Review for Accuplacer

Arithmetic

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1. Introduction

I have found that often students enter our College Transitions program with a basic understand of algebra, but have forgotten some of the arithmetic. This guide will help students refresh those skills before the move on to more the advanced mathematics, which will be required for their success in college level math courses.

This is not designed to be a comprehensive lesson plan to teach all of the skills needed to pass the Accuplacer Arithmetic test. It is merely a study guide and set of refresher lessons that will help them remember the skills they have learned but may not have used in a while. Students who require in-depth work in their foundation skills will need more than this guide is designed to provide.

2. Operations with Whole Numbers

a. Addition

Adding numbers with more than one digit is similar to adding a single column of numbers more than once. Start from the column on the right (the "ones" column).

174 + 425 =

Add the Ones

	Hundreds	Tens
	1	7
+	4	2

Add the Tens

	Hundreds	**Tens**
	1	**7**
+	4	**2**
		9

Add the Hundreds

	Hundreds	Tens
	1	7
+	**4**	2
	5	9

232 + 647

	Hundreds	Tens
	2	3
+	6	4

	Hundreds	**Tens**
	2	**3**
+	6	**4**
		7

	Hundreds	Tens
	2	3
+	**6**	4
	8	7

When the sum of a column is greater than 9, you must regroup that sum into the next column before continuing.

487 + 645 =

Add the Ones

	Hundreds	Tens
		1
	4	8
+	6	4

Since 7+5 is 12, the 2 stays in the Ones column, but the 1 moves to the top of the Tens column.

Add the Tens

	Hundreds	**Tens**
	1	**1**
	4	**8**
+	6	**4**
		3

Since 1+8+4 is 13, the 3 stays in the Tens column, but the 1 moves to the top of the Hundreds column.

Add the Hundreds

	Hundreds	Tens
	1	1
	4	8
+	**6**	4
1	**1**	3

Since 1+4+6 is 11, the 1 stays in the Hundreds column, but the second 1 moves to the top of the Thousands column.

155 + 376 =

	Hundreds	Tens
		1
	1	5
+	3	7

Since 5+5 is 11, the 1 stays in the Ones column, but the 1 moves to the top of the Tens column.

	Hundreds	**Tens**
	1	**1**
	1	**5**
+	3	**7**
		3

Since 1+5+7 is 13, the 3 stays in the Tens column, but the 1 moves to the top of the Hundreds column.

	Hundreds	Tens
	1	1
	1	5
+	**3**	7
	5	3

b. Subtraction

Subtracting numbers with more than one digit is similar to subtracting a single column of numbers more than once. Start from the column on the right (the "ones" column).

989 – 756 =

Subtract the Ones

	Hundreds	Tens	**Ones**
	9	8	**9**
-	7	5	**6**
			3

Subtract the Tens

	Hundreds	**Tens**	O
	9	**8**	9
-	7	**5**	6
		3	3

Subtract the Hundreds

	Hundreds	Tens	O
	9	8	9
-	**7**	5	6
	2	3	3

346 – 236 =

	Hundreds	Tens	**O**
	3	4	**6**
-	2	3	**6**
			0

	Hundreds	**Tens**	O
	3	**4**	6
-	2	**3**	6
		1	0

	Hundreds	Tens	Ones
	3	4	6
-	**2**	3	6
	1	1	0

Sometimes when you try to subtract from a column, you do not have enough to subtract. You must regroup from another column to get enough.

52 – 16 =

Can you subtract the Ones? No

	Tens	Ones
	5	2
-	1	6

Regroup

	Tens	Ones
	4	**12**
	5	2
-	1	6

Subtract the Ones

	Tens	Ones
	4	**12**
	5	2
-	1	**6**
		6

Subtract the Tens

	Tens	
	4	1
	5	2
-	**1**	6
	3	6

Can you subtract the Ones? No

	Tens	Ones
	7	1
-	5	8

Regroup

	Tens	Ones
	6	**1**
	7	1
-	5	8

Subtract the Ones

	Tens	Ones
	6	**1**
	7	1
-	5	**8**
		3

Subtract the Tens

	Tens	
	6	1
	7	1
-	**5**	8
	1	3

Sometimes you may need to regroup more than once.

