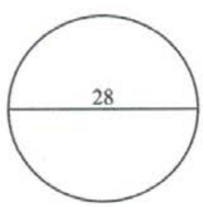
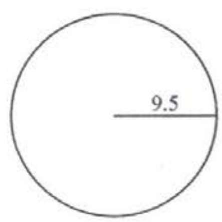


8)



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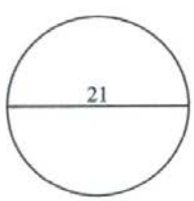

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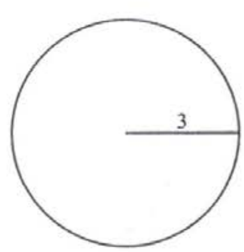


Answers

1a. **380.13**
1c. **69.12**
2a. **78.54**
2c. **31.42**
3a. **38.48**
3c. **21.99**
4a. **660.52**
4c. **91.11**
5a. **706.86**
5c. **94.25**
6a. **490.87**
6c. **78.54**
7a. **615.75**
7c. **87.96**
8a. **283.53**
8c. **59.69**
9a. **346.36**
9c. **65.97**
10a. **28.27**
10c. **18.85**

1-10	95	90	85	80	75	70	65	60	55	50
11-20	45	40	35	30	25	20	15	10	5	0

5 How to · · · · · · · · · · · · · · · · Solve Word Problems with Geometry

Facts to Know

Basic Geometric Formulas

Perimeter

- Perimeter is the length around a closed shape. It is computed by adding the length of all the sides of the figure.
- The formula for finding the perimeter of rectangles and other parallelograms is $P = (l + w) \times 2$ or $P = 2l + 2w$

Area

The area of a flat surface is a measure of how much space is covered by that surface. Area is measured in square units.

- **Area of a Rectangle**

  The area of a rectangle is computed by multiplying the width of one side times the length of the adjoining side.
  $A = l \times w$

  The area of a rectangle can also be determined by multiplying the base times the height.
  $A = b \times h$

- **Area of a Triangle**

  A triangle is always one half of a rectangle or a parallelogram. The area of a triangle is computed by multiplying 1/2 of the base times the height of a triangle.
  $A = \frac{1}{2} b \times h$

- **Area of a Parallelogram**

  The area of a parallelogram is computed by multiplying the base times the height.
  $A = b \times h$

- **Area of a Circle**

  To find the area of a circle, multiply π (3.14) times the radius times the radius again.
  $A = \pi r^2$

Circumference

The circumference is the distance around a circle. To find the circumference of a circle, multiply π (which always equals 3.14) times the diameter or multiply 2 times π (3.14) times the radius.

$$C = \pi d \text{ or } C = 2\pi r$$

Volume

- The formula for finding the volume of a rectangular prism, such as a box, is to multiply the length times the width times the height. $V = l \times w \times h$
- The formula for finding the volume of a cylinder is to multiply π (3.14) times the radius squared times the height. $V = \pi \times r^2 \times h$
- Volume is always computed in cubic units. Use cubic inches or centimeters when determining volume for small prisms and cylinders, and cubic feet or meters for larger ones.

5 Practice · · · · · · · · · · · · · **Solving Word Problems with Geometry**

Geometry at Home

Geometry is a very important aspect of math around the home. Houses and property are measured in geometric terms. Floor and wall coverings, heating systems, and the water supply all have a geometric component.

For this practice page, you need to know the following:

- Wallpaper is sold in double rolls totaling 44 square feet.
- Carpeting is priced by the square yard.
- There are 9 square feet in 1 square yard.
- You cannot buy partial rolls of carpeting or wallpaper.

Directions: Use the formulas and information on page 21 and the information above to help you solve these word problems.

1. Your mother said you can have new carpeting in your room if you compute the amount of carpeting needed and the cost. The length of your room is $18\frac{1}{2}$ feet and the width is 17 feet. The cost of one medium grade of carpeting is $20.00 per square yard.
   A. Compute the number of square feet in the room: ________________
   B. Convert square feet to square yards (divide by 9): ________________
   C. Compute the cost of carpeting needed (multiply by $20.00): _________
2. You want to cover one wall of your room with neon-colored wallpaper that costs $25.00 for a double roll containing 44 square feet. The wall is $18\frac{1}{2}$ feet long and 10 feet high.
   A. Compute the area of your wall in square feet. ________________
   B. Determine how many rolls of wallpaper you need: ____________
   C. Compute the cost of the wallpaper: ________________________
3. Your friend decided to paint the walls and the ceiling of her room with a lovely lavender paint. One gallon of this paint will cover only 400 square feet and costs $17.99 a gallon. These are the dimensions of her room:
   - Wall 1—$21\frac{1}{4}$ feet long and $11\frac{1}{2}$ feet high
   - Wall 3—$21\frac{1}{4}$ feet long and $11\frac{1}{2}$ feet high
   - Wall 2—20 feet long and $11\frac{1}{2}$ feet high
   - Wall 4—20 feet long and $11\frac{1}{2}$ feet high
   - Ceiling—$21\frac{1}{4}$ feet long and 20 feet wide

   A. Compute the area of each wall and ceiling in square feet.
   Wall 1 ______ Wall 2 ______ Wall 3 ______ Wall 4 ______ Ceiling _________
   B. Compute the total area in square feet: ________________________________
   C. Determine how many gallons of paint are needed: _____________________
   D. Compute the total cost of the paint: ___________________________________

1. A. 314.5 sq. ft. B. 34.9 or 35 sq. yd. C. $698.00 or $700.00 2. A. 185 sq. ft. B. 5 rolls C. $125
3. A. 244 3/8 sq. ft. 230 sq. ft.; 244 3/8 sq. ft.; 230 sq. ft.; 425 sq. ft. B. 1,373 3/4 sq. ft. or 1,374 sq. ft. C. 4 gallons D. $71.96

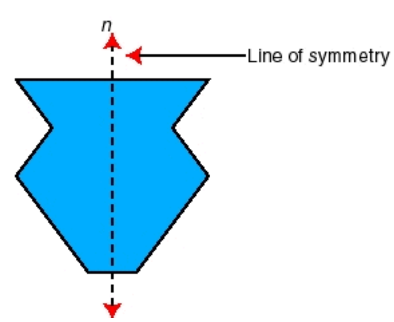
X. Geometry

1. Reason with Shapes and their Attributes

a. Recognize and draw shapes having specified attributes, such as a given number of equal faces; identify triangles, quadrilaterals, pentagons, hexagons, and cubes
b. Partition circles and rectangles into two, three, four or more equal shares, describe the shares using the words halves, thirds, half of, a third of, etc. and describe the whole as two halves, three thirds, etc.
c. Understand that shapes in different categories, such as rhombuses, rectangles, and others, may share attributes, such as having four sides, and that shared attributes can define a larger category, such as quadrilaterals
d. Partition shapes into parts with equal areas; express the area of each part as a unit fraction of the whole
    - *Example: partition a shape into four parts with equal area, and describe the area of each part as* $\frac{1}{4}$ *of the area of the shape*

2. Draw and Identify Lines and Angles, and Classify Shapes by Properties of their Lines and Angles

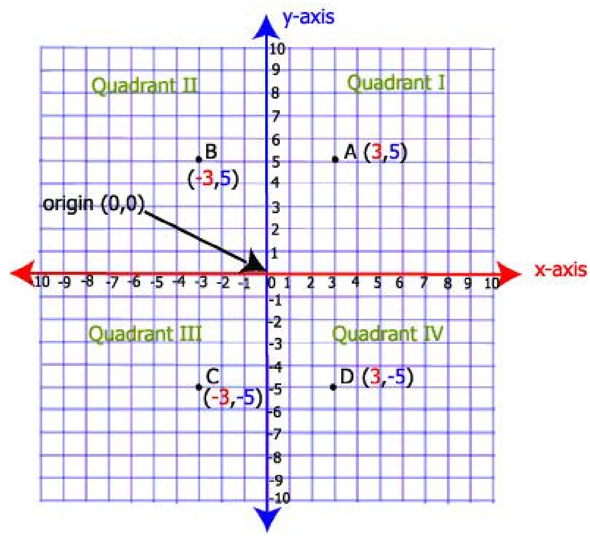
a. Draw points, lines, line segments, rays, angles (right, acute and obtuse), and perpendicular and parallel lines; identify these in two-dimensional figures
b. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles
c. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts



3. **Graph Points on a Coordinate Plane to Solve Real-World and Mathematical Problems**

   a. Use a pair of perpendicular lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinate. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (x-axis and x coordinate, y-axis and y-coordinate)

   b. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and intercept coordinate values of points in the context of the situation



   c. Determine the slope of lines in the coordinate plane. Slope is defined as the rate of change for a function or rise/run

4. **Classify Two-Dimensional Figures into Categories based on their Properties**

   a. Understand attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category
      - *Example: all rectangles have four right angles and squares are rectangles, so all squares have four right angles*

5. **Solve Real-World and Mathematical Problems involving Area, Surface Area, and Volume**

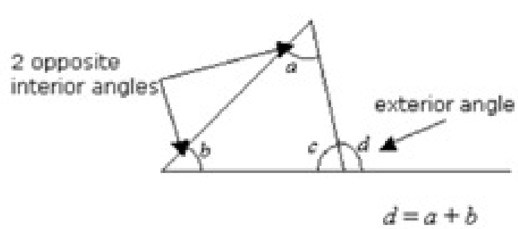
a. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world mathematical problems
b. Draw polygons in the coordinate grid plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems

6. Solve Real-Life Problems and Mathematical Problems involving Angle, Measure, Area, Surface Area, and Volume

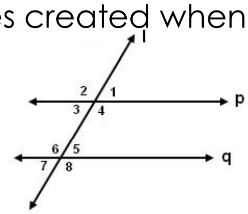
a. Use the correct formulas to find the area and circumference of a circle
b. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure
c. Solve real-world and mathematical problems involving angle, area, volume and surface area of two and three dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms

7. Understand Congruence and Similarity

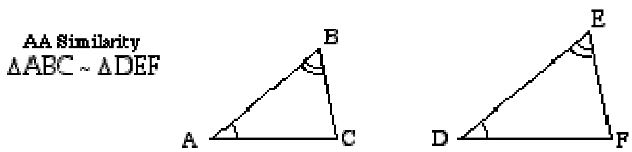
a. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them
b. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them
c. Understand facts about:
    - the angle sum and exterior angle of triangles



- the angles created when parallel lines are cut by a transversal



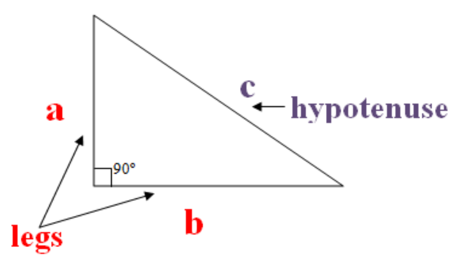
- the angle-angle criterion for similarity of triangles



For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so

8. Understand and Apply the Pythagorean Theorem

a. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensional problems
b. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system



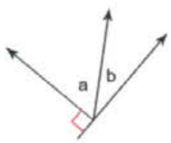
$$a^2 + b^2 = c^2$$

Geometry Name________________________

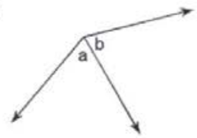
Date______________

Name the relationship: complementary, supplementary, vertical, or adjacent.

1)



2)



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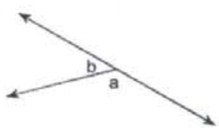
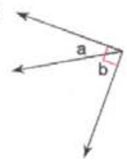
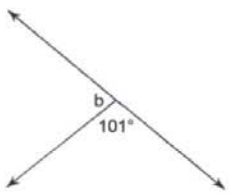


4)



Find the measure of angle b.

5)



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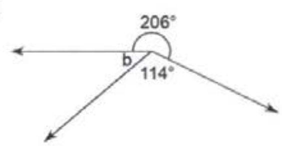

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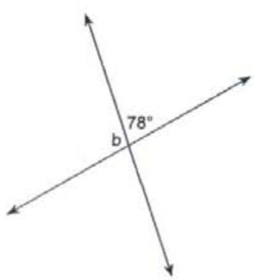
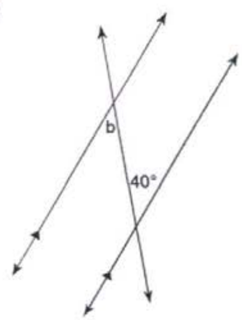
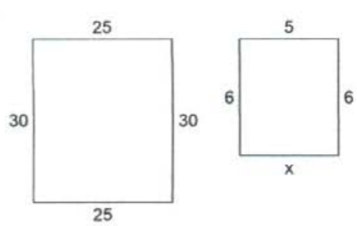


8)

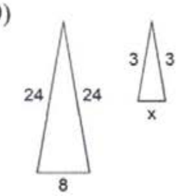


Each pair of figures is similar. Find the missing side.

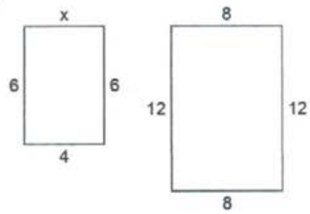
9)



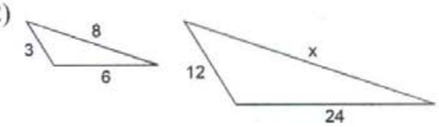
10)



11)

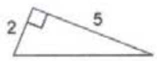


12)

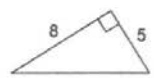


Find each missing length to the nearest tenth.

13)



14)



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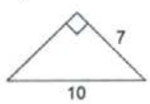
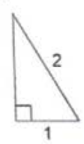
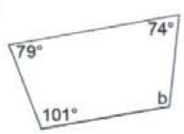


16)



Find the measure of angle b.

17)



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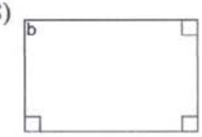
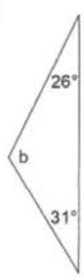
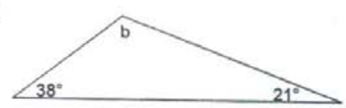


19)

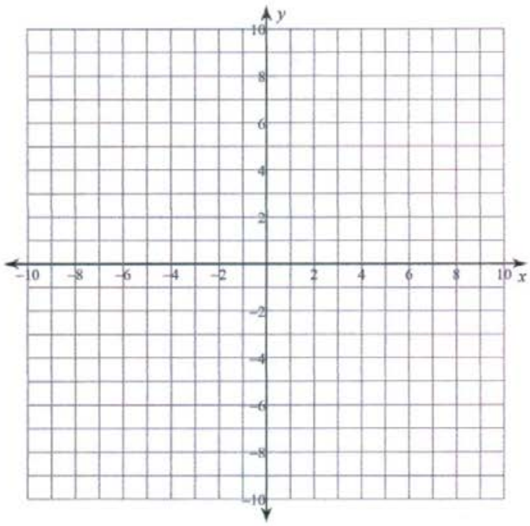


20)



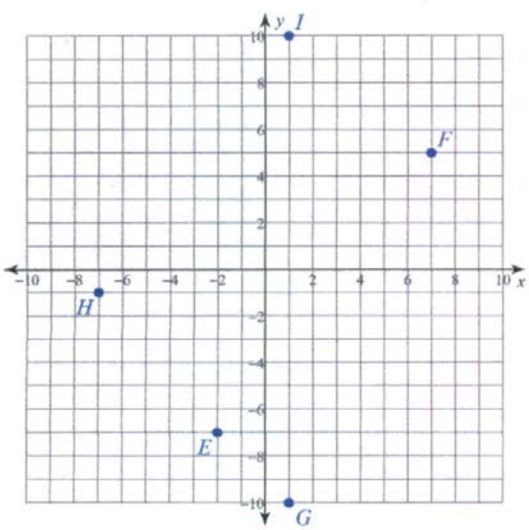
Plot each point.

21) $A(8, -5)$ $B(-4, 10)$ $C(10, 0)$
$D(-6, 10)$ $E(0, -5)$



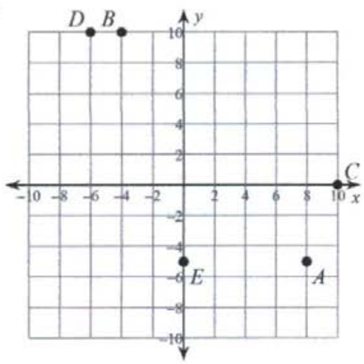
State the coordinates of each point.