530 – 258 =

Regroup the Ones, then subtract

	Hundreds	Tens	O
		2	**1**
	5	3	**0**
-	2	5	**8**
			2

Regroup the Tens, then subtract

	Hundreds	**Tens**	O
		12	
		2	1
	5	**3**	0
-	2	**5**	8
		7	2

Regroup the Ones and Tens, then subtract

	Hundreds	Tens	O
		9	
		10	**1**
	8	0	**0**
-	6	8	**1**
			9

Next, subtract from the Tens that is regrouped

	Hundreds	**Tens**	O
		9	
	7	**10**	1
	8	**0**	0
-	6	**8**	1
		1	9

		2	7	2

	Hundreds	Tens	Ones
		9	
	7	10	1
	8	0	0
-	**6**	8	1
	1	1	9

Finally, Subtract the Hundreds

	Hundreds	Tens	Ones
		12	
	4	2	1
	5	3	0
-	**2**	5	8

Finally, subtract the hundreds

c. Multiplication

When multiplying large numbers, use the multiplication facts more than once. Always start with the Ones column, and remember to keep all products aligned correctly. (Note: sometimes it is useful to draw the column lines so it is easier to keep the numbers straight.)

72 x 13 =

Multiply the Ones by Ones

	Tens	Ones
	7	**2**
x	1	**3**
		6

Multiply the Ones by the Tens

	Tens	Ones
	7	2
x	1	**3**
2	**1**	6

Multiply the Tens by the Ones (add a 0 as a placeholder with the Ones)

	Tens	
	7	**2**
x	**1**	3
2	1	6
	2	**0**

Multiply the Tens by the Tens, then add the partial product

		Tens
		7
	x	**1**
	2	1
+	**7**	2
	9	**3**

Sometimes it is necessary to regroup when multiplying. This is similar to what happened with addition.

Multiply the One by the Ones. Regroup.

	Tens	Ones
	7	
	5	**9**
x	2	**8**
		2

Multiply the Ones by the Tens, then add the 7.

	Tens	Ones
	7	
	5	9
x	2	**8**
4	**7**	2

Multiply the Tens by the Ones . Regroup.

	Tens	Ones
	1	
	7	
	5	**9**
x	**2**	8
4	7	2
	8	**0**

Multiply the Tens by the Tens, then add the 1. Finally, add the partial products.

		Tens	Ones
		1	
		7	
		5	9
	x	**2**	8
	$^{1}4$	7	2
+ 1	**1**	8	0
1	**6**	**5**	**2**

d. Division

Division is the opposite of multiplication. To divide a number (the divisor) into a multi-digit number (the dividend), just remember 4 steps: Divide, Multiply, Subtract, Bring Down. Start at the left and work your way across.

836 ÷ 4 =

Divide: 4 goes into 8, 2 times

Multiply: 2 x 4 = 8

Subtract: 8 – 8 = 0

Bring Down: the 3

	2		
4	8	3	6
	-8	↓	
	0	3	

Divide: 4 does not go into 3

Multiply: 0 x 4 = 0

Subtract: 3 – 0 = 3

Bring Down: the 6

	2	0	9
4	8	3	6
	-8		↓
	0	3	↓
		-0	↓
		3	6

Divide: 4 goes into 36, 9 times

Multiply: 9 x 4 = 36

Subtract: 36-36 = 0

	2	0	9
4	8	3	6
	-8		
	0	3	
		-0	
		3	6
		-3	6
			0

Most of the time, the divisor will not go into the dividend evenly. Write the remainder with the quotient.

789 ÷ 5 =

Divide: 5 into 7

Multiply: 1 x 5

Subtract: 7 – 5

Bring Down: 8

	1		
5	7	8	9
	-5	↓	
	2	8	

Divide: 5 into 28

Multiply: 5 x 5

Subtract: 28 – 25

Bring Down: 9

	1	5	
5	7	8	9
	-5		↓
	2	8	↓
	-2	5	↓
		3	9

Divide: 5 into 29

Multiply: 7 x 5

Subtract 39 – 35

The 4 is the remainder

				r
	1	5	7	4
5	7	8	9	
	-5			
	2	8		
	-2	5		
		3	9	
		-3	5	
			4	

To divide a 2-digit divisor, use the same steps as before, just be careful to place your quotient in the proper column. It will go in the same column as the last digit of the dividend. (Division with a 3-digit divisor follows the same pattern; however, any division on the Accuplacer, which is that complex will allow a calculator.)

828 ÷ 35

Divide: 35 into 82 (3 goes into 8 about twice so 35 will go into 82 about twice)

Multiply: 35 x 2 = 70

Subtract: 82 – 70

Bring Down: the 8

Divide: 35 into 128 (3 goes into 12 exactly 4 times, so it is best to try 3)

Multiply: 35 x 3 = 105

Subtract: 128 – 105

The 23 is the remainder

		2	3
3			
5	8	2	8
	-7	0	↓
	1	2	8

			r
			2
	2	3	3
3 5 | 8	2	8	
-7	0	↓	
1	2	↓	
-1	0	5	
	2	3	

e.