22)



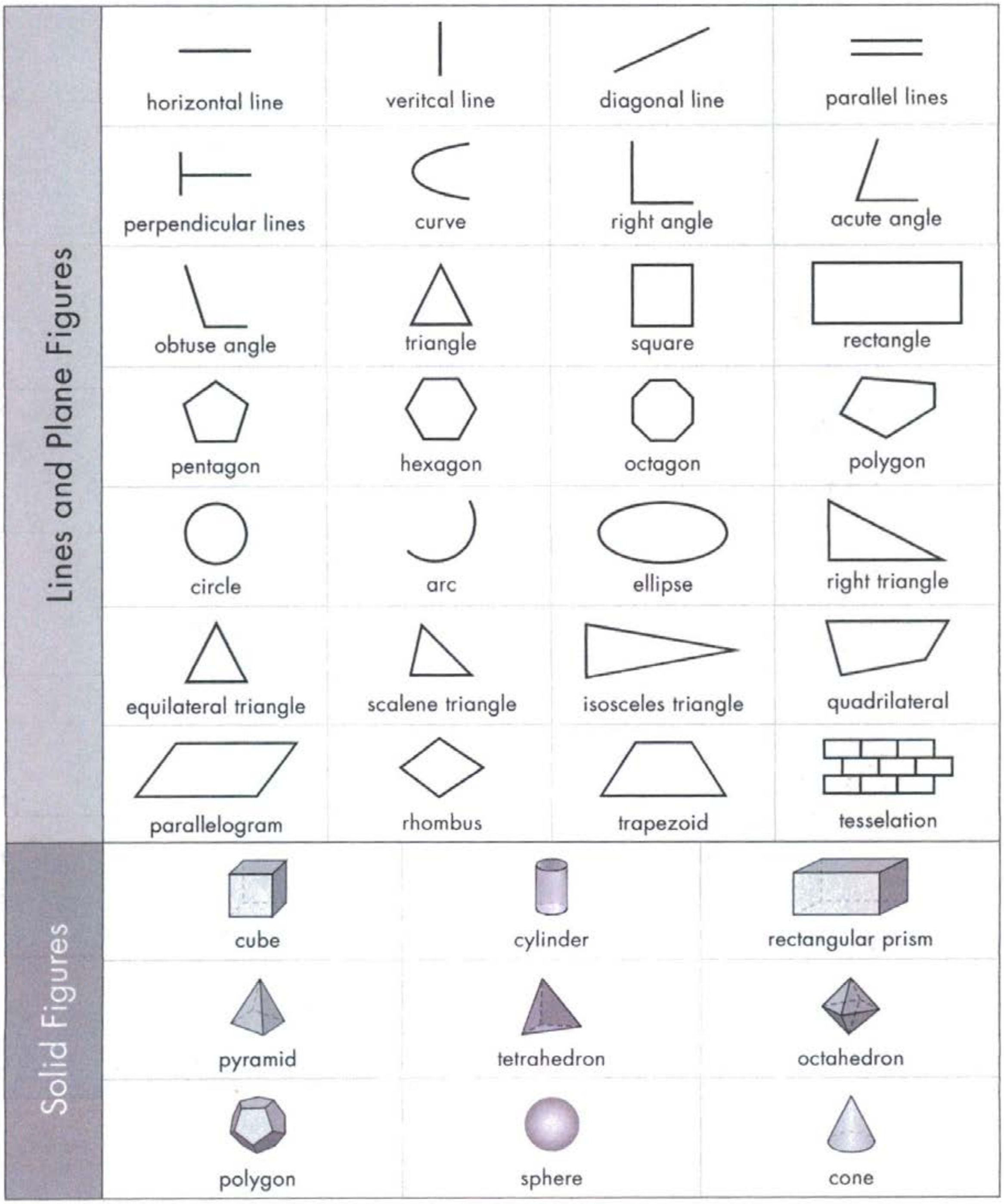
Answers to

1) complementary
2) adjacent
3) supplementary
4) complementary
5) 79°
6) 40°
7) 102°
8) 40°
9) 5
10) 1
11) 4
12) 32
13) 5.4
14) 9.4
15) 7.1
16) 1.7
17) 106°
18) 90°
19) 123°
20) 121°
21)



22) $I(1, 10)$ $H(-7, -1)$ $G(1, -10)$
$F(7, 5)$ $E(-2, -7)$

GEOMETRIC SHAPES CHART



front 1

Point

back 1

•*A*

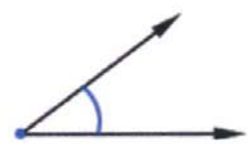
point *A*

An exact location in space, usually represented by a dot

front 2

Angle

back 2

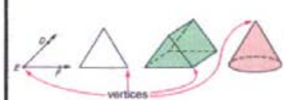


A figure formed by two rays that have a common endpoint

front 3

Vertex

back 3



The point where two or more rays meet; the point of intersection of two sides of a polygon; the point of intersection of three or more edges of a solid figure; the top point of a cone; the plural of vertex is vertices

front 4

Circle

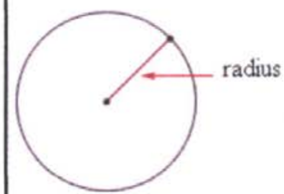
back 4

A closed plane figure with all points on the figure the same distance from the center

front 5

Radius

back 5



A line segment with one endpoint at the center of a circle and the other endpoint on the circle

front 6

Obtuse Triangle

back 6

A triangle that has one obtuse angle

front 7

Polygon

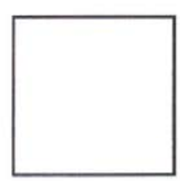
back 7

A closed plane figure formed by three or more line segments

front 8

Square

back 8

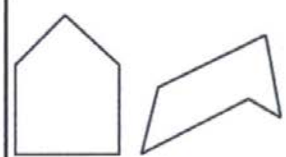


A rectangle with 4 equal sides

front 9

Pentagon

back 9



A polygon with five sides

front 10

Line

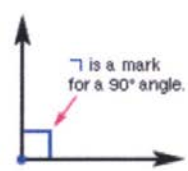
back 10

A straight path in a plane that goes on forever in opposite directions

front 11

Right Angle

back 11

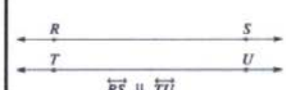


An angle formed by perpendicular lines, line segments, or rays and with a measure of 90°

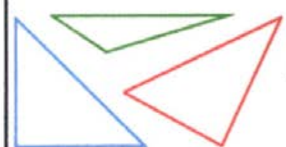
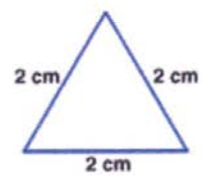
front 12

Parallel Line

back 12



Lines in a plane that never intersect

front 13 Circumference	back 13  The distance around a circle
front 14 Triangle	back 14  A polygon with three sides
front 15 Equilateral Triangle	back 15  A triangle with three congruent sides
front 16 Parallelogram	back 16  A quadrilateral whose opposite sides are parallel and congruent

front 17

Trapezoid

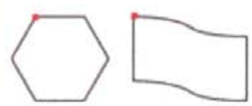
back 17

A quadrilateral with one pair of parallel sides

front 18

Closed Figure

back 18



A figure that begins and ends at the same point

front 19

Line Segment

back 19

line segment *AB* or line segment *BA*

A part of a line that includes two points, called endpoints, and all of the points

front 20

Acute Angle

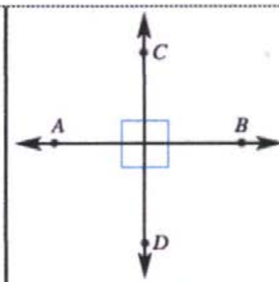
back 20

An angle that has a measure less than a right angle (less than 90°)

front 21

Perpendicular Lines

back 21



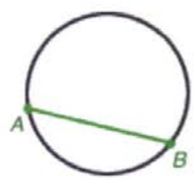
$\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

Two lines that intersect to form four right angles

front 22

Chord

back 22



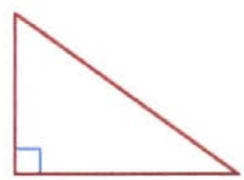
$\overline{AB}$ is a chord

A line segment with its endpoints on a circle

front 23

Right Triangle

back 23

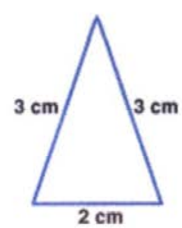


A triangle with one right angle

front 24

Isosceles Triangle

back 24

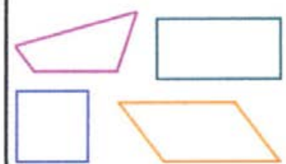


A triangle with two congruent sides

front 25

Quadrilateral

back 25



A polygon with four sides

front 26

Hexagon

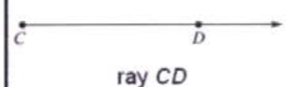
back 26

A polygon with six sides and six angles

front 27

Ray

back 27



A part of a line, with one endpoint, that continues without end in one direction

front 28

Obtuse Angle

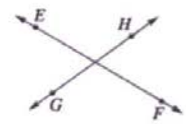
back 28

An angle whose measure is greater than 90° and less than 180°

front 29

Intersecting Lines

back 29



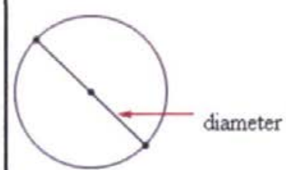
Line *EF* intersects line *GH*

Lines that cross at exactly one point

front 30

Diameter

back 30

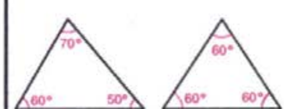


A line segment that passes through the center of a circle and has its endpoints on the circle

front 31

Acute Triangle

back 31

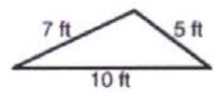


A triangle in which all three angles are acute

front 32

Scalene Triangle

back 32



A triangle with no congruent sides

STRING SHAPES

PURPOSE

To practice geometry vocabulary in a fun (and funny!) way.

YOU NEED:

- ☆ *a 20-foot-long piece of string for each group.*

Number of players: 3 or more

Geometric Shapes

triangle
right triangle
acute triangle
obtuse triangle
isosceles triangle
equilateral triangle
scalene triangle
square
rectangle
parallelogram
rhombus
trapezoid
pentagon
hexagon
two similar triangles
a concave polygon
add your own!

This activity helps students practice geometry-related vocabulary, and it never fails at getting everyone laughing.

1. Tie the ends of a 20-foot-long piece of string together to form a big loop. Make a loop like this for each team.
2. There should be at least 3 people on each team. If there are not enough students to form more than one team, you can play with just one team. It's no longer a race, but it's still good vocab practice and good fun.
3. One person does not play. This person is the "Caller" and the "Judge". This can be the teacher or a student.
4. Members of a team stand in a circle and hold the large loop of string with both hands.
5. The "Caller and Judge" calls out the name of a geometric figure. The players on each team have to work cooperatively to form this shape with their string. The first group to do this successfully scores a point. The judge makes this decision. If a group's shape doesn't quite match the criteria closely enough, she can say "Not quite!" and the play continues.
6. To score a point, students must each have two hands on the string when they announce that they've completed a shape. (If there are 5 or more students on a team, you can change the rules so that each student can hold the string with only one hand.)
7. Play 7 rounds. The group with the most points at the end is the winner.

Variations: **(a)** The geometric shapes can be written on cards, and teams can take turns picking a card and forming the shape they pick. The other group times how long it takes them. The team with the shortest total time after five cards is the winner. **(b)** One team can make a shape for the other team to guess and vice versa. **(c)** Instead of loops of string, each group can have two pieces of string and asked to form different types of lines and angles.

Active Math

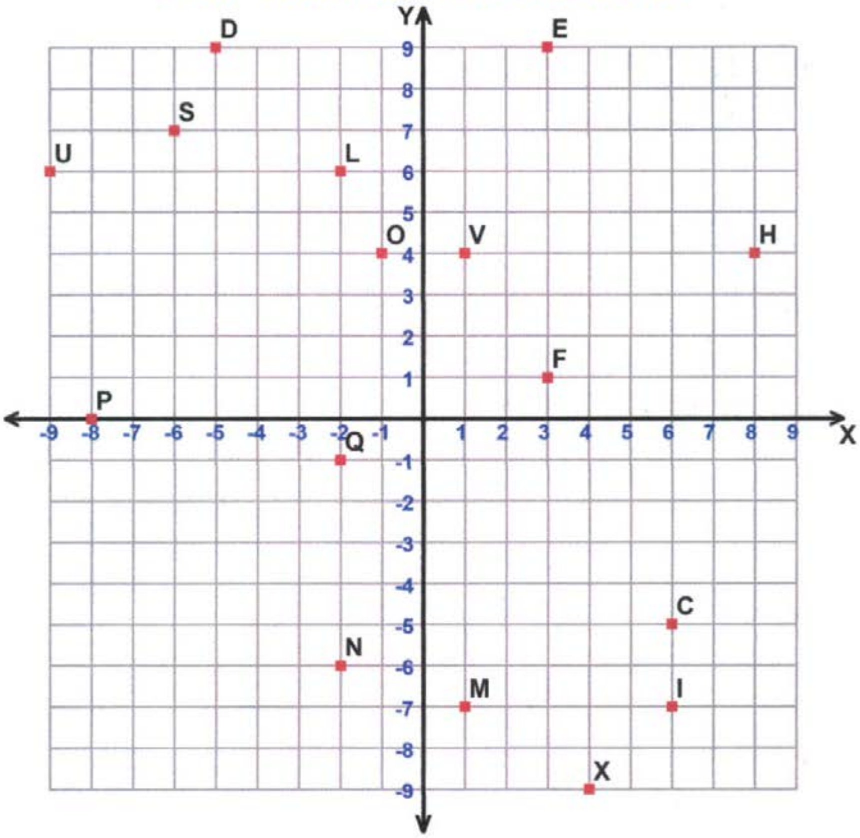
TEACHING IDEAS FOR ADULT EDUCATION TEACHERS OF MATHEMATICS

New Hampshire Bureau of Adult Education — Ruth Estabrook

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

Four Quadrant Ordered Pairs



Tell what point is located at each ordered pair.

1) (-8,+0) _____ 3) (-5,+9) _____ 5) (+6,-7) _____ 7) (-2,+6) _____

2) (+6,-5) _____ 4) (-2,-1) _____ 6) (-1,+4) _____ 8) (+3,+1) _____

Write the ordered pair for each given point.

9) **S** _______ 11) **H** _______ 13) **U** _______ 15) **N** _______

10) **M** _______ 12) **V** _______ 14) **E** _______ 16) **X** _______

Plot the following points on the coordinate grid.

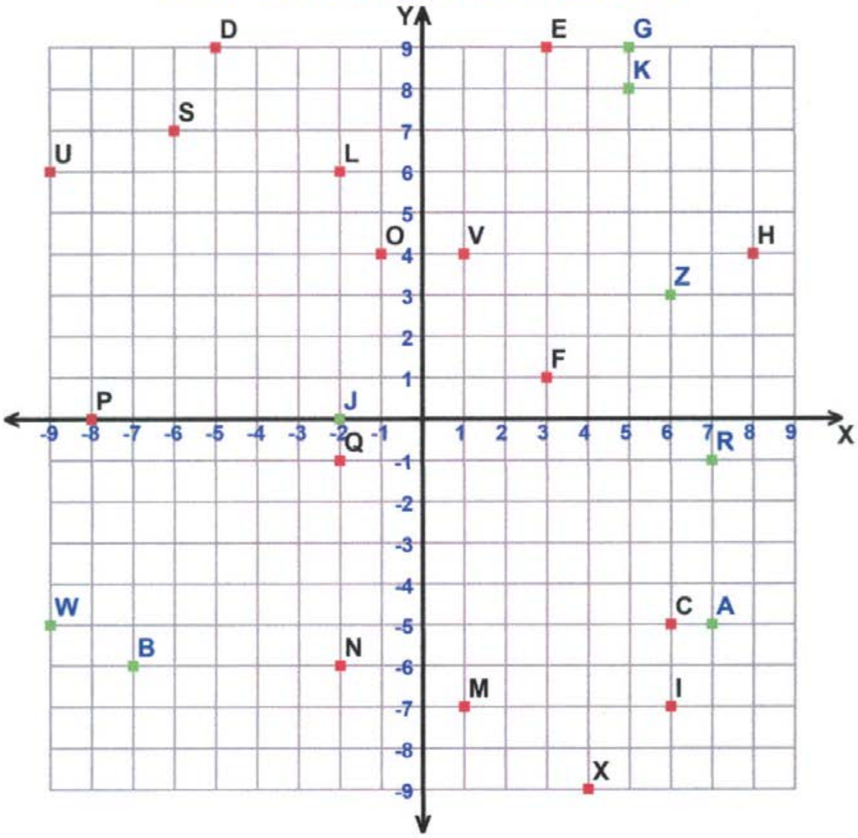
17) **B** (-7,-6) 19) **J** (-2,+0) 21) **R** (+7,-1) 23) **W** (-9,-5)

18) **Z** (+6,+3) 20) **G** (+5,+9) 22) **A** (+7,-5) 24) **K** (+5,+8)

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Four Quadrant Ordered Pairs



Tell what point is located at each ordered pair.

1) (-8,+0) P
3) (-5,+9) D
5) (+6,-7) I
7) (-2,+6) L
2) (+6,-5) C
4) (-2,-1) Q
6) (-1,+4) O
8) (+3,+1) F

Write the ordered pair for each given point.

9) **S** (-6,+7)
11) **H** (+8,+4)
13) **U** (-9,+6)
15) **N** (-2,-6)
10) **M** (+1,-7)
12) **V** (+1,+4)
14) **E** (+3,+9)
16) **X** (+4,-9)

Plot the following points on the coordinate grid.

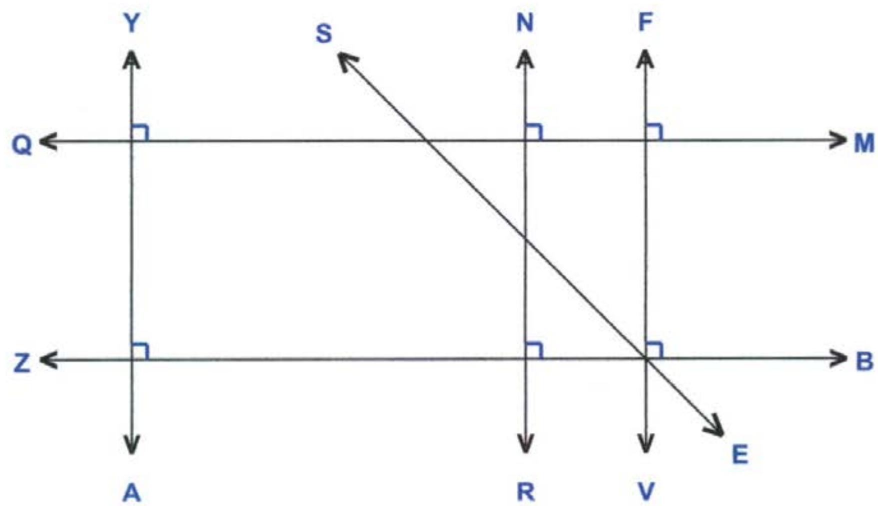
17) **B** (-7,-6)
19) **J** (-2,+0)
21) **R** (+7,-1)
23) **W** (-9,-5)
18) **Z** (+6,+3)
20) **G** (+5,+9)
22) **A** (+7,-5)
24) **K** (+5,+8)

Name : ______________________________ Score : __________

Teacher : ____________________________ Date : __________

Identify Parallel, Perpendicular, and Intersecting Lines

Identify the given pair of lines as either parallel, perpendicular, or intersecting.



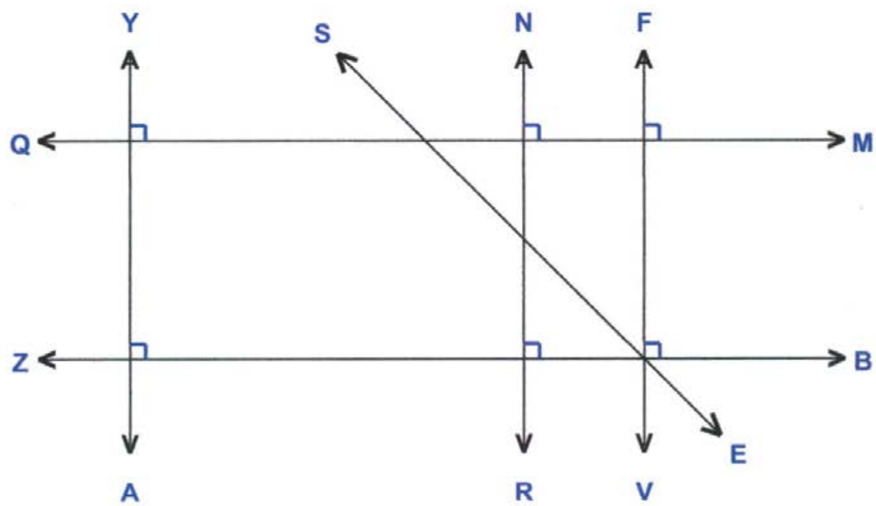
1) Line QM and Line NR are ______________ lines.	6) Line QM and Line SE are ______________ lines.
2) Line YA and Line NR are ______________ lines.	7) Line FV and Line NR are ______________ lines.
3) Line QM and Line FV are ______________ lines.	8) Line QM and Line YA are ______________ lines.
4) Line YA and Line FV are ______________ lines.	9) Line ZB and Line SE are ______________ lines.
5) Line SE and Line NR are ______________ lines.	10) Line ZB and Line NR are ______________ lines.

Name : ______________________ Score : ________

Teacher : ______________________ Date : ________

Identify Parallel, Perpendicular, and Intersecting Lines

Identify the given pair of lines as either parallel, perpendicular, or intersecting.



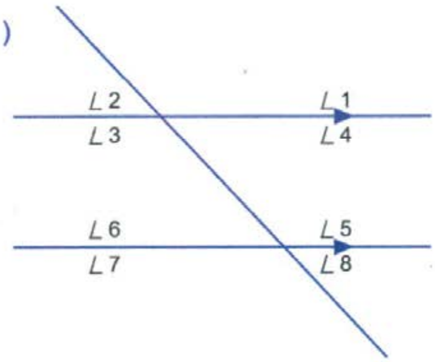
1) Line QM and Line NR are Perpendicular lines.	6) Line QM and Line SE are Intersecting lines.
2) Line YA and Line NR are Parallel lines.	7) Line FV and Line NR are Parallel lines.
3) Line QM and Line FV are Perpendicular lines.	8) Line QM and Line YA are Perpendicular lines.
4) Line YA and Line FV are Parallel lines.	9) Line ZB and Line SE are Intersecting lines.
5) Line SE and Line NR are Intersecting lines.	10) Line ZB and Line NR are Perpendicular lines.

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

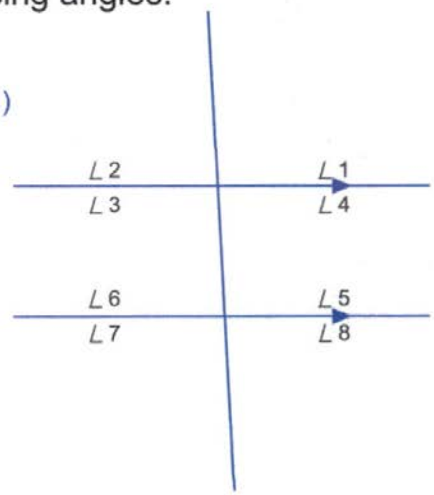
Find all of the missing angles.

1)



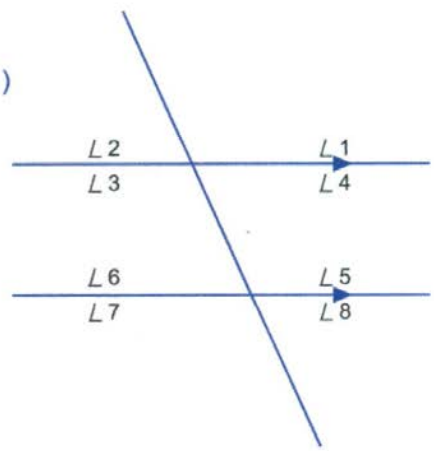
∠1 = 133°
∠2 = ____
∠3 = ____
∠4 = ____
∠5 = ____
∠6 = ____
∠7 = ____
∠8 = ____

2)



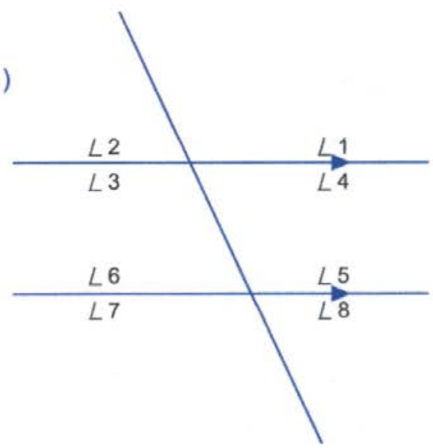
∠1 = ____
∠2 = ____
∠3 = ____
∠4 = ____
∠5 = ____
∠6 = 87°
∠7 = ____
∠8 = ____

3)



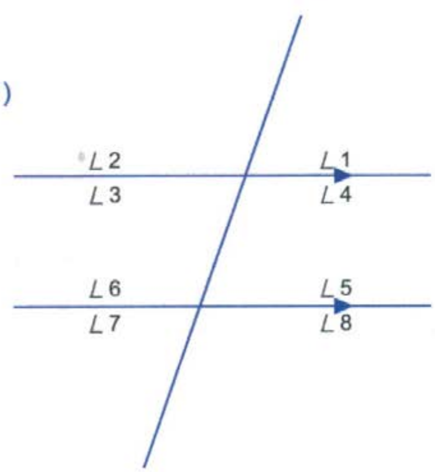
∠1 = ____
∠2 = ____
∠3 = ____
∠4 = ____
∠5 = 114°
∠6 = ____
∠7 = ____
∠8 = ____

4)



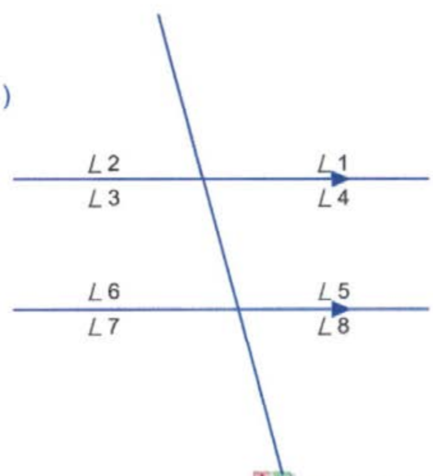
∠1 = ____
∠2 = ____
∠3 = ____
∠4 = ____
∠5 = ____
∠6 = ____
∠7 = ____
∠8 = 65°

5)



∠1 = ____
∠2 = ____
∠3 = 71°
∠4 = ____
∠5 = ____
∠6 = ____
∠7 = ____
∠8 = ____

6)



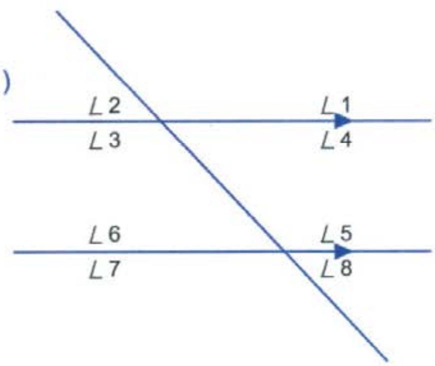
∠1 = ____
∠2 = 75°
∠3 = ____
∠4 = ____
∠5 = ____
∠6 = ____
∠7 = ____
∠8 = ____

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

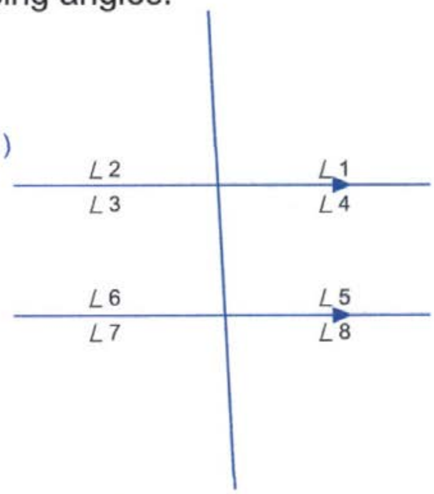
Find all of the missing angles.

1)



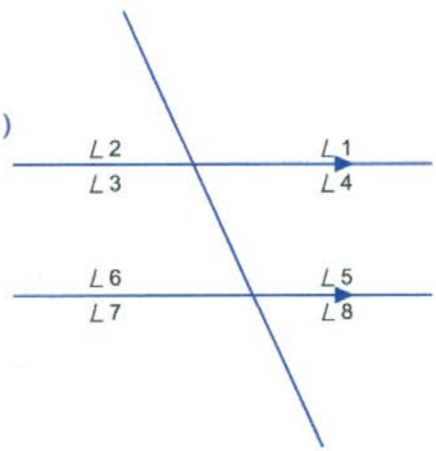
∠1 = 133°
∠2 = 47°
∠3 = 133°
∠4 = 47°
∠5 = 133°
∠6 = 47°
∠7 = 133°
∠8 = 47°

2)



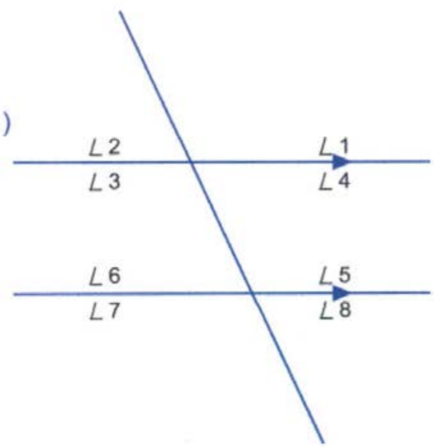
∠1 = 93°
∠2 = 87°
∠3 = 93°
∠4 = 87°
∠5 = 93°
∠6 = 87°
∠7 = 93°
∠8 = 87°

3)



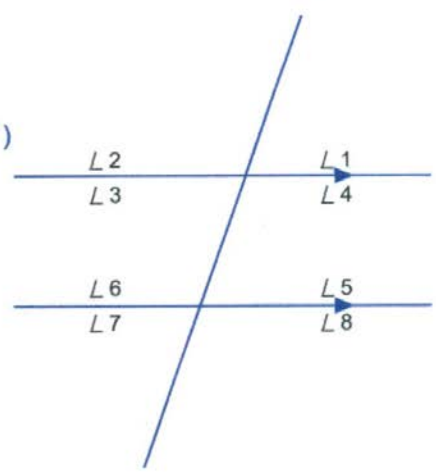
∠1 = 114°
∠2 = 66°
∠3 = 114°
∠4 = 66°
∠5 = 114°
∠6 = 66°
∠7 = 114°
∠8 = 66°

4)



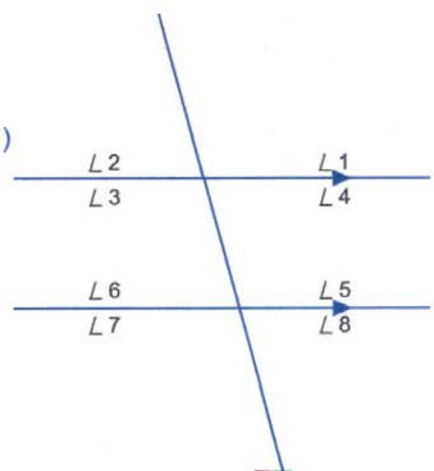
∠1 = 115°
∠2 = 65°
∠3 = 115°
∠4 = 65°
∠5 = 115°
∠6 = 65°
∠7 = 115°
∠8 = 65°

5)



∠1 = 71°
∠2 = 109°
∠3 = 71°
∠4 = 109°
∠5 = 71°
∠6 = 109°
∠7 = 71°
∠8 = 109°

6)

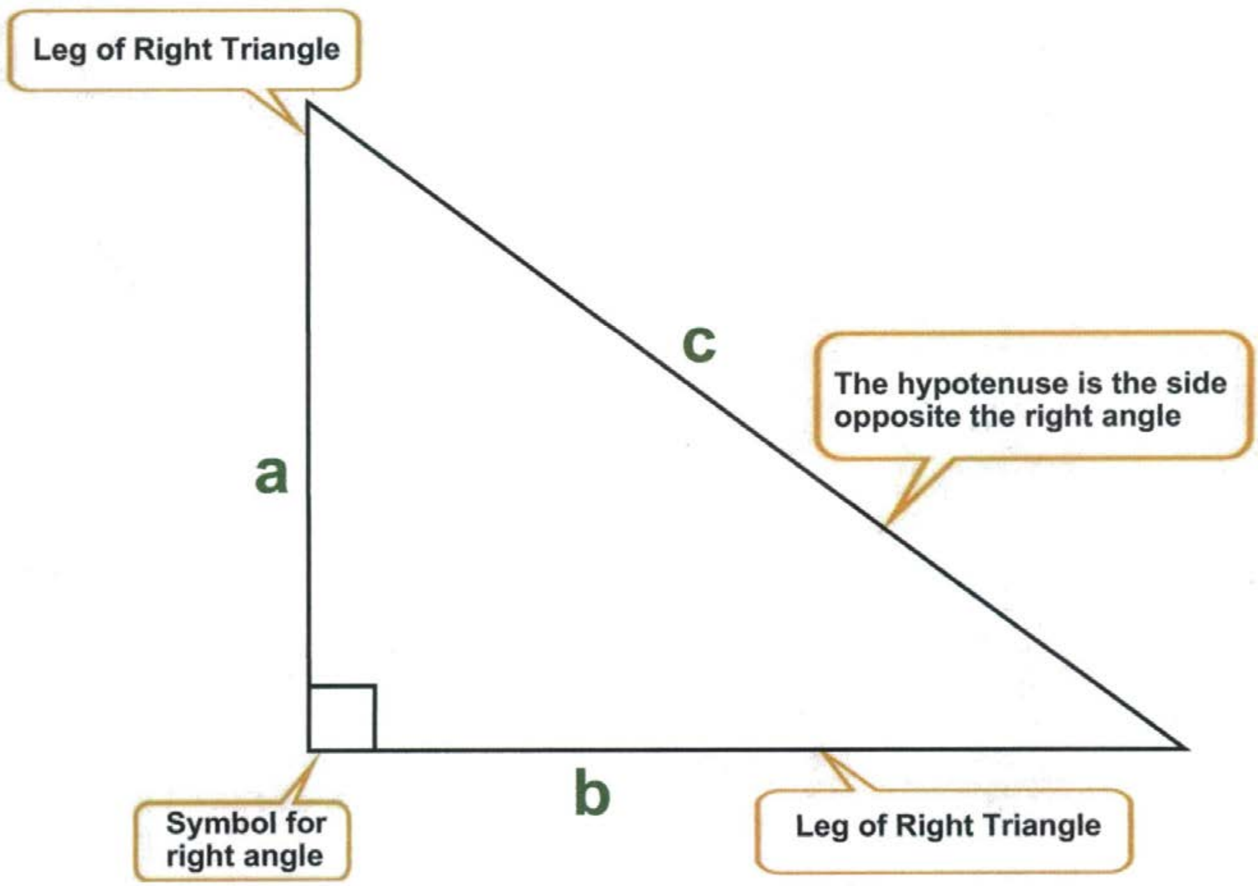


∠1 = 105°
∠2 = 75°
∠3 = 105°
∠4 = 75°
∠5 = 105°
∠6 = 75°
∠7 = 105°
∠8 = 75°

Pythagorean Theorem

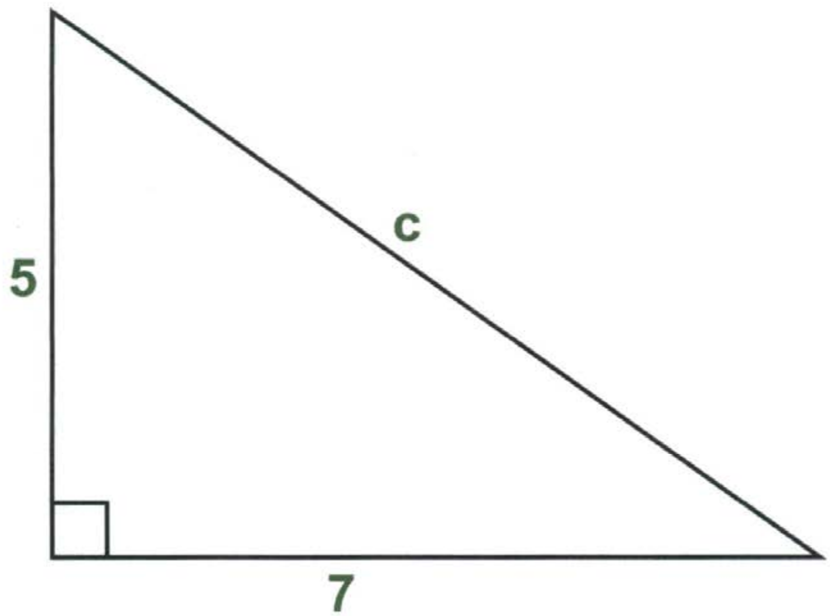
The Pythagorean Theorem describes the relationship between the lengths of the legs and the hypotenuse of a right triangle.

$$a^2 + b^2 = c^2$$



Pythagorean Theorem

The Pythagorean Theorem will work for any right triangle.



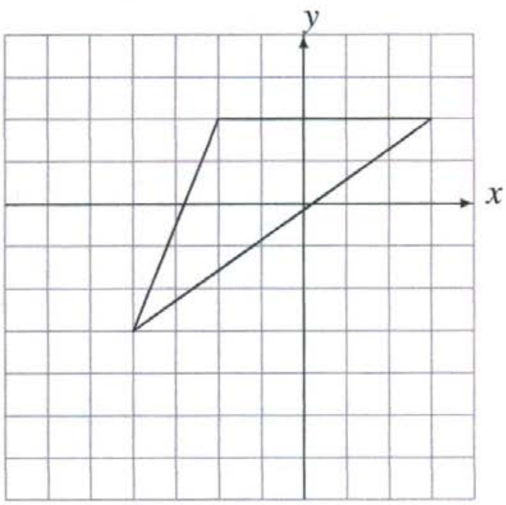
$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 7^2$$
$$c^2 = 25 + 49$$
$$c^2 = 74$$
$$c = \sqrt{74}$$
$$c \approx 8.6023$$

Math-Aids.Com

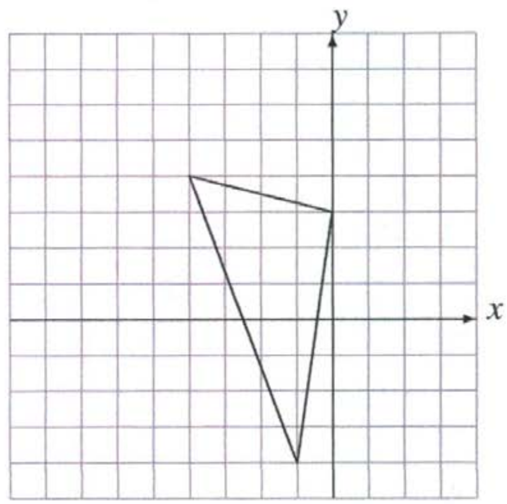
Translations (A)

Draw each translated image.

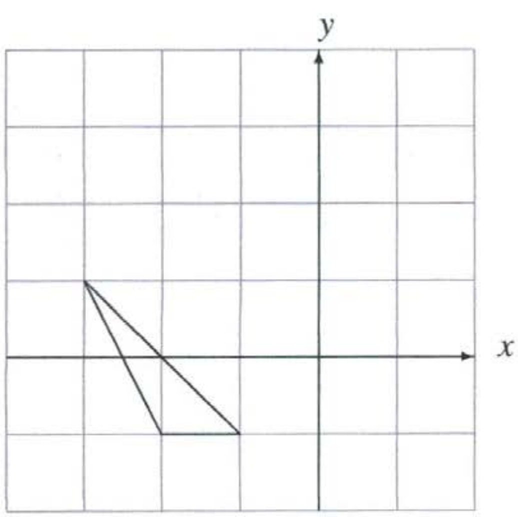
Translate by (-2, -3).



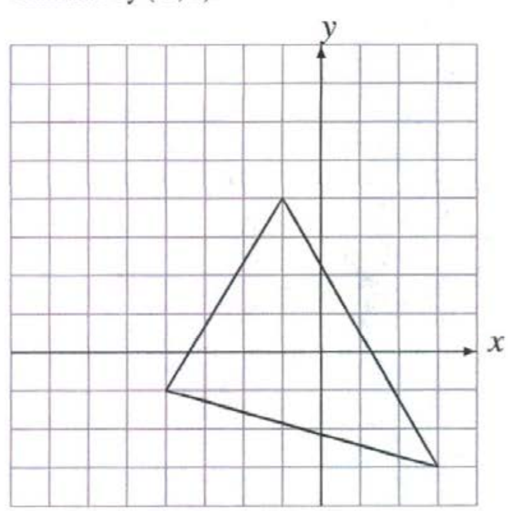
Translate by (-1, 3).



Translate by (2, 2).



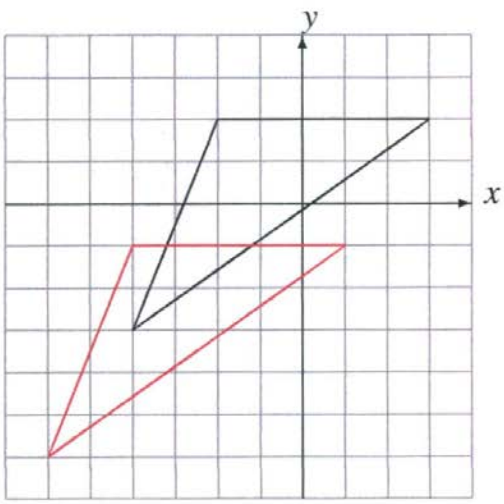
Translate by (-3, 2).



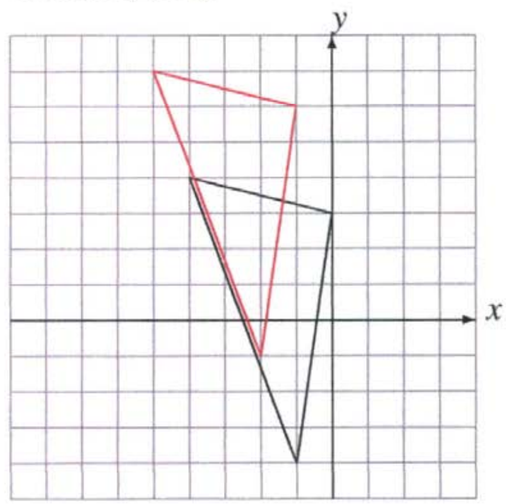
Translations (A) Answers

Draw each translated image.

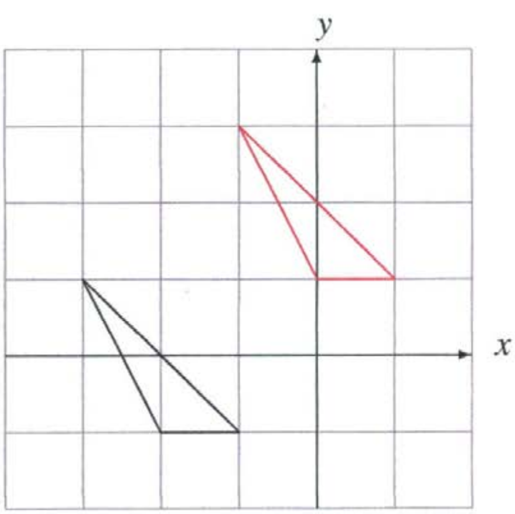
Translate by (-2, -3).



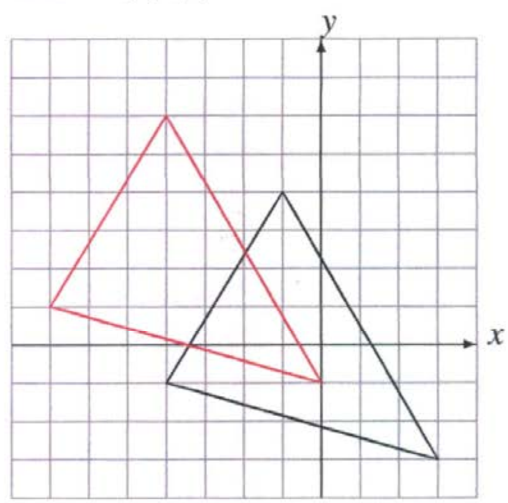
Translate by (-1, 3).



Translate by (2, 2).



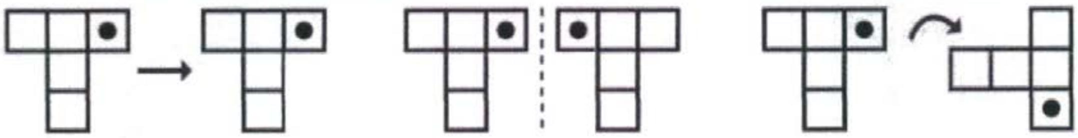
Translate by (-3, 2).



Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

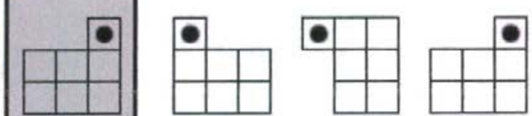
Translation, Rotation, and Reflection



Translation **Reflection** **Rotation**

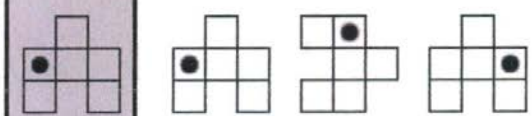
Identify each shape as translation, rotation, and reflection.

1)



________ ________ ________

2)

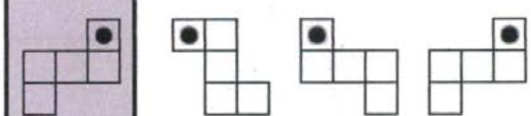


________ ________ ________

3)

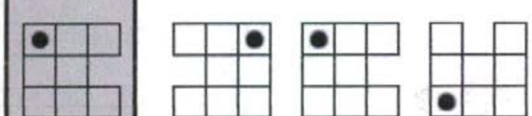
________ ________ ________

4)



________ ________ ________

5)

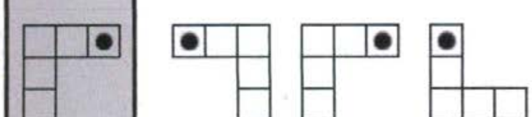


________ ________ ________

6)

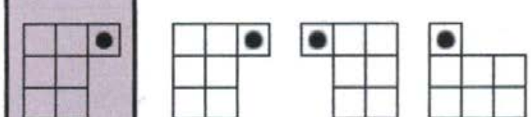
________ ________ ________

7)



________ ________ ________

8)

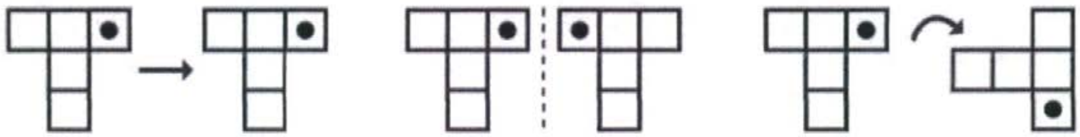


________ ________ ________

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

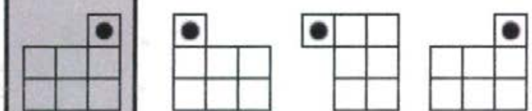
Translation, Rotation, and Reflection



Translation **Reflection** **Rotation**

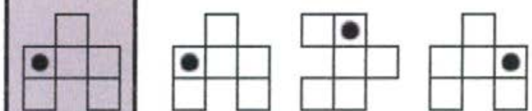
Identify each shape as translation, rotation, and reflection.

1)



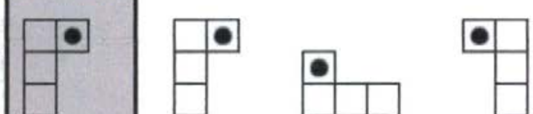
Reflection Rotation Translation

2)



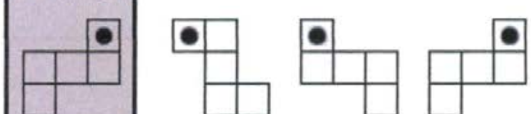
Translation Rotation Reflection

3)



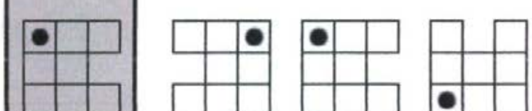
Translation Rotation Reflection

4)



Rotation Reflection Translation

5)



Reflection Translation Rotation

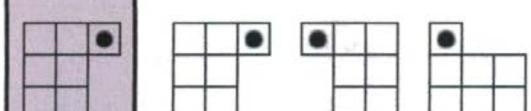
6)

Rotation Reflection Translation

7)

Reflection Translation Rotation

8)



Translation Reflection Rotation

XI. Algebraic Thinking, Expressions and Equations

1. Develop and Apply Understanding of Integers

a. Locate integers (*positive and negative whole numbers*) on a number line
b. Apply understanding of integers to describe real life situations such as above/below sea level, credit/debit, and temperature above/below 0
c. Compare and order integers
d. Understand that absolute value is the distance between a number and zero *Distance is never negative, so absolute value is always a positive number and is represented using vertical bars* $|+4| = 4$ *and* $|-4| = 4$
e. Find and position integers on a coordinate plane
f. Use a number line to find the sum of two or more integers
g. Use integer addition rules to find the sum of two or more integers
h. Use a number line to subtract integers
i. Use addition of opposite integers to find the difference between two integers
j. Multiply and divide integers
k. Use knowledge of integers to solve real life problems

2. Develop Understanding of Factors and Multiples

a. Find all factor pairs for a whole number in the range of 1-100
b. Recognize that a whole number is a multiple of each of its factors
c. Determine whether a given whole number in the range of 1-100 is a multiple of a given one-digit number
d. Determine whether a given whole number in the range of 1-100 is prime or composite

3. Generate and Analyze Patterns

a. Generate a number pattern that follows a given rule
   - *Example: the pattern 2, 4, 6, 8… follows the rule 2n where n is the number of the term 1, 2, 3…*
   - *Example: 5, 8, 11, 14,… rule: each term is three more than the previous term*
b. Identify apparent features of a pattern that were not explicit in the rule itself
   - *Example: given the rule "add 3" and starting at the number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*

4. Write and Interpret Numerical Expressions

a. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols

- *Examples:* $5 + [2 + (3 - 8)]$ $\quad\quad$ $-9 + \{2 - [1 + (3 - 9)]\}$
  Note: work from innermost grouping symbols to outermost while following order of operations

b. Follow the "Order of Operations"

Order of Operations	
( ) , [ ] , { }	Parentheses, Brackets, Braces
x^a , $\sqrt{}$	Exponents, radicals
$\times$, $\div$	Multiplication, Division
$+$, $-$	Addition, Subtraction

c. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them

- *Add 5 and 7, then multiply by 4 is expressed as 4(5+7) or 4 X (5 + 7)*
- *Recognize that 3(20,500 + 55,000) is three times larger than 20,500 + 55,000*

5. Apply and Extend Previous Understandings of Arithmetic to Algebraic Expressions

An expression is a mathematical phrase that combines numbers and/or variables using mathematical operations such as 2(50 -n) + $\frac{10-5}{2}$

a. Write and evaluate numerical expressions involving whole number exponent

- $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ $\quad\quad$ $5^2 + (100 - 2^3) = 25 + 92 = 117$

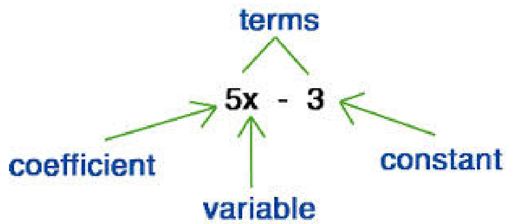
b. Read, write, and evaluate expressions in which letters stand for numbers

- *Twice a number increased by 5 can be written 2x + 5*
- *John has twice as many pencils as Jen. If Jen has 'x' pencils, represent John's amount as 2x. If Jen has 5 pencils, how many does John have?*
- *What is the value of* $5n - (3^2 + 6)$ *if n = 6?*

c. Write expressions that record operations with numbers and with letters standing for numbers

- *The product of 2 and a number can be expressed as 2n*
- *2 less than the quotient of n and 8 can be expressed as* $\frac{n}{8} - 2$

d. Identify parts of an expression using mathematical terms such as *sum, product, term, factor, coefficient* and view one or more parts of an expression as a single entity



- *2(3 + 5) is described as a product of two factors; view (3 + 5) as a single entity and the sum of two terms*
- *In the expression, identify the quotient:* $\frac{12}{3} + 6$*; the expression is a sum*
- *In the expression, identify the product:* $4 \cdot 7 - 5$*; the expression is a difference*

e. Evaluate algebraic expressions and " real world" formulas using the Order of Operations
   - *Use the formulas* $C = \frac{5}{9}(F - 32)$ *and* $F = \frac{9}{5}C + 32$ *to convert Celsius temperature to Fahrenheit*
   - $h = -16t^2$ *where h is height of an object dropped after t seconds*

f. Apply the properties of operations to generate equivalent expressions
   - *Apply the distributive property to the expression 5(2 + x) to produce the equivalent expression 10 + 5x*
   - *apply the properties of operations to n + n + n to produce the equivalent expression 3n*

g. Identify when two expressions are equivalent
   - *n + n + n = 3n regardless of which number n represents*

6. Reason About and Solve One and Two-Variable Equations and Inequalities

a. Understand solving an equation or inequality as a process of answering a question: *which values from a specified set, if any, make the equation or inequality true*
   - *Given the equation* $3x = -6$*, which values in the set {0, 1,-1,-2} make the equation true?*
   - *Given the inequality* $3x < -3$*, which values in the set {-3,-2,-1, 0, 1, 2, 3} make the inequality true?*

b. Use substitution to determine whether a given number makes an equation or inequality true
   - *Given the equation* $3x = -6$*, since* $3(-2) = -6 \textit{ so that } -6 = -6$*, then {-2} is the solution to the equation*
   - *Given the inequality* $3x < -3$*, since* $3(-3) < -3 \textit{ and } 3(-2) < -3$*, {-3,-2} are solutions. Since* $3(-1) < -3 \textit{ is } false$*, {-1, 0,1,2,3} are not solutions*

c. Use variables to represent numbers and write expressions to solve real world or mathematical problems
    - *George has 14 more pepper plants than Sue. Represent Sue's plants with 'x' and George's plants as 'x + 14'. If Sue has 5 pepper plants, how many does George have?*

d. Use algebra to solve addition and subtraction equations using inverse operations
    - *Solve $x + 9 = 2$ using inverse operations:* x + 9 – 9 = 2 – 9*, x = -7*

e. Use algebra to solve multiplication and division equations, use inverse operations
    - *Solve 3m = 27 using inverse operations* $\frac{3m}{3} = \frac{27}{3}$ $m = 9$

f. Write algebraic inequalities and represent solutions on a number line
    - *The distance (d) I drive each day is at least 20 miles d ≥ 20*

g. Write an inequality of the form x > c or x < c to represent a constraint or condition in a real world or mathematical problem
    - *Fred has more than 2 cars:*$C > 2$
    - *Nate has less than 10 dollars:* $N < 10$

h. Translate verbal sentences into a one-step inequality
    - *The number of students increased by 5 is less than 15:* $T + 5 < 15$

i. Solve one-step inequalities

$$\begin{aligned} T + 5 &< 15 \\ -5 \quad &\;\; -5 \\ T &< 10 \end{aligned}$$

   - *Note of caution: Solving inequalities by multiplication or division: when multiplying or dividing an inequality by a negative number, the order of the inequality is reversed. Example:* $-2x > 14$ *solve by dividing by -2:* $\frac{-2x}{-2} < \frac{14}{-2}$ *produces:* $x > -7$

Multiplication by positives or negatives:	
$2 < 3$	$2 < 3$
$2 \cdot 2 < 3 \cdot 2$	$2 \cdot -2 < 3 \cdot -2$
$4 < 6$	$-4 < -6$
True!	*False!*

j. Translate verbal sentences into two-step equations
    - *I have 5 more chocolates than twice the number that Peter has. If I have 19 chocolates, how many does Peter have?*
      Translate: 5 more than twice Peter is 19 → $5 + 2P = 19$

k. Solve two-step equations using inverse operations

$$\begin{aligned} 5 + 2n &= 19 \\ -5 \quad & \quad -5 \\ 2n &= 14 \\ \frac{2n}{2} &= \frac{14}{2} \\ n &= 7 \end{aligned}$$

l. Translate verbal statements into two-step inequalities
   - *three times a number is increased by 5. The result is less than 26*
     Translate: $3x + 5 < 26$

m. Solve two-step inequalities

$$\begin{aligned} 3x + 5 &< 26 \\ -5 \quad & \quad -5 \\ \frac{3x}{3} &< \frac{21}{3} \\ x &< 7 \end{aligned}$$

n. Identify, represent, and generalize patterns using expressions and equations

7. Represent and Analyze Quantitative Relationships between Dependent and Independent Variables

a. Write an equation to represent the relationship between an independent variable and a dependent variable (*the value of the dependent variable changes according to the value of the independent variable)*
   - *Jake rides the bus almost every day. He pays $2.50 for each bus ride. c = 2.5r, c = total cost and r = the number of rides he takes, the value of c (dependent variable) will change when the value of r (independent variable) changes*

b. Use tables and equations to represent the relationship between two quantities
   - *Joey buys a subscription to Netflix at $7 per month. Make a table representing M as the number of months and C as the cost. Describe how the cost relates to the number of months subscribed. Write an equation representing the cost in terms of the number of months subscribed.*

8. Work with Radicals and Integer Exponents

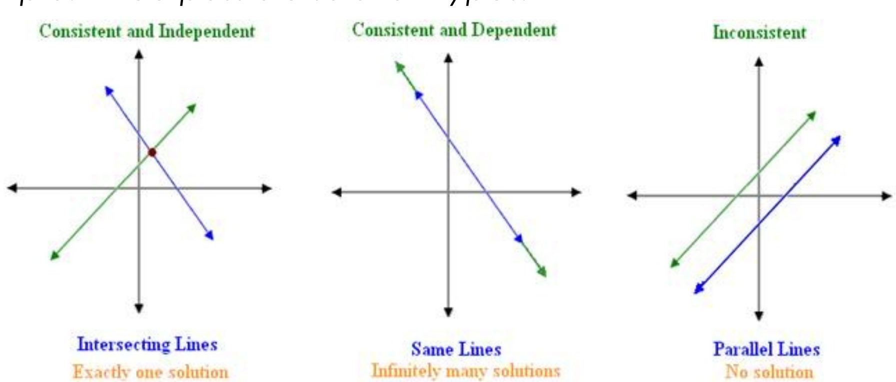
a. Know and apply the properties of integer exponents to generate equivalent numerical expressions such as

$$3^3 \cdot 3^{-5} = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

b. Evaluate square roots of small perfect squares and cube roots of small perfect cubes
c. Develop and use an understanding of large numbers and translate between numbers written in scientific notation and standard notation
Example: $63,000,000 = $\$6.3\ x\ 10^7$
d. Develop and use an understanding of small quantities and translate between numbers written in scientific notation and standard notation
Example: 0.0000009 cm = $9\ x\ 10^{-7}$*cm*
e. Perform operations with numbers expressed in scientific notation
    - *The current national debt is $18.1 trillion or* $1.81\ x10^{13}$ *and the US population is 321 million or* $3.21\ x\ 10^8$. *What is the cost of the National Debt per person?* $1.81\ x10^{13} \div 3.21\ x\ 10^8 = \$5.6\ x\ 10^4\ per\ person.$

9. Analyze and Solve Linear Equations and Pairs of Simultaneous Linear Equations

a. Solve linear equations *(an equation whose graph is a line)* in one variable
b. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers)
    - *one solution:* $3x = 15$, *infinite solutions:* $3x + 5 = 5x - 2x + 5$, *no solutions:* $3x - 5 = 3x + 10$
c. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms
    - *Example:* $2(x + 5) - 4x = 20$ *2x + 10 – 4x = 20 -2x + 10 = 20 x = -5*
d. Analyze and solve pairs of simultaneous linear equations
    - *Understand the solution to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously*
    Example: Three possible solution types:



e. Solve systems of two linear equations in two variables algebraically [through substitution and elimination (or addition) methods], and estimate solutions by graphing equations. Solve simple cases by inspection

- *For example: 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6*
- *Substitution method:*

$$2x + 3y = 6$$
$$x - 2y = 10$$

Solve by substitution: First solve for x or y, then substitute!

$$Solve\ x - 2y = 10\ for\ x: \quad x = 2y + 10$$
$$Sub: \ 2(2y + 10) + 3y = 6$$
$$4y + 20 + 3y = 6$$
$$7y + 20 = 6$$
$$7y = -14$$
$$y = -2$$

Back-substitute to find x:

$$x - 2y = 10$$
$$x - 2(-2) = 10$$
$$x + 4 = 10$$
$$x = 6$$
$$solution\ to\ system: \ (6, -2)$$

- *Elimination (or addition) method:*

$$2x + 3y = 6$$
$$x - 2y = 10$$

One variable is eliminated by adding the equations when the variable has equal but opposite coefficients:
First multiply to achieve equal, opposite coefficients:

$$2(2x + 3y = 6)$$
$$\underline{3(x - 2y = 10)}$$
$$4x + 6y = 12$$
$$\underline{3x - 6y = 30}$$
$$Add: 7\,x = 42$$
$$x = 6$$

Back substitute to find y:

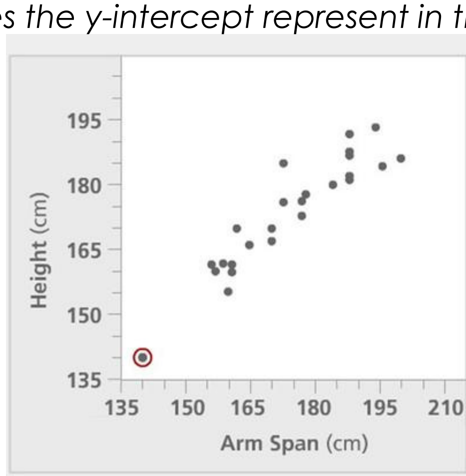
$$2x + 3y = 6$$
$$2(6) + 3y = 6$$
$$12 + 3y = 6$$
$$3y = \ -6$$
$$y = \ -2$$
$$solution\ (6, -2)$$

f. Solve real world and mathematical problems leading to two variables
   - *For example: given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair {can be solved graphically by plotting the pairs of points and looking for the solution by inspection}*

g. Graph proportional relationships, interpreting the unit rate as the slope of the graph
   - *Slope: Slope is defined as the rate of change for a line. Slope has many real world applications such as pitch of a roof, the speed at which we travel, and the cost of running a business. Slope can be determined from a graph using the phrase "rise over run" or $\frac{rise}{run}$, where rise represents the change in the y-coordinate from one point to the next, and run represents the change in the x-coordinate from one point to the next. Slope can also be calculated using the symbolic formula*

$$\text{Slope} = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

   - *Speed and distance traveled: You are traveling at 70mph. Show a graph of distance traveled after 1 hr., 2 hrs. 3 hrs. What is the rate of change of the distance? (This rate of change is the slope of the line – solution is speed:70)*
   - *Cell phone plans: A cell phone plan charges \$20 monthly for data which includes 500 minutes. Any minutes used above 500 will cost \$0.10/min. The equation that models this relationship is C = 0.10T + 20 (T is minutes over 500). Graph the line by substituting T = 10,20,30,40. What is the rate of change for this graph? Compare the rate of change to slope-intercept form, y=mx+b. What does the m represent? What does b represent?*
   - *Use data and scatter plots to interpret slope:*
     Given a data set, find the line of best fit.
     What does the slope of the line represent in the problem?
     What does the y-intercept represent in the problem?

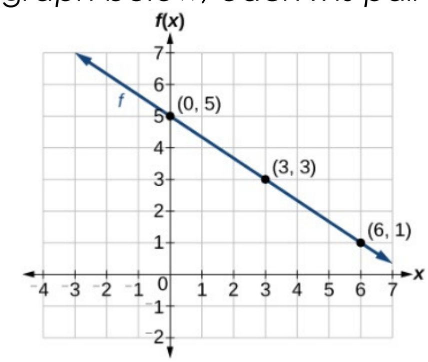


Person	Arm Span	Height
1	156	162
2	157	160
3	159	162
4	160	155
5	161	160
6	161	162
7	162	170
8	165	166
9	170	170
10	170	167
11	173	185
12	173	176
13	177	173
14	177	176
15	178	178
16	184	180
17	188	188
18	188	187
19	188	182

FUNCTIONS

10. Define, Evaluate, and Compare Functions

a. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output
   - *Examples: The 'birthday' function: every person (input) has exactly one birthday (output). Washington's birthday is Feb 16; Kennedy's birthday is May 29; Lincoln's birthday is Feb 12*
   - *{(1,5),(2,10),(3,15)} each x is paired to one y*
   - *In the graph below, each x is paired to one y*



b. Compare properties of two functions each represented in a different way: algebraically, graphically, numerically in tables, or by verbal descriptions
   - *For example, given linear functions represented by algebraic expressions, determine which function has the greater rate of change*
     $y = 2x \text{ compared to } y = 10x$

c. Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear
   - *For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2,4), and (3,9), which are not on a straight line*
   - *In contrast, the function D=55T giving the distance traveled over time at a speed of 55 mph, is linear because it contains the points (0,0), (1,55), (2,110) which lie on a straight line*

11. Use Functions to Model Relationships between Quantities

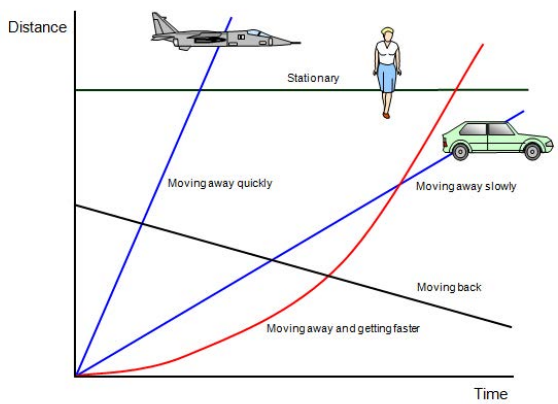
a. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values

- *Example: Given the data below, what is the rate of change (slope)? What is the cost if the number of items = 0? (find initial value or y-intercept)*

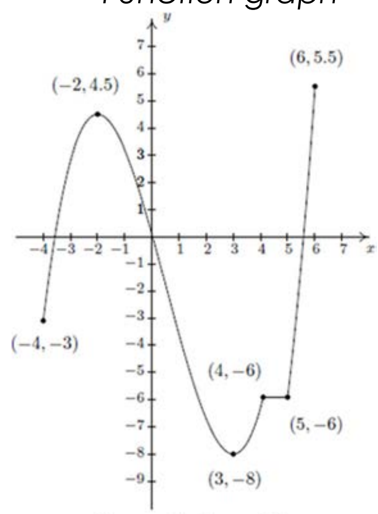
Items	*2*	*3*	*7*	*10*
Cost	*$20*	*$30*	*$70*	*$100*

b. Describe quantitatively the functional relationship between two quantities by analyzing a graph, *such as where the function is increasing or decreasing, linear or nonlinear.* Sketch a graph that exhibits the qualitative features of a function that has been described verbally

Examples: from Physics –motion graph



Function graph



The graph of $y = f(x)$

Algebra Assessment

Name______________________

Date__________

Evaluate each expression.

1) $(-4)+(-1)+(-2)$

2) $1-(-3)+4$

3) $5-5+5$

4) $(-2)-(-1)+3$

Find each product.

5) $-8\times -9\times -2$

6) $8\times -8\times -3$

Find each quotient.

7) $\frac{27}{3}$

8) $\frac{-18}{-2}$

Evaluate each using the values given.

9) $\frac{p+q}{4}$; use $p=6$, and $q=2$

10) x^2y; use $x=4$, and $y=3$

11) $y+yx$; use $x=2$, and $y=6$

12) $h+hj$; use $h=4$, and $j=6$

Solve each equation.

13) $\frac{13}{18}=\frac{b}{18}$

14) $a+4=-11$

15) $4=\frac{n}{10}$

16) $-\frac{1}{2}=\frac{v}{12}$

17) $-4-n=7$

18) $x+(-14)=-29$

19) Eugene ran 25 miles more than Lisa last week. Eugene ran 42 miles. How many miles did Lisa run?

20) Chelsea and eight of her friends went out to eat. They decided to split the bill evenly. Each person paid \$16. What was the total bill?

Solve each equation.

21) $\frac{2+x}{3}=4$

22) $\frac{1+x}{3}=-3$

23) $5+\frac{n}{3}=8$

24) $\frac{-2+b}{4}=-3$

25) Jasmine spent half of her weekly allowance buying pizza. To earn more money her parents let her weed the garden for \$5. What is her weekly allowance if she ended with \$10?

26) Kristin wanted to make note cards by cutting pieces of paper in half. Before starting she got three more pieces to use. When she was done she had 14 half-pieces of paper. With how many pieces did she start?

27) Danielle spent half of her weekly allowance at the movies. To earn more money her parents let her clean the gutters for \$7. What is her weekly allowance if she ended with \$10?

28) Eduardo had some paper with which to make note cards. On his way to his room he found seven more pieces to use. In his room he cut each piece of paper in half. When he was done he had 22 half-pieces of paper. With how many sheets of paper did he start?

Solve each equation.

29) $x+1=11-x$

30) $5v+3v=-3+5v$

31) $1-x=x+1$

32) $5n+5=-1-n$

Simplify each expression.

33) $-7v-3v$

34) $-9+4x-4$

35) $-2(5a+6)$

36) $6(3+9b)$

37) $2(1+7v)-4v$

38) $4b-2(9-7b)$

Simplify. Your answer should contain only positive exponents.

39) $2v \cdot 5v^2$

40) $2p^3 \cdot 6p^3$

41) $\frac{v}{v^3}$

42) $\frac{6a^2}{2a^2}$

43) $\dfrac{6n^{-2} \cdot 4n}{5n}$

44) $\dfrac{x^3}{2xx^{-3}}$

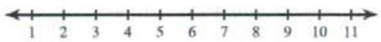
Write the prime-power factorization of each.

45) $60x$

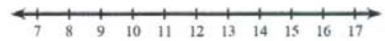
46) $100ab$

Solve each inequality and graph its solution.

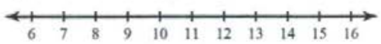
47) $p + 10 < 15$



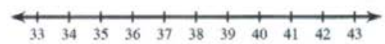
48) $-10x \geq -90$



49) $4b < 40$

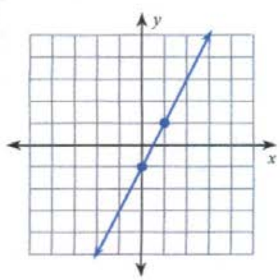


50) $18 > \dfrac{n}{2}$

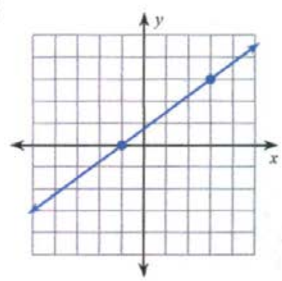


Find the slope of each line.

51)



52)



Find the slope of the line through each pair of points.

53) $(-11, 16), (17, 18)$

54) $(-15, -1), (-16, -8)$

Write each number in scientific notation.

55) 0.591

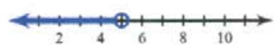
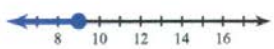
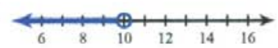
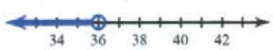
56) 0.0000332

Write each number in standard notation.

57) 9.3×10^{0}

58) 2.68×10^{-5}

Answers to

1) -7	2) 8	3) 5	4) 2
5) -144	6) 192	7) 9	8) 9
9) 2	10) 48	11) 18	12) 28
13) $\{13\}$	14) $\{-15\}$	15) $\{40\}$	16) $\{-6\}$
17) $\{-11\}$	18) $\{-15\}$	19) 17	20) \$144
21) $\{10\}$	22) $\{-10\}$	23) $\{9\}$	24) $\{-10\}$
25) \$10	26) 4	27) \$6	28) 4
29) $\{5\}$	30) $\{-1\}$	31) $\{0\}$	32) $\{-1\}$
33) $-10v$	34) $-13+4x$	35) $-10a-12$	36) $18+54b$
37) $2+10v$	38) $18b-18$	39) $10v^3$	40) $12p^6$
41) $\frac{1}{v^2}$	42) 3	43) $\frac{24}{5n^2}$	44) $\frac{x^5}{2}$
45) $2^2 \cdot 3 \cdot 5 \cdot x$	46) $2^2 \cdot 5^2 \cdot a \cdot b$	47) $p<5$: 	
48) $x \leq 9$: 		49) $b<10$: 	
50) $n<36$: 		51) 2	52) $\frac{3}{4}$
53) $\frac{1}{14}$	54) 7	55) 5.91×10^{-1}	56) 3.32×10^{-5}
57) 9.3	58) 0.0000268		

Name:________________________

Positive and Negative Integers with Operations

Adding Rules:

Positive + Positive = Positive	5 + 4 = 9
Negative + Negative = Negative	(- 7) + (- 2) = - 9
	(- 7) + 4 = -3
Sum of a negative and a positive number	6 + (-9) = - 3
Use the sign of the larger number and subtract	(- 3) + 7 = 4
	5 + (-3) = 2.

Subtracting Rules:

Negative - Positive = Negative	(- 5) - 3 = -5 + (-3) = -8
Positive - Negative = Positive + Positive = Positive	5 - (-3) = 5 + 3 = 8
Negative - Negative = Negative + Positive =	(-5) - (-3) = (-5) + 3 = -2
Use the sign of the larger number and subtract *(Change double negatives to a positive shown in red.)*	(-3) - (-5) = (-3) + 5 = 2

Multiplying Rules

Positive x Positive = Positive	3 x 2 = 6
Negative x Negative = Positive	(-2) x (-8) = 16
Negative x Positive = Negative	(-3) x 4 = -12
Positive x Negative = Negative	3 x (-4) = -12

Dividing Rules

Positive ÷ Positive = Positive	12 ÷ 3 = 4
Negative ÷ Negative = Positive	(-12) ÷ (-3) = 4
Negative ÷ Positive = Negative	(-12) ÷ 3 = -4
Positive ÷ Negative = Negative	12 ÷ (-3) = -4

Scientific Notation

SCIENTIFIC NOTATION is a way of writing very large numbers or very small decimals. The numbers are expressed as a product of a number between 1 and 10 and a power of 10.

1. **Example:** Write 12,300,000 in scientific notation.
   Move the decimal point to the <u>left</u> until it lands between the 1 and 2.
   Answer: 1.23×10^7 because the decimal point moved <u>left</u> 7 places.

   PRACTICE: Write each number in scientific notation:

   1) 456,000 2) 25,000,000 3) ten million

   4) 200,000 human cells could fit on the head of a pin. Write this number in scientific notation.

2. **Example:** Write .00012345 in scientific notation.
   Move the decimal point to the <u>right</u> until it lands between 1 and 2.
   Answer: 1.2345×10^{-4} because the decimal point moved right four places.

   PRACTICE: Write each number in scientific notation:

   5) .00005 6) .000012 7) .0000123

   8) Human hair grows .0000000108 miles per hour. Write this number in scientific notation.

3. **Example:** Write 7.2×10^5 in standard notation
   Answer: 720,000 because the decimal point moved <u>right</u> five places.

   PRACTICE: Write each number in standard notation:

   9) 4.7×10^6 10) 7.123×10^3

   11) A super computer can perform 2.5×10^9 operations per second. Write this number in standard notation.

4. **Example:** Write 9×10^{-5} in standard notation.
   Answer: .00009 because the decimal point moved to the left 5 places.

   PRACTICE: Write each number in standard notation:

   12) 5.17×10^{-4} 13) 1.9×10^{-6}

   14) The diameter of a flu virus is approximately 6.047×10^{-5}. Write in standard notation.

Adapted from floridatechnet.org

Name ______________________________ Date ____________________

Scientific Notation

In the first part, write the number in scientific notation.
In the second part, write the scientific notation number in standard form.

1. 718,900	2. 0.0035
3. 900,000	4. 0.009
5. 12,000	6. 83,470
7. 0.0025	8. 990,000
9. 2,900,000	10. .00025
11. 0.05	12. 0.2400

13. 4.4×10^5	14. 3.65×10^4
15. 8.5×10^3	16. 1.5×10^{-2}
17. 4.4×10^5	18. 6×10^{-2}
19. 9.2×10^5	20. 2.9×10^5
21. 6.98×10^3	22. 3×10^{-3}
23. 2.2×10^{-1}	24. 3.7×10^3

Distributive Property (A)

Use the distributive property to simply each expression.

$2(4 + 9w)$	$-8(6x + 3)$
$-4(-4d - 5)$	$-6(8p + 3)$
$2(3v - 8)$	$(2 - 5m)(-5)$
$4(-6z + 4)$	$-9(n - 4)$
$(-5d + 1)(-2)$	$-4(9k + 9)$
$2(-5 - 7j)$	$(3b - 2)(-3)$
$-3(3 - 8j)$	$-(-5 - 3v)$
$-8(2 + 9v)$	$-9(8 - 2h)$
$(-5f + 8)4$	$(7x - 8)(-1)$
$-(6 - 4p)$	$9(8 + 5t)$

Distributive Property (A) Answers

Use the distributive property to simply each expression.

$2(4 + 9w)$
$18w + 8$

$-4(-4d - 5)$
$16d + 20$

$2(3v - 8)$
$6v - 16$

$4(-6z + 4)$
$-24z + 16$

$(-5d + 1)(-2)$
$10d - 2$

$2(-5 - 7j)$
$-14j - 10$

$-3(3 - 8j)$
$24j - 9$

$-8(2 + 9v)$
$-72v - 16$

$(-5f + 8)4$
$-20f + 32$

$-(6 - 4p)$
$4p - 6$

$-8(6x + 3)$
$-48x - 24$

$-6(8p + 3)$
$-48p - 18$

$(2 - 5m)(-5)$
$25m - 10$

$-9(n - 4)$
$-9n + 36$

$-4(9k + 9)$
$-36k - 36$

$(3b - 2)(-3)$
$-9b + 6$

$-(-5 - 3v)$
$3v + 5$

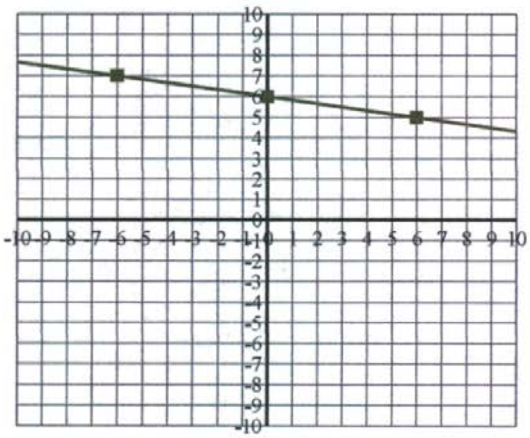
$-9(8 - 2h)$
$18h - 72$

$(7x - 8)(-1)$
$-7x + 8$

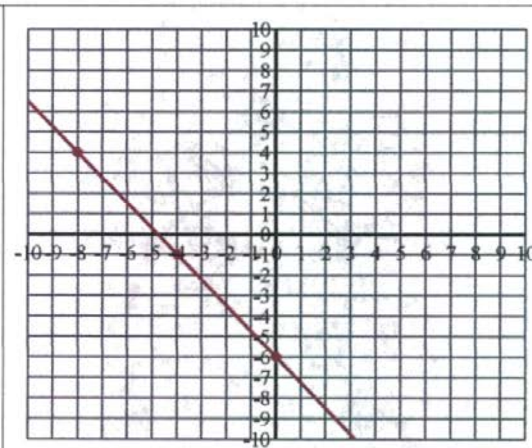
$9(8 + 5t)$
$45t + 72$

Linear Equation Graphs (A)

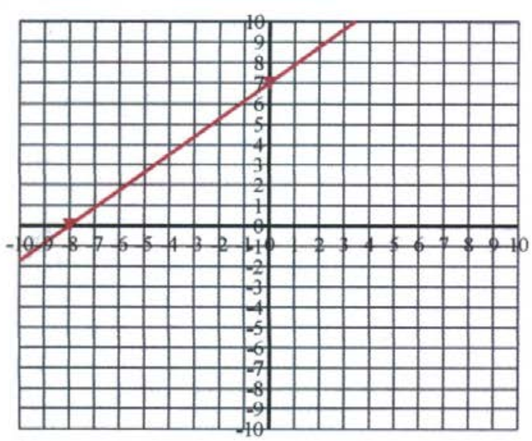
Find the slope for each line.



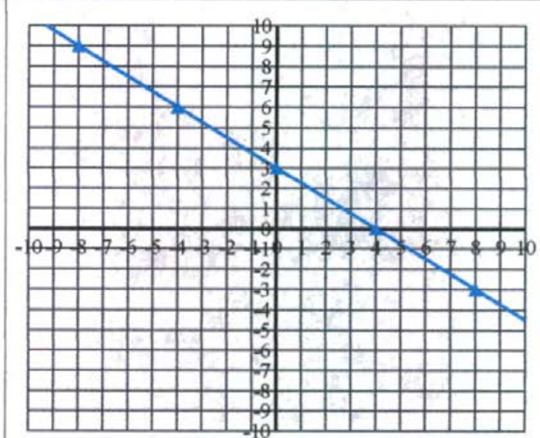
slope:



slope:



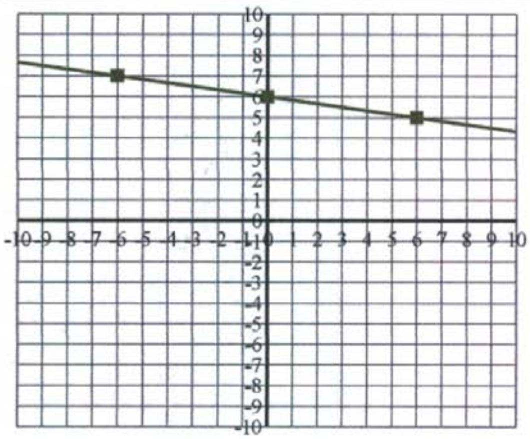
slope:



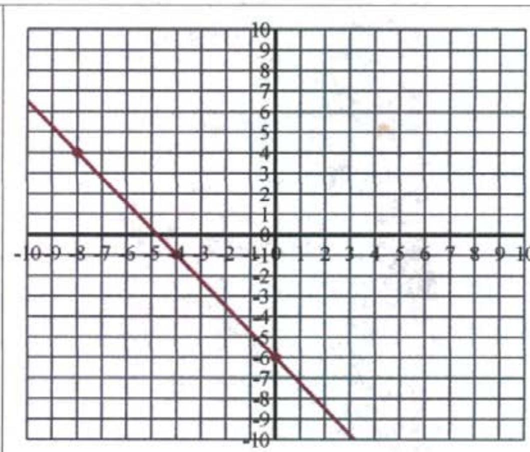
slope:

Linear Equation Graphs (A) Answers

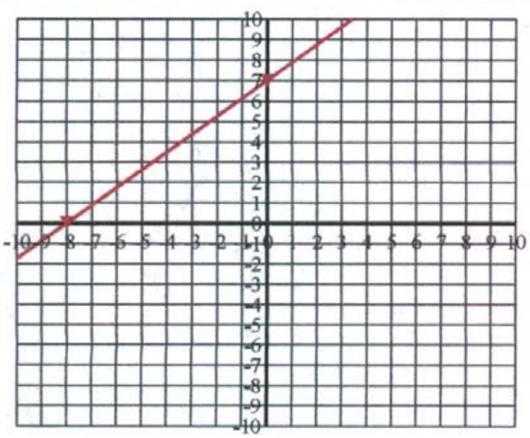
Find the slope for each line.



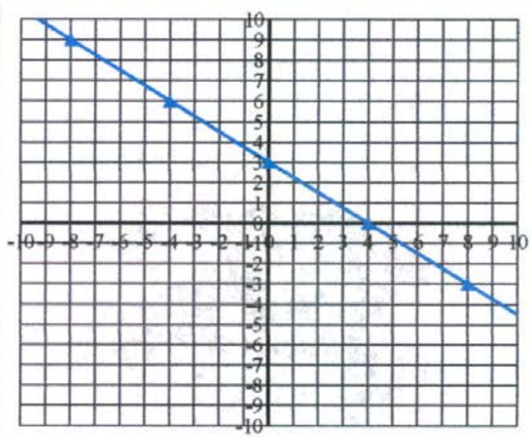
slope: -1/6



slope: -5/4



slope: 7/8



slope: -3/4

Using Substitutions to Solve Problems. Name:

Determine which option(s) the variable 'e' could be. If none of the options could be the variable write 'none'.