3. Operations with Fractions

Operations with fractions can include recognizing equivalent fractions and simplifying fractions, converting Improper fractions into mixed numbers (or the reverse), and the arithmetic operations of addition, subtraction, multiplication, and division.

a. Recognizing Equivalent Fractions

Equivalent fractions have the same value but are written with different denominators. To change a fraction into an equivalent in higher terms, multiply both the numerator (the top number of the fraction) and the denominator (the bottom number of the fraction) by the same number:

To change a fraction from a higher term to its lowest terms is called **simplifying**. To simplify a fraction, divide both the numerator and the denominator by the greatest number possible.

b. Converting Improper Fractions and Mixed Numbers

A **proper fraction** will have a smaller numerator than its denominator. An **improper fraction** will have a lager numerator than its denominator. A **mixed number** is a combination of a whole number and a fraction.

To convert an improper fraction into a mixed number, divide the numerator by the denominator. The quotient will be the whole number while the remainder will be the new numerator.

$$
\begin{array}{r|rr}
 & & 4 \\
\hline
3 & 1 & 3 \\
 & -1 & 2 \\
\hline
 & & 1
\end{array}
$$

$$
\begin{array}{r|rr}
 & & 1 \\
\hline
1 & & \\
2 & 1 & 9 \\
 & -1 & 2 \\
\hline
 & & 1
\end{array}
$$

c. Addition

1. Adding Fractions with the Same Denominator

To add fractions with the same denominator:

2. Adding Fractions with Different Denominators

To add (or subtract) fractions, they MUST have the same denominator. If they have different denominators, a **common denominator** must be found before they can be combined.

3. Adding Mixed Numbers, Whole Numbers, and Fractions

To add mixed numbers, whole numbers, and fractions, first check for like denominators (and find them if necessary). Next, add the fractions, then the whole numbers. Simplify if necessary.

d. Subtraction

1. Subtracting Fractions with the Same Denominator

When the denominators are the same, subtraction works the same as addition: combine the numerators and keep the denominator.

2. Subtracting Fractions with Different Denominators

Subtracting fractions with different denominators is also very similar to addition: Find the common denominator, then subtract the numerators, and finally, simplify if possible.

3. Subtracting Fractions with Regrouping

Sometimes when subtracting fractions, it is necessary to regroup the top fraction in order to subtract the bottom one. Remember, a common denominator is still required.

e. Multiplication

1. Basic multiplication

To multiply fractions, multiply the numerators then multiply the denominators. Simplify if possible.

2. Multiplication with Cancellation

Sometimes simplification can happen before you multiply in a process called cancellation. If the numerator of one fraction and the denominator of the other fraction have a common factor, you can divide both by that factor and simplify before you multiply. This often makes multiplication easier. (Note: cancellation can be done in one or both fractions, but ONLY if there is a common factor. Sometimes there will be no common factors.)

3. Multiplying Fractions, Whole Numbers, and Mixed Numbers

To multiply a fraction by a whole number or mixed number, the whole number or mixed number must be converted into an improper fraction first.

f. Division

To divide a fraction by a fraction, you must first find the reciprocal of the second fraction. To find the reciprocal, invert the fraction placing the numerator where the denominator is and the denominator where the numerator is. After finding the reciprocal, multiply the two fractions.

All multiplication rules apply once you have found the reciprocal and changed to multiplication.

4. Operations with Decimals

The key to working with decimals is to remember what the decimal point is doing in each type of operations. As a quick reminder, here is a place value diagram including the digits after the decimal point.

a. Addition

To add decimals, line up the decimal points. Write zeros in any place that has a gap. Be sure to write the decimal point in the sum (final answer).

32.2 + 5.71 =

Line up the decimal points

	3	2	.	2	
+		5	.	7	1

Write in zeros

	3	2	.	2	**0**
+		5	.	7	1

Add

	3	2	.	2	0
+		5	.	7	1
	3	7	.	9	1

8 + 2.08 + 0.5

Line up the decimal points

	8	.			
	2	.	0	8	
+	0	.	5		

Write in zeros

	8	.	**0**	**0**	
	2	.	0	8	
+	0	.	5	**0**	

Add

	8	.	0	0	
	2	.	0	8	
+	0	.	5	0	
1	0	.	5	8	

b. Subtraction

Subtracting decimals is similar to addition since you must also line up the decimal points, write in any zeros to fill the gaps, and then subtract as you would with whole numbers. All rules concerning regrouping all to decimals just as they do with whole numbers.