Ex) $10e + 3 < 92$
A. 10
B. 4
C. 6
D. 2

1) $2 < 17 \div e$
A. 3
B. 1
C. 7
D. 4

2) $7e + 10 < 82$
A. 1
B. 10
C. 9
D. 5

3) $6e + 3 < 30$
A. 7
B. 6
C. 4
D. 2

4) $8 \times e > 71$
A. 10
B. 3
C. 8
D. 4

5) $107 \div e > 3$
A. 5
B. 8
C. 10
D. 1

6) $e \times 2 < 5$
A. 9
B. 1
C. 8
D. 7

7) $2 \times e > 26$
A. 7
B. 9
C. 1
D. 3

8) $7e - 6 > 53$
A. 7
B. 8
C. 6
D. 9

9) $8 + 4e < 47$
A. 5
B. 3
C. 10
D. 7

10) $6e + 5 > 45$
A. 6
B. 1
C. 5
D. 7

11) $10 \times e > 29$
A. 2
B. 9
C. 1
D. 4

Answers

Ex. B,C,D
1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______
11. ______

1-10	91	82	73	64	55	45	36	27	18	9
11	0									

Using Substitutions to Solve Problems. Name: **Answer Key**

Determine which option(s) the variable 'e' could be. If none of the options could be the variable write 'none'.

Ex) $10e + 3 < 92$
A. 10
B. 4
C. 6
D. 2

1) $2 < 17 \div e$
A. 3
B. 1
C. 7
D. 4

2) $7e + 10 < 82$
A. 1
B. 10
C. 9
D. 5

3) $6e + 3 < 30$
A. 7
B. 6
C. 4
D. 2

4) $8 \times e > 71$
A. 10
B. 3
C. 8
D. 4

5) $107 \div e > 3$
A. 5
B. 8
C. 10
D. 1

6) $e \times 2 < 5$
A. 9
B. 1
C. 8
D. 7

7) $2 \times e > 26$
A. 7
B. 9
C. 1
D. 3

8) $7e - 6 > 53$
A. 7
B. 8
C. 6
D. 9

9) $8 + 4e < 47$
A. 5
B. 3
C. 10
D. 7

10) $6e + 5 > 45$
A. 6
B. 1
C. 5
D. 7

11) $10 \times e > 29$
A. 2
B. 9
C. 1
D. 4

Answers

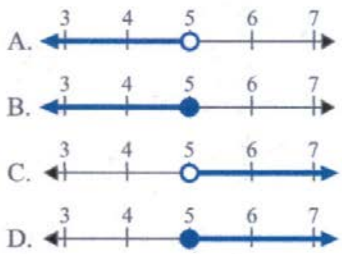
Ex. B,C,D
1. A,B,C,D
2. A,B,C,D
3. C,D
4. A
5. A,B,C,D
6. B
7. none
8. D
9. A,B,D
10. D
11. B,D

1-10	91	82	73	64	55	45	36	27	18	9
11	0									

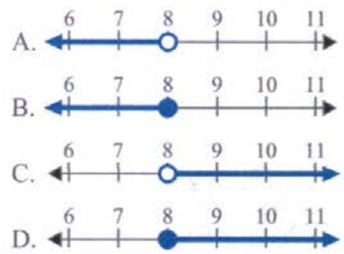
Matching Inequalities to Numberlines Name:

Solve each problem.

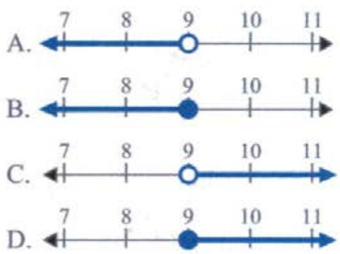
1) Which option best shows $X < 5$



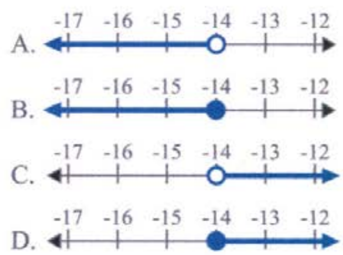
2) Which option best shows $X < 8$



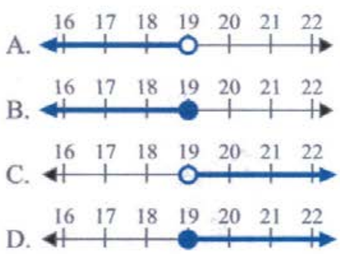
3) Which option best shows $X \geq 9$



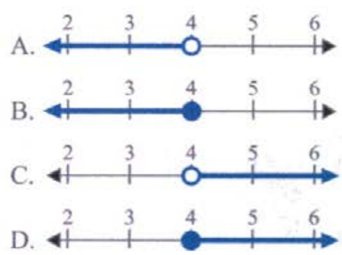
4) Which option best shows $X > -14$



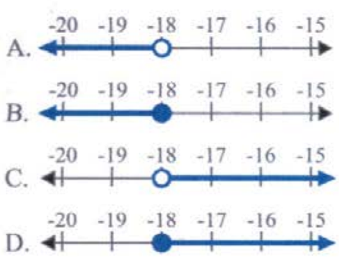
5) Which option best shows $X < 19$



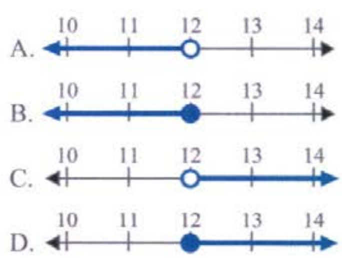
6) Which option best shows $X < 4$



7) Which option best shows $X > -18$



8) Which option best shows $X > 12$



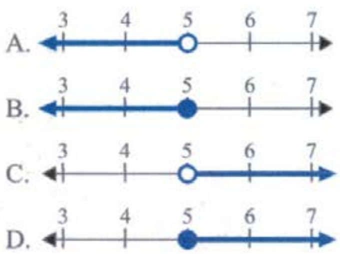
Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________

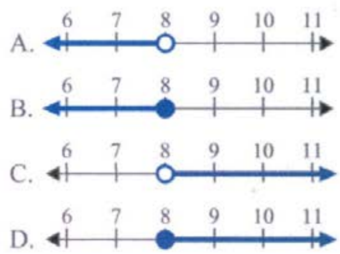
Matching Inequalities to Numberlines Name: **Answer Key**

Solve each problem.

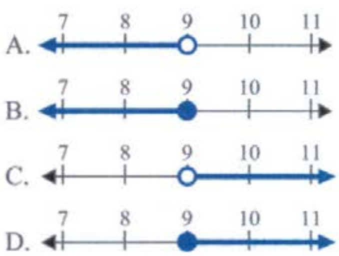
1) Which option best shows $X < 5$



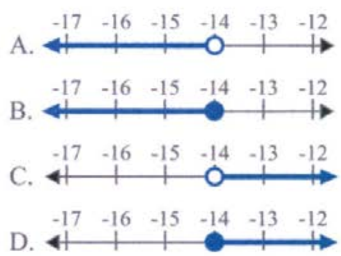
2) Which option best shows $X < 8$



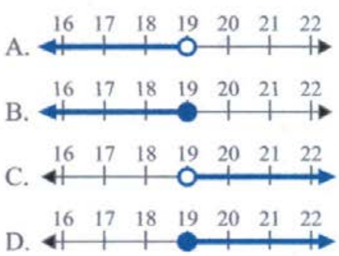
3) Which option best shows $X \geq 9$



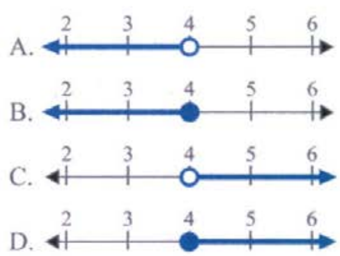
4) Which option best shows $X > -14$



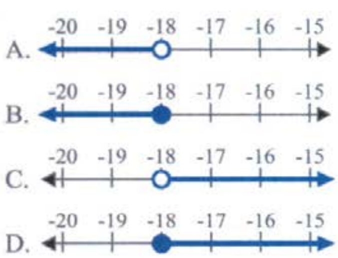
5) Which option best shows $X < 19$



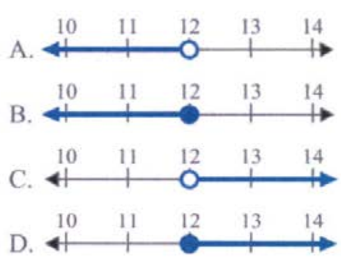
6) Which option best shows $X < 4$



7) Which option best shows $X > -18$



8) Which option best shows $X > 12$



Answers

1. A
2. A
3. D
4. C
5. A
6. A
7. C
8. C

1-8	88	75	63	50	38	25	13	0

Expressing Inequalities on a Numberline

Name:

Use the numberline to express the inequality.

Ex) X ≥ 9 — 4 5 6 7 8 9 10 11 12 13 14 15 16

X

1) X ≥ 195 — 170 175 180 185 190 195 200 205 210 215 220 225 230

2) X > 16 — 10 11 12 13 14 15 16 17 18 19 20 21 22 23

3) X ≥ -40 — -75 -70 -65 -60 -55 -50 -45 -40 -35 -30 -25 -20 -15

4) X ≥ 120 — 70 80 90 100 110 120 130 140 150 160 170 180

5) X ≥ 18 — 12 13 14 15 16 17 18 19 20 21 22 23 24 25

6) X ≥ 60 — 0 10 20 30 40 50 60 70 80 90 100 110 120

7) X > 75 — 50 55 60 65 70 75 80 85 90 95 100

8) X ≥ -100 — -170 -160 -150 -140 -130 -120 -110 -100 -90 -80 -70 -60 -50

9) X ≤ -15 — -21 -20 -19 -18 -17 -16 -15 -14 -13 -12 -11 -10

10) X < 170 — 110 120 130 140 150 160 170 180 190 200 210 220 230

11) X > -7 — -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1

12) X > -5 — -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1

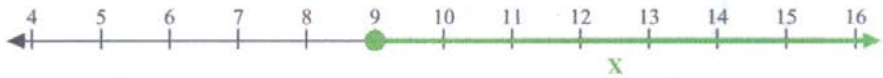
13) X ≤ 11 — 4 5 6 7 8 9 10 11 12 13 14 15 16 17

1-10	92	85	77	69	62	54	46	38	31	23
11-13	15	8	0							

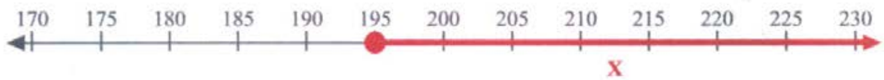
Expressing Inequalities on a Numberline Name: **Answer Key**

Use the numberline to express the inequality.

Ex) $X \geq 9$

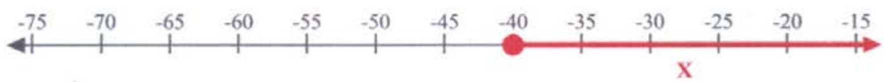


1) $X \geq 195$



2) $X > 16$

3) $X \geq -40$



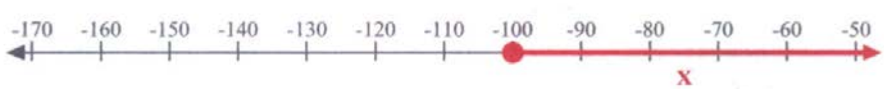
4) $X \geq 120$

5) $X \geq 18$

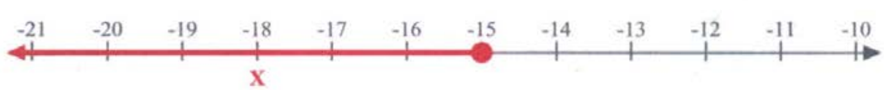
6) $X \geq 60$

7) $X > 75$

8) $X \geq -100$

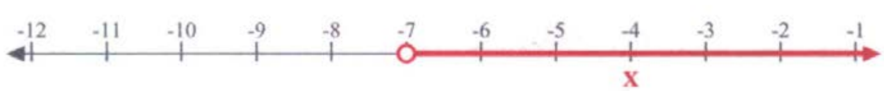


9) $X \leq -15$

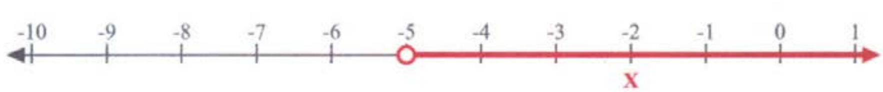


10) $X < 170$

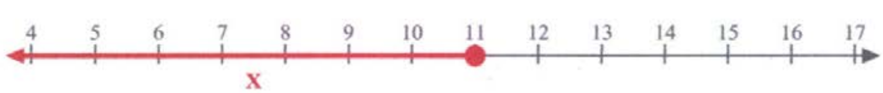
11) $X > -7$



12) $X > -5$



13) $X \leq 11$



1-10	92	85	77	69	62	54	46	38	31	23
11-13	15	8	0							

Solving Two-Step Equations

Multiplication & Division - No Negative Coefficients

Name: ____________________ **Date:** ____________________

Solve the equations.

(1) $8x - 17 = 31$

(2) $\frac{x}{5} - 5 = 2$

(3) $\frac{x}{3} + 1 = 5$

(4) $39 + 6x = 135$

(5) $^{-}35 + 6x = 19$

(6) $3 + \frac{x}{10} = 7$

(7) $81 + 12x = 237$

(8) $\frac{x}{3} + 7 = 19$

(9) $4x + 11 = 59$

(10) $12x - 122 = 58$

(11) $1 + \frac{x}{12} = 4$

(12) $\frac{x}{2} + 1 = 8$

(13) $\frac{x}{13} - 1 = 2$

(14) $60 + 12x = 180$

(15) $12x + 70 = 190$

Solving Two-Step Equations

Multiplication & Division - No Negative Coefficients

ANSWER KEY

Solve the equations.

(1) $8x - 17 = 31$

$8x = 48$

$x = 6$

(2) $\frac{x}{5} - 5 = 2$

$\frac{x}{5} = 7$

$x = 35$

(3) $\frac{x}{3} + 1 = 5$

$\frac{x}{3} = 4$

$x = 12$

(4) $39 + 6x = 135$

$6x = 96$

$x = 16$

(5) $-35 + 6x = 19$

$6x = 54$

$x = 9$

(6) $3 + \frac{x}{10} = 7$

$\frac{x}{10} = 4$

$x = 40$

(7) $81 + 12x = 237$

$12x = 156$

$x = 13$

(8) $\frac{x}{3} + 7 = 19$

$\frac{x}{3} = 12$

$x = 36$

(9) $4x + 11 = 59$

$4x = 48$

$x = 12$

(10) $12x - 122 = 58$

$12x = 180$

$x = 15$

(11) $1 + \frac{x}{12} = 4$

$\frac{x}{12} = 3$

$x = 36$

(12) $\frac{x}{2} + 1 = 8$

$\frac{x}{2} = 7$

$x = 14$

(13) $\frac{x}{13} - 1 = 2$

$\frac{x}{13} = 3$

$x = 39$

(14) $60 + 12x = 180$

$12x = 120$

$x = 10$

(15) $12x + 70 = 190$

$12x = 120$

$x = 10$

Name : ______________________ Score : __________

Teacher : ______________________ Date : __________

Word Problems

1) Sam had 133 dollars to spend on 8 books. After buying them he had 13 dollars. How much did each book cost ? __________

2) The sum of three consecutive even numbers is 156. What is the smallest of the three numbers ? __________

3) The sum of three consecutive odd numbers is 153. What is the smallest of the three numbers ? __________

4) Mary sold half of her comic books and then bought 9 more. She now has 12. How many did she begin with ? __________

5) On Monday, 412 students went on a trip to the zoo. All 9 buses were filled and 7 students had to travel in cars. How many students were in each bus ? __________

6) Oceanside Bike Rental Shop charges a 13 dollar fixed fee plus 8 dollars an hour for renting a bike. Jessica paid 77 dollars to rent a bike. How many hours did she pay to have the bike checked out ? __________

7) Jason bought a soft drink for 4 dollars and 6 candy bars. He spent a total of 34 dollars. How much did each candy bar cost ? __________

8) Sam bought 5 new baseball trading cards to add to his collection. The next day his dog ate half of his collection. There are now only 28 cards left. How many cards did Sam start with ? __________

9) Sara spent half of her allowance going to the movies. She washed the family car and earned 7 dollars. What is her weekly allowance if she ended with 16 dollars ? __________

10) The sum of three consecutive numbers is 144. What is the smallest of the three numbers ? __________

Name : ____________________ Score : __________

Teacher : ____________________ Date : __________

Word Problems

1) Sam had 133 dollars to spend on 8 books. After buying them he had 13 dollars. How much did each book cost ? 15 dollars

2) The sum of three consecutive even numbers is 156. What is the smallest of the three numbers ? 50

3) The sum of three consecutive odd numbers is 153. What is the smallest of the three numbers ? 49

4) Mary sold half of her comic books and then bought 9 more. She now has 12. How many did she begin with ? 6 comic books

5) On Monday, 412 students went on a trip to the zoo. All 9 buses were filled and 7 students had to travel in cars. How many students were in each bus ? 45 students

6) Oceanside Bike Rental Shop charges a 13 dollar fixed fee plus 8 dollars an hour for renting a bike. Jessica paid 77 dollars to rent a bike. How many hours did she pay to have the bike checked out ? 8 hours

7) Jason bought a soft drink for 4 dollars and 6 candy bars. He spent a total of 34 dollars. How much did each candy bar cost ? 5 dollars

8) Sam bought 5 new baseball trading cards to add to his collection. The next day his dog ate half of his collection. There are now only 28 cards left. How many cards did Sam start with ? 51 cards

9) Sara spent half of her allowance going to the movies. She washed the family car and earned 7 dollars. What is her weekly allowance if she ended with 16 dollars ? 18 dollars

10) The sum of three consecutive numbers is 144. What is the smallest of the three numbers ? 47

1	**Product Rule** (multiplying same base, add exponents)	$a^m \bullet a^n = a^{m+n}$
2	**Power Rule** (raising a base and exponent to a power)	$(a^m)^n = a^{m*n}$
3	**Power of a Product** (distribute an exponent over multiplication)	$(ab)^n = a^n \bullet b^n$
4	**Power of a Quotient** (distribute an exponent over division)	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5	**Quotient Rule** (dividing same base, subtract exponents)	$\frac{a^m}{a^n} = a^{m-n}$
6	**Zero Exponent Rule** (anything raised to an exponent of 0 equals 1)	$a^0 = 1,\ a \neq 0$
7	**Negative Exponent Rule** (negative exponent becomes positive when moved to the opposite location in a fraction) MT108	$a^{-m} = \frac{1}{a^m}$ or $\frac{1}{a^{-m}} = \frac{a^m}{1}$

Math Lesson Plans, Videos, & Resources

www.learnzillion.com/lessons

- Learn Zillion is a learning platform that combines video lessons, interactive lessons, printable practice worksheets, lesson slides, lesson commentary and guided practice
- A Common Core navigator is available; click on a standard and be directed to lessons, video instruction, and practice

www.tv411.org/math

- TV 411 is an excellent website designed for adult education students. Short and interesting videos as well as interactive web lessons and worksheets on a variety of math topics are available for free. The videos really help students see how math is applied to real life in a fun way. Highly recommended!

www.pbslearningmedia.org

- Hundreds of math videos, lessons, interactive activities and support materials
- You can search by grade or topic and match to Common Core standards
- A student view is available

www.illuminations.nctm.org

- Resources for teaching math from the National Council of Teachers of Math
- Search for lessons and interactives by grade, topic or Common Core standard

www.illustrativemathematics.org

- Find illustrative tasks and other resources for each math standard
- Find videos and tasks illustrating the progression of mathematical topics

www.khanacademy.org

- Ask a question and find a video explanation and interactive activities
- Set up a simple, free account to track learning, helpful for both students and teachers to develop math skills

www.nald.ca/library/learning/mathman/mathman.pdf

- A Manual for Teaching Basic Math to Adults: ***Changing the Way We Teach Math*** by Kate Nonesuch
- A free 117 page PDF file
- Chapters include: Hands-On Learning, Group Work, and Activities for Students

www.cord.org/contextual-classroom-resources

- Find contextualized teaching resources including ***Teaching Mathematics Contextually***
- Free resources from the nonprofit Center for Occupational Research (CORD)

Worksheets

www.worksheetworks.com

- Customize worksheets using a worksheet generator and create PDF worksheets with answers
- Some topics include good problem solving worksheets

www.math-drills.com

- Generate hundreds of worksheets on different topics including algebra, geometry, order of operations and integers with answers included
- Because the difficulty of each worksheet can be controlled, this site would be especially useful for multilevel classrooms

www.math-aids.com

- This site contains 90+ different math topics
- The math worksheets are randomly generated. This allows you to make and customize an unlimited number of printable math worksheets to your specifications instantly

www.CommonCoreSheets.com

- Printable multi-level worksheets aligned with common core standards

Student Math Website Recommendations

www.tv411.org/math

- Videos, web lessons, and interactive activities on different math topics
- This site is designed for adult students, is free, and has no ads

www.aaamath.com

- Select a math SUBJECT (not grade) and find instruction, practice, and games on many different math topics and levels

www.aplusmath.com

- This site has flashcards and games for whole numbers, fractions, geometry and algebra
- Create your own worksheets to complete on line

www.ixl.com/math

- Provides unlimited interactive practice on a lot of math topics
- If you get a question wrong, an explanation is provided
- Ignore the listed grade levels and focus on the skill you want to practice

www.mathisfun.com

- Be sure to choose a math SUBJECT and not a grade
- Good explanations are provided followed by practice and answers
- Practice skills using puzzles and games

www.gcflearnfree.org

- An ad free nonprofit learning website designed for adults
- Click on math and choose from topics such as percent and algebra
- Includes instruction, videos, games, and interactive practice

www.mathplayground.com

- Provides math instruction on many different topics
- Practice math skills through a variety of games without ads
- Videos and interactive activities included

www.khanacademy.org

- Video instruction on all math topics

Recommended Mathematic Materials

Active Math by Ruth Estabrook

An excellent variety of math activities written by our own former Mentor Teacher, Ruth Estabrook. Order it free at *www.nhadulted.org*. There are also other excellent mini-grant math projects available for download or order from the above website.

Math for All Learners *by Pam Meader and Judy Storer*

Titles include Pre-Algebra and Geometry. Order from www.walch.com or other outlets such as Amazon.com.

Common Core Basics: Mathematics

Building Essential Test Readiness Skills for High School Equivalency Exams by McGraw Hill. Order from *www.mheonline.com*.

Contemporary Number Power, A Real World Approach to Math

This series includes topics such as: Pre-Algebra, Word Problems, Analyzing Data, and Measurement. Order the latest editions (2011+) from *www.mheonline.com*.

Math Sense: Focus on Operations, Focus on Problem Solving, Focus on Analysis

Math for the High School Equivalency Tests. Order the 2015 edition of this three book series from *www.newreaderspress.com*.

Scoreboost Mathematics/ Decimals, Fractions, Proportions, and Percent

Scoreboost Mathematics/Measurement and Data Analysis

Scoreboost Mathematics/Algebra and Geometry

A series of three practice HiSET booklets by New Readers Press. Order the latest editions (2014+) from *www.newreaderspress.com*.

Math1: Whole Numbers, Decimals, Fractions, Percents, and Measurement

Math 2: Algebraic Thinking, Data Analysis, and Probability

A series of two Pre-High School Equivalency booklets by New Readers Press. Order from *www.newreaderspress.com*.

The Official Guide to the HiSET Exam

This is the only official ETS sanctioned book so far. Order from McGraw Hill at *www.mheonline.com* or other outlets such as *www.amazon.com*.

Mathematics Guidebook Glossary

Absolute value: distance of a number from zero

Acute angle: an angle whose measure is less than 90 degrees

Acute triangle: a triangle that has three acute angles

Addend: the numbers being added in an addition problem.

Addition: finding a sum of two or more numbers

Algorithm: a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer

Angle: two rays with the same endpoint or vertex

Area: the amount of space inside a shape or a two-dimensional figure, measured in square units

Average: the number obtained as a result of adding two or more quantities and dividing the sum by the numbers of quantities

Axis: one of the reference lines in a coordinate system

Bar graph: a graph that make use of bars in order to give a visual representation that can be used compare data or amounts of sizes

Base: in a polygon, the base represents one side of a polygon used to find area

Base: In percentage the base represents the amount you are taking a part of

Base: In multiplication with exponents, the base represents the number being multiplied or a factor

Bisect: In geometry, bisect is the process by which one uses a ruler and a compass top cut an angle in half

Capacity: the amount a container or a unit will hold when full

Centimeter: A measure of length approximately equal to a fingernail

Circle: a plane figure whose points are located at a fixed distance from the center

Circle graph: a pictorial way to compare amounts using circles or segments of a circle

Circumference: the perimeter or distance around a circle

Commission: the amount of the total money paid for a service

Common denominator: common multiple of one or two denominators located at a fixed distance from the center

Common factors: for two numbers, the common factor is a number that can divide the two numbers evenly

Composite number: a number that has more than 2 factors

Cone: a solid figure with a circular based plane, connected to a point called the vertex

Conversion: the action of changing a unit to a different unit of measure

Conversion factor: a number you multiply by to change to another unit of measure

Cross product: the answer obtained by multiplying the numerator of one fraction by the denominator of another fraction

Cube: a prism with square sides and faces

Cylinder: a solid figure with two congruent circular bases that are parallel.

Data: information that we collect

Decimal places: the positions to the right of the decimal point

Decimal number: all numbers in the base 10 number system that have one or more numbers in the decimal places

Decimal point: in a decimal number, it is a period that is used to separate the whole number from the numbers in the decimal places

Degree: a measure of angles. It is equal to 1/360 of a circle

Denominator: In a fraction, It is the number below the fraction bar

Diameter: the distance across a circle through the center

Difference: the answer to a subtraction problem

Dimensions: length, width, or height of the size of an geometric figure

Discount: a reduction made from the regular price

Discount rate: the percent that the price is reduced

Dividend: the number being divided

Divisibility: able to be divided evenly

Divisible: able to be divided without a remainder

Division: the process of dividing two numbers to find how many times one number is contained into another number

Divisor: the number by which you are dividing

Equation: two mathematical expressions that are separated by an equal sign

Equidistant: same distance

Equilateral triangle: a triangle that has three equal sides

Equivalent fractions: fractions that are equal in values but have different numerators and denominators

Estimate: an approximation for the real value

Exponent: the number that tells how many times the base is multiplied by itself

Even number: a number that has no remainder when divided by 2

Face: any of the plane surfaces of a solid

Factor: a number that is being multiplied in a multiplication problem

Fibonacci numbers: a sequence in which, except for the first and the second number, each number is the sum of the two preceding numbers (1, 1, 2, 3, 5, 8....)

Figure: a two or three-dimensional figure such as a square, a cube, or a sphere

Finite: finite means that your set, may it be numbers or objects, has an end or definable limits

Fluid ounce: one-sixteenth of a pint

Formula: mathematical equation that states a general fact, principle, or rule

Fraction: a part of a whole number

Fractional form: a number expressed as a fraction

Gallon: a unit of liquid capacity that is equal to 4 quarts or 3.785 liters

Geometry: the study of size and shape of points, lines, angles, surfaces, and solid figures

Gram: a metric measure of mass that is approximately equal to the weight of a penny

Graph: a visual display of information

Greater than: bigger or larger than

Greatest common factor: the largest factor of two or more numbers (GCF)

Greatest common divisor: the greatest factor that divides two or more numbers evenly. The greatest common factor is called greatest common divisor if it is used to simplify fractions

Height: the distance from bottom to top

Heptagon: a polygon that has seven sides

Hexagon: a polygon with six angles and six sides

Hexagonal prism: a prism that has hexagonal faces

Histogram: a histogram is a graphical way to display information or data using bar

Horizontal: a line that has no slope

Horizontal axis: one of the axis in the coordinate system that has a slope of zero

Hypotenuse: the longest side in a right triangle. The longest side is the one opposite to the right angle

Improper fraction: a fraction with a bigger numerator than a denominator

Infinite: with no end or limit.

Integers: the set of all whole numbers and their opposites

Intersecting lines: lines that meet or cross in the same plane

Invert: in a fraction, it means to switch the position of the numerator with the denominator

Irrational number: a number that cannot be expressed as a fraction

Isosceles triangles: a triangle that has two equal sides.

Kilogram: the measure of mass that is equal to 1000 grams or has a weight approximately equal to 1 liter of water or 4 rolls of quarters

Kiloliter: the measure of capacity that is equal to 1000 liters or approximately to a small wading pool

Kilometer: a measure of distance that is equal to 1000 meters, a little more than half a mile

Least common denominator: the smallest denominator that is a multiple of two or more denominators (LCD)

Least common multiple: the smallest number that two or more number will divide(LCM)

Length: the distance from end to end

Like denominators: fractions with the same denominators

Length: the distance from end to end

Linear equation: an equation whose graph is a straight line

Linear measurement: measurement of distance or length

Line graph: a graph made up line segments that are connected together

Line segment: part of a line with two endpoints

Liter: a metric measure of capacity that is approximately equal to a little bit more than one-fourth of a gallon

Mean: the average

Median: the middle number in an ordered set

Metric System: a measurement system based on units of 10 used in most of the world except the U.S. where customary measurement is used

Mixed number: a number with both a whole number and a fractional part such as 1¾

Mode: the number that occurs most often in a set

Multiple: the product of a whole number and any other whole number (multiples of 5— 5,10,15, 20, 25, 30, 35…)

Multiplicand: the top # in a multiplication problem (the number to be multiplied)

Multiplier: the bottom # in a multiplication problem (number by which another # is multiplied)

Negative number: any number less than zero

Numerator: top number in a fraction; # that tells how many parts of the entire item you have

Obtuse angle: an angle that is greater than 90º but less than 180º

Order of Operation: when performing more than one operation in a numerical expression there is a special order to follow; sometimes called *PEMDAS* (parentheses, exponents, multiplication, division, addition, subtraction) or *PERMDAS* (parentheses, exponents, roots, multiplication, division, addition, sub)

Ordinal number: numbers that tell a position or place like first, second, third and so on

Origin: point where the x and y axis intersect (0, 0)

Outlier: data that are far apart from the rest of the data

Parallel line: two or more lines, line segments or rays that lie in the same plane, are the same distance apart, and never intersect

Perimeter: the distance around the outside of a figure

Permutation: an arrangement or listing of objects in which order is important

Pi: the Greek letter "π" is the ratio of the circumference to the diameter of a circle. This irrational number is commonly expressed as the fraction 22/7 or rounded to 3.14

Place value: the place that a digit occupies within a number tells that digit's value (in "24", there are 4 ones and 2 tens)

Plane figure: a geometric figure that lies on one plane (or flat surface). It has no depth, just length and width; it gets its name from the number of its sides (three sides - triangle; four sides - quadrilateral; five sides - pentagon, etc.)

Prime factorization: the process of breaking down the factors of a product until all the factors are prime numbers

Prime number: any number with only 2 factors, 1 and itself. All primes are odd except for the number 2

Percent: "out of 100"; uses the % sign instead of decimal notation

Perpendicular lines: two lines, line segments or rays that intersect at right angles

Polygon: a closed figure that has at least three sides

Pictograph: a graph that compares data using symbols to show how many

Probability: that ratio of the # of ways an event can occur to the # of possible outcomes

Product: the answer to a multiplication problem

Properties of Operations: characteristics that are always true when you perform on of the four main mathematical operations

Proportion: equation that shows 2 ratios are equivalent

Quadrant: one of the four regions of a coordinate graph

Quadrilateral: a four sided polygon with four sides and four angles

Quotient: the answer to a division problem (on top of the bar)

Radical sign: the symbol for square root $\sqrt{}$

Radius: a line segment from the center of a circle to any point of the circle

Random: a sample is random if the members of the sample are selected purely on the basis of chance

Range: the difference between the greatest and least number in a set

Rate: a ratio of 2 measurements with different units (price per gallon, miles per hour)

Ratio: a comparison of 2 numbers

Rational number: any number that can be expressed in fraction or decimal form

Ray: a part of a line; it has one endpoint and goes in one direction forever

Rectangular prism: a space figure that has two rectangular parallel bases

Rectangular pyramid: a space figure that has a rectangle as a base and triangular faces that meet at a vertex

Right angle: an angle of 90°

Root: the root of a number x is another number, which when multiplied by itself a given number of times, equals x; the second root is usually called the "square root", the third root of a number is usually called the "cube root", and after that, they are called the nth root

Rounding: the most accurate way of estimating

Sample space: the set of all possible outcomes

Scale: all possible values of a given measurement

Scalene Triangle: a triangle with three unequal sides and three unequal angles

Scientific notation: a shorthand way of writing very large or small numbers, so to avoid writing many digits

Sequence: list of #'s in a specific order

Similar Triangle: triangles that have the same shape, but not the same size

Space figure: another name for a three-dimensional figure where the points of a figure lie on more than one plane; sometimes space figures are called solids

Sphere: a three-dimensional figure in which all points are the same distance from the radius, but they are not on the same plane (balls, bubbles, planets)

Square root: one of two equal factors of a number; a square root of 81 is 9 because 9x9=81

Standard form: the usual way of writing the name of a number using digits

Straight angle: an angle that is equal to 180º

Substitute: to replace, to use instead of

Sum: the answer to an addition problem

Supplementary angles: any two angles that have a combined measure of 180º

Surface area: the sum of the areas of all the surfaces of a three-dimensional figure

Symmetry: a figure has symmetry if it can be folded along a line to form two identical parts

Unit rate: the rate at which the denominator is one unit (20mpg)

Variable: a symbol that stands for a number; usually a letter like *n* or x

Vertex: the point where two lines meet in a single plane or where two edges meet in a three-dimensional figure

Volume: the amount of space measured in cubic units occupied by a three dimensional figure

Whole number: any number, including zero, without a decimal or fraction

X-axis: the horizontal number line on a coordinate graph

Y-axis: the vertical number line on a coordinate graph