16.2 – 8.04

Line up the decimal points

	1	6	.	2
-		8	.	0

Write in zeros

	1	6	.	2
-		8	.	0

Subtract

				1
	1	6	.	2
-		8	.	0
		8	.	1

20.05 – 0.306

Line up the decimal points

	2	0	.	0	5	
-		0	.	3	0	6

Write in zeros

	2	0	.	0	5	**0**
-		0	.	3	0	6

Subtract

	1	9		1 0	4	1 0
	2	0	.	0	5	0
-		0	.	3	0	6
	1	9	.	7	4	4

c. Multiplication

To multiply decimals, multiply the way you would with whole numbers. Place the decimal point by counting the number of decimal places in each factor. The product will have the same number of decimal places.

16.2 x 11

	1	6	.	2
x			1	1
		1	6	2
+	1	6	2	0
	1	7	8.	2

	0	.	0	5	2
x			0	.	4
0	.	0	2	0	8

d. Division

To divide by a decimal, count the number of decimal places in the divisor, then move the decimal point in the dividend that number of places.

$$
\begin{array}{rr|l}
 & 7\ 5 & \\
\hline
0.7 & 5.\ 2\ 5 & \\
 & -4\ \ 9 & \\
\hline
 & 3\ 5 & \\
 & -3\ 5 & \\
\hline
 & 0 &
\end{array}
$$

$$
\begin{array}{rr|l}
 & 2\ 9.\ 9 & \\
\hline
0.16 & 4.\ 7\ 8\ 4 & \\
 & -3\ 2 & \\
\hline
 & 1\ 5\ 8 & \\
 & -1\ 4\ 4 & \\
\hline
 & 1\ 4\ 4 & \\
 & -1\ 4\ 4 & \\
\hline
 & 0 &
\end{array}
$$

5. Operations with Fractions, Decimals and Percents

Percent (%) means *per one hundred*, or *out of a hundred.* So, 37% means 37 out of 100. This can be written three ways:

As a percent:

37%

As a decimal:

0.37

Or as a fraction:

To convert a decimal to a percent, move the decimal point two places to the left:

0.18 = 18%

1.34 = 134%

0.06 = 6%

0.009 = 0.9%

To convert a percent to a decimal, move the decimal point two places to the right:

29% = 0.29

145% = 1.45

3% = 0.03

.25% = 0.0025

To convert a fraction to a percent, first change the fraction to a decimal by dividing the numerator into the denominator, then move the decimal point two places to the left.

To convert a percent to a fraction, place the percent into the numerator of the fraction and use 100 as the denominator. Simplify if possible.

6. Operations with Percents

a. Finding the Percent of a Number

Percents use the basic formula: **rate X whole = part**, where the rate is the percent (converted to a fraction or decimal for calculation), the whole is the base number, and the part is the fraction of the original base number.

What is 25% of 84?

Convert to an equation:

25% X 84 =

Change the percent to a decimal, then multiply:

0.25 x 84 = 21

What is 20% of 50?

Convert to an equation:

20% X 50

Change the percent to a fraction, then multiply:

b. Finding What Percent of a One Number is Another Number

Sometimes the missing piece is the percent. Use the formula: **rate = part ÷ whole**.

What percent of 75 is 60?

Convert to an equation:

?% = 60 ÷ 75

?% = 0.8

Convert the decimal to a percent:

0.8 = 80%

What percent of 140 is 168?

Convert to an equation:

?% = 168 ÷ 140

?% = 1.2

Convert the decimal to a percent:

1.2 = 120%

c. Finding the Whole When the Percent and Part are Known

When the rate and the part are known, use the formula: **whole = part ÷ rate**

12% of what number is 18?

Convert to an equation, convert percent to a decimal, and calculate

$$? = 18 \div 0.12$$

$$150 = 18 \div 0.12$$

17 is 25% of what number?

Convert to an equation, convert percent to a decimal, and calculate

$$? = 17 \div 0.25$$

$$68 = 17 \div 0.25$$

7. Applications and Problem Solving

The problem solving sections of the Accuplacer will involve problems with rate, percents, basic geometry, and fractional parts. Each of these will be examined separately.

The key to all of the application and problem solving questions will be to examine the question carefully using the following five steps:

1) Understand the Question
    a) Read the problem carefully
    b) Watch for tricky wording
    c) Pay careful attention to the final line of the problem. That is usually where they are asking you to do something. Make sure you understand what they want you to do and find.
2) Find the information
    a) Look for labels and units
    b) Find the numbers
        b.i) Sometimes the numbers are written as words
        b.ii) Sometimes they "hide" the numbers in words like *a dozen* (meaning 12) or *twice* (meaning two time)
    c) Be careful with extra information: they may provide numbers in the problem that will not be used to find your answer
    d) Sometimes there will not be enough information to answer the question
3) Make a plan
    a) Sort the problem out with words
    b) Create an equation or select a formula
4) Solve the equation
5) Check your answer
    a) Did you answer the question?
    b) Is your answer reasonable?

c)

a. Rate

Rate problems tend to involve using the distance formula: distance = rate X time.

Jon drove east for 1 hour at 50mph, and then increased his speed to 60mph for another 2 hours. How far has Jon traveled?

d = r X t

In this problem there are two rates and times that must be combined to find the final total.

d = (50 X 1) + (60 X 2)

d = 50 + 160

d = 210 miles

Emily left school traveling at 50 mph. Sometime later Beth left traveling at 60 mph. After two hours, Beth caught up with Emily. How long had Emily been traveling when Beth caught up?

Sometimes the time looks hidden, but can be found by comparing their travel:

d = r X t

Beth's d = 50 X t

Emily's d = 60 X 2

Since Emily caught up, that means they traveled the same distance so

Beth's d = Emily's d

50 X t = 60 X 2

50t = 120

t = 120 ÷ 50

t = 2.4 hours

b. Percents

Percent problems may involve discounts, mark ups, or simple interest.

If Elizabeth bought a television for $230 that was on sale for 20% off, how much did she pay for the television?

To find the discount, use the percent formula:

230 X 0.2 = 46

Then subtract the discount from the original price:

230 – 46 = $184

Juan bought a car for $6000. He paid 10% down, and then financed the rest for 4 years at 8% interest. How much will his monthly payments be?

This problem has several step. First find out how much his down payment was and subtract that from the principle amount.

6000 X .1 = 600

6000 – 600 = 5400

Next, use the interest formula interest = principle X rate X time *to find out how much he will pay in interest.*

I = 5400 X 0.08 X 4

I = 1728

Finally, add that to the principle, and divide by the number of months to find his monthly payment.

(5400 + 1728) ÷ (4 X 12)

7128 ÷ 48 = $148.50

c. Geometry

Geometry on the Accuplacer used very basic formulas for perimeter, area, and volume. The formula will not usually be given in the problem.

Perimeter of a:

Square	P = 4 X side
Rectangle	P = 2 X length + 2 X width
Triangle	P = $side_1$ + $side_2$ + $side_3$

Circumference of a circle C = π X diameter; π = approximately 3.14

Area of a:

Square	A = $side^2$
Rectangle	A = length X width
Parallelogram	A = base X height
Triangle	A = ½ X base X height
Trapezoid	A = ½ X ($base_1$ + $base_2$) X height
Circle	A = π X $radius^2$; π = approximately 3.14

Volume of a:

Cube	V = $edge^3$
Rectangular prism	V = length X width X height
Square pyramid	V = 1/3 X $(base\ edge)^2$ X height
Cylinder	V = π X $radius^2$ X height; π = approximately 3.14
Cone	V = 1/3 X $radius^2$ X height; π = approximately 3.14

d. Fractional Parts

These word problems look at situations where multiple people are doing a single task. You are asked to break the whole into parts for each participant or combine their parts to make a whole.

In one week, JoAnn, Sara, Adam and Ron volunteered 84 hours of their time to the animal shelter. JoAnn and Ron each volunteered one-fourth of the hours, Sara volunteered three-eighths of the hours, and Adam volunteered the remaining hours. How many hours did each person give?

JoAnn and Ron

Sara

Adam

Total hours – (JoAnn's + Ron's + Sara's)

84 – (21 + 21 + 31.5)

84 – 73.5= 10.5

JoAnn = 21 hours, Ron = 21 hours, Sara = 31.5 hours, and Adam = 10.5 hours.

Working along, Denise can paint a room in 2 hours. Eric can paint the same room in 4 hours. How hours would it take for them to paint the room together?

Hours to complete the job:

Denise = 2

Eric = 4

Total = t

Denise paints at a rate of of a room per hour.

Eric paints at a rate of of a room per hour.

Total =

If you combine their rates:

Invert

Divide which is 4 ÷ 3 = 1hours

To convert that to minutes, multiply the fraction by 60

So the pair can paint the room in 1 hour and 20 minutes.

8. Problem Sets

1 Adding with Whole Numbers
2 Subtraction with Whole Numbers
3 Multiplication with Whole Numbers
4 Division with Whole Numbers
5 Simplifying Fractions
